Superfluid Pairing Gap in Strong Coupling

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The zero-temperature pairing gap is a fundamental property of interacting Fermions, providing a crucial test of many-body theories in strong coupling. We analyze recent cold-atom experiments on imbalanced Fermi systems using Quantum Monte Carlo results for the superfluid and normal phases. Through this analysis we extract, for the first time, the experimental zero-temperature pairing gap in the fully paired superfluid state at unitarity where the two-body scattering length is infinite. We find that the zero-temperature pairing gap is greater than 0.4 times the Fermi energy E_F, with a preferred value of $(0.45 \pm 0.05)\ E_F$. The ratio of the pairing gap to the Fermi Energy is larger here than in any other Fermi system measured to date.

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The pairing gap in strongly coupled Fermi systems is fundamental in diverse areas of physics including condensed matter physics, the physics of atomic nuclei, nuclear matter in neutron stars and the phase structure of dense QCD. In conventional superfluids or superconductors where weak-coupling methods are reliable, the pairing gap is very small, of order 0.1% the Fermi energy (E_F). In contrast, in cold atoms systems with large scattering length and other strongly paired fermion systems such as cuprate superconductors, dense nuclear and quark matter, the gap is a substantial fraction of the Fermi energy, and no obvious expansion parameter can be identified. A crucial test of many-body theories for systems in strong coupling is their ability to predict the pairing gap at $T = 0$, which will allow us to understand how the corrections to BCS theory evolve from weak coupling, where they are very strong, to the BEC limit where they are relatively modest.

At the BCS-BEC transition point, which is also called the unitarity regime, the short-range interaction is tuned to infinite scattering length. Here all measurable quantities including the gap are related to their free Fermi-gas counterparts by universal constants since the interaction does not present an energy scale. Our extracted value of the gap is 0.45 E_F implies that corrections beyond BCS theory are reduced at unitarity, $\Delta/\Delta_{\rm BCS} \approx 0.65$, compared to weak coupling where the Gorkov result is $\Delta/\Delta_{\rm BCS} = (1/4e)^{1/3} \approx 0.45$ [1]. A precise knowledge of the pairing gap in this strong coupling regime is crucial to understanding many physical processes which are controlled by the magnitude of the pairing gap, such as the cooling of neutron stars.

Experiments that trap and cool fermion atoms are now providing new insights [2–6]. Experiments to date have studied systems containing two hyperfine states of ^{6}Li, which we label $|\uparrow\rangle$ and $|\downarrow\rangle$ for convenience. These experiments can tune both the number asymmetry (polarization) and the interaction strength and thus have the potential to probe new phases of superfluid matter that are expected on theoretical grounds. Experiment done in the unitarity regime indicate that two phases are certain to exist involving three fundamental constants. The superfluid state at $T = 0$ and number density n can be characterized by the ground-state energy $E_{\rm SF} = \xi(3/5)E_F$ and the pairing gap $\Delta = \delta E_F$ with $E_F = (3\pi^2 n)^{2/3}/2$ m. The normal state can be characterized by the binding of a minority spin particle to the Fermi sea of majority particles, $E_N - E_0 = -\chi E_{f,N}$, where E_N is the energy of the normal state, E_0 is the energy of noninteracting particles, and $E_{f,N}$ is the Fermi energy of the majority spin population $E_{f,N} = (6\pi^2 n)^{2/3}/2$ m.

The universal constants have been calculated using quantum Monte Carlo techniques,[7–11] yielding: $\xi = 0.42(01)$, $\delta = 0.50(03)$, and $\chi = 0.60(.01)$. More recent calculations of ξ using both diffusion Monte Carlo and auxiliary field Monte Carlo [12] indicate that ξ may be slightly smaller, $\xi = 0.40(.01)$. While these calculations are approximate, calculations of the respective ground-state energies are upper bounds and hence provide an apparently accurate upper bound to the parameter ξ and lower bound to χ [13]. Measurements of ξ and χ appear to be consistent within errors with QMC results as shown below. In contrast the pairing gap parameter δ is obtained from the difference between even- and odd-particle number simulations, and hence is not a bound on the true value.

In this Letter we show that the recent measurement of the polarization density in the MIT experiment published in Ref. [6] can be used to extract the $T = 0$ pairing gap rather precisely. Previously the pairing gap has been extracted from studies of the RF response [14–16]. While RF spectroscopic studies are very intriguing the extracted gaps are smaller and inconsistent with the present value. Final-state interactions must be taken into account in any realistic study of the RF response [17].

QMC calculations of the superfluid and normal state imply that simple descriptions based upon a quasiparticle picture and these universal parameters are valid over a wide range of polarizations between zero and one. At

$T = 0$ the superfluid + quasiparticle picture appears to work reliably up to polarization $\sigma = (n_\uparrow - n_\downarrow)/(n_\uparrow + n_\downarrow) \leq 0.2$ [10], while in the normal state the simple quasiparticle picture appears to work reliably for polarizations $\sigma \geq 0.4$ [11]. The phase transition between these two states is first-order at low temperature. At $T = 0$ the transition occurs between an unpolarized superfluid state and a normal state at a polarization of approximately 0.4 [11,18]. At the transition $\delta\mu = \delta\mu_c \simeq \mu$, where $\delta\mu = (\mu_\uparrow - \mu_\downarrow)/2$ and $\mu = (\mu_\uparrow + \mu_\downarrow)/2$ [10,11]. Here the spin-up and spin-down particle chemical potentials are $\mu_\uparrow$ and $\mu_\downarrow$, respectively. The pairing gap is only slightly larger than the value required for the stability of a gapless superfluid with finite polarization at zero temperature. At finite temperature, superfluid quasiparticles will be thermally excited, polarizing the superfluid; this polarization is exploited to extract the pairing gap.

The first-order phase transition appears to have been observed in recent MIT [3] and Rice [4] experiments. Significant differences remain, however. In particular the Rice experiment finds a transition directly from zero to full polarization. Such a transition *cannot* happen in a bulk system if the binding of spin-down particles in the normal state ($\chi = 0.6$) is large [18]. Hence the observed polarization must be related to the unique geometry of the trap. The MIT experiment has been performed for larger numbers of particles and more spherical traps, and hence the simple local density approximation employed here is better suited for an analysis of these experiments.

The three fundamental constants at zero temperature uniquely predict the polarization as a function of density for small enough temperatures and superfluid polarization. The polarization in the two phases are

Superfluid state with a small polarization.—In the universal regime the energy density, pressure and the chemical potential of the unpolarized superfluid state are given by $\epsilon_{\rm SF} = \xi k_F^5/(10\pi^2 m)$, $P_{\rm SF} = \xi k_F^5/(15\pi^2 m)$, and $\mu = \xi k_F^2/2$ m, respectively. Here k_F is the Fermi momentum $k_F = (3\pi^2 n)^{1/3}$. In Ref. [10] we calculated the quasiparticle (qp) dispersion relation by introducing additional fermions to the unpolarized superfluid state. The dispersion curve for qp's measured relative to μ is given by

$$\omega_{\rm qp}(k) = \Delta\sqrt{(1 + a_2(x - x_0)^2 + \mathcal{O}\{(x - x_0)^4\})}, \qquad (1)$$

where $x = k^2/k_F^2$, $x_0 \simeq 0.83(3)$, and $a_2 \simeq 1.3(2)$. The higher order terms are negligible at small polarization.

The energy to add an additional spin-up fermion to the unpolarized system in the vicinity of the normal-superfluid transition is anomalously small in strong coupling because $\delta\mu_c \sim \Delta$ at the first-order transition [10]. Zero-temperature QMC calculations suggests that interactions between quasiparticles in the superfluid are weak at low polarization. Hence the number density and pressure of additional spin-up quasiparticles are given by the noninteracting Fermi-gas expressions with the dispersion relation in Eq. (1)

$$n_{\rm qp\uparrow} = \int \frac{d^3k}{(2\pi)^3}\left[1 + \exp\left(\frac{\omega_{\rm qp}(k) - \delta\mu}{T}\right)\right]^{-1}, \qquad (2)$$

$$P_{\rm qp\uparrow} = T\int \frac{d^3k}{(2\pi)^3}\log\left[1 + \exp\left(\frac{\omega_{\rm qp}(k) - \delta\mu}{T}\right)\right]. \qquad (3)$$

In contrast, $n_{\rm qp\downarrow} \simeq 0$, since the energy to add additional spin-down particles is $\omega_{\rm qp\downarrow} \simeq (\delta\mu_c + \Delta) \gg T$. Hence the polarization density $n_\uparrow - n_\downarrow \simeq n_{\rm qp\uparrow}$.

We anticipate that the gap will decrease due to both finite temperature and polarization. For the present calculations we assume that for low temperature the pairing gap decreases slowly according to $\Delta(T) = \Delta(0) \times \sqrt{1 - (T/t_c E_F)^2}$. The coefficient t_c characterizes the transition temperature. The zero-temperature pairing gap is a constant fraction δ of the Fermi energy: $\Delta(0) = \delta E_F$. The temperature dependence of the gap is determined by t_c and the polarization. Its calculation is beyond the scope of this work, a mean-field treatment of the polarized finite temperature superfluid phase is discussed in Ref. [19]. Here we take t_c in the range 0.05–0.25; even this huge range has a modest effect on the extracted pairing gap at zero temperature.

Normal state at high polarization.—Through QMC studies we have determined the energetics of the normal state at high polarization. Near unit polarization, the energy of a spin-down particle is given by

$$E_\downarrow(k) = \frac{k^2}{2\text{ m}} - \chi\frac{k_{F_\uparrow}^2}{2\text{ m}}, \qquad (4)$$

where $\chi = 0.6$ characterizes the strength of the energy shift due to interactions. Earlier work has shown that this provides a good description of QMC results for the energy at zero temperature [11,18]. At finite temperature we adopt an independent-particle model in which the interactions modify the single-particle levels and are able to fit the QMC results. A symmetric form for the dispersion relation that also fits the QMC results is given by

$$E_\uparrow(k) = \frac{k^2}{2\text{ m}} - \chi\frac{k_{F_\downarrow}^3}{4\text{ m}\tilde{k}_F}; \qquad E_\downarrow(k) = \frac{k^2}{2\text{ m}} - \chi\frac{k_{F_\uparrow}^3}{4\text{ m}\tilde{k}_F}, \qquad (5)$$

where $\tilde{k}_F = (k_{F_\uparrow}^6 + k_{F_\downarrow}^6)^{1/6}$. We note that Eq. (4) and (5) predict the same energy density in the limit $k_{F_\downarrow} \to 0$.

We assume a harmonic trap and work in the local density or Thomas-Fermi approximation. The physical state at any location in the trap is completely determined by the local chemical potentials $\mu_\uparrow = \mu + \delta\mu$ and $\mu_\downarrow = \mu - \delta\mu$. The chemical potential $\mu = \lambda - V_{\rm Trap}(r)$, where $V_{\rm Trap}(r) = 0.5\hbar\omega(r/r_0)^2$ and $r_0 = \hbar/\sqrt{\hbar\omega m}$ while $\delta\mu$ is constant since the trap does not distinguish between the different fermion species. Using the notation of reference [6] distances are scaled relative to the radius $R_\uparrow$ at which the zero-temperature spin-up density vanishes. The radius $R_\downarrow$ is similarly defined as the radius at which the zero-temperature spin-down density vanishes. The temperature T' is scaled to the Fermi energy of a noninteracting Fermi gas at $r = 0$ with the same density profile as the exterior cloud. Similarly the densities are measured in units of n_0, where n_0 is the density of a noninteracting Fermi gas at $r = 0$ with the same density profile as the exterior cloud. At low temperature the density profiles predicted by the theory discussed above are shown in Fig. 1. The inner superfluid region and the outer normal region separated by a first-order transition are clearly seen. The transition is characterized by σ_s and σ_n—the polarization's in the superfluid and normal phase, respectively. As mentioned earlier, at low temperature $\sigma_N \simeq 0.4$, while σ_s increases rapidly from zero at zero-temperature with increasing temperature due to the low excitation energy of spin-up quasiparticles in the vicinity of the transition. In our implementation of the equation of state for the normal phase, thermal effects on the density profiles are negligible except close to $r = R_\downarrow$ and $r = R_\uparrow$.

The density distribution of the spin-up particles (majority species) between $R_\downarrow$ and $R_\uparrow$ provides a measure of the temperature and the chemical potential of the spin-up particles. Similarly, the density distribution of the unpolarized superfluid state at the origin provides an independent measure of the average chemical potential μ assuming the calculated value of ξ. Since $\chi = 0.6$ describes the normal state near unit polarization, the radius $R_\downarrow$ can independently provide a measure of the spin-down chemical potential. Thus, the two radii, the density at large radii, and the central density measure the two chemical potentials and the temperature and provide a consistency check between the calculated values of ξ and χ in extracting μ and $\delta\mu$.

In order to compare with experimental results, we assume a specific ratio of the density at the origin to the maximum density of a cloud of free fermions with the same density distribution for $r > R_\uparrow$, which is equivalent to assuming a specific average chemical potential μ. The values of the ratio $R_\downarrow/R_\uparrow$, the total polarization $P_{\rm tot}$ in the trap, the transition radius R_c, and the polarization as a function of radius are then predicted for various values of the pairing gap and the temperature. We find that present measurements provide strong constraints on the pairing gap even though the temperature is not very precisely determined in the lowest temperature measurements.

Figure 2 compares the calculated spin-up and spin-down densities as a function of radius. We used a normalized temperature $T' = 0.03$ and a normalized density at the origin $n_\uparrow(r=0)/n_0 = n_\downarrow(r=0)/n_0 = 1.72$ consistent with the experiment. This simple model reproduces the radius of the transition, the radius where the spin-down density goes to zero, and the overall polarization in the trap. The calculated overall polarization in the trap is 0.44 for these values of the parameters, and the measured value is 0.44(0.04) [6].

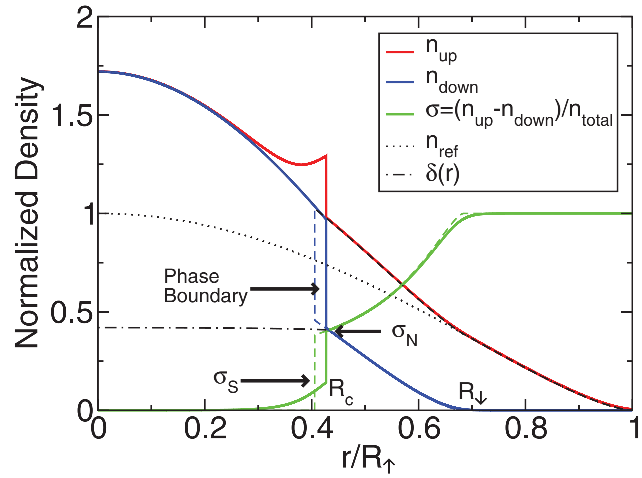

FIG. 1 (color). Spin-up and spin-down densities and polarization versus radius predicted by theory for $\delta = 0.43$ at $T' = 0$ (dashed) and $T' = 0.03$ (solid) are shown. The universal parameters ξ and χ are taken as 0.4 and 0.6, respectively. For $T' = 0.03$ the local value of δ shown by the dot-dashed curve was obtained using $t_c = 0.1$.

In Fig. 3 we compare the measured and calculated polarization at $T' = 0.03$ and $T' = 0.05$. Results for different values of δ and t_c are shown. At $T' = 0.03$ the polarization is approximately 0.12 at the interface at $r/R_\uparrow = 0.43$, consistent with the experimental results. The qualitative and quantitative features of the measured polarization at $T' = 0.03$ are captured by the normal phase at $r/R_\uparrow \gtrsim 0.45$ and a thermally polarized superfluid phase for $r/R_\uparrow \lesssim 0.4$—consistent with a first-order transition somewhere in between. In contrast, the comparison be-

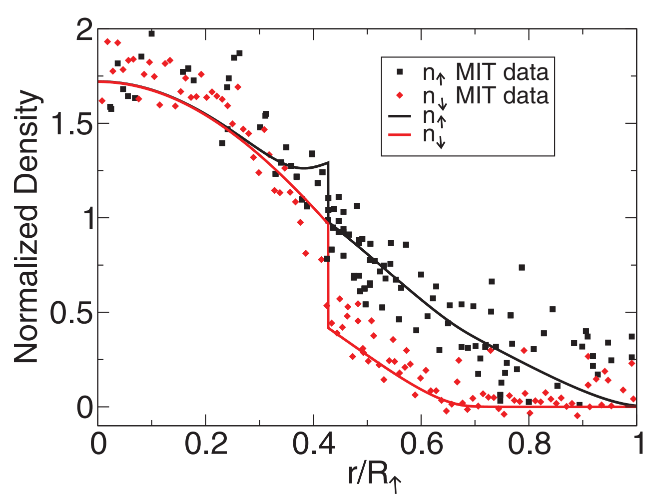

FIG. 2 (color). Spin-up and spin-down densities theory and experiment for $\delta = 0.43$, $\xi = 0.4$ and $\chi = 0.6$ at $T' = 0.03$.

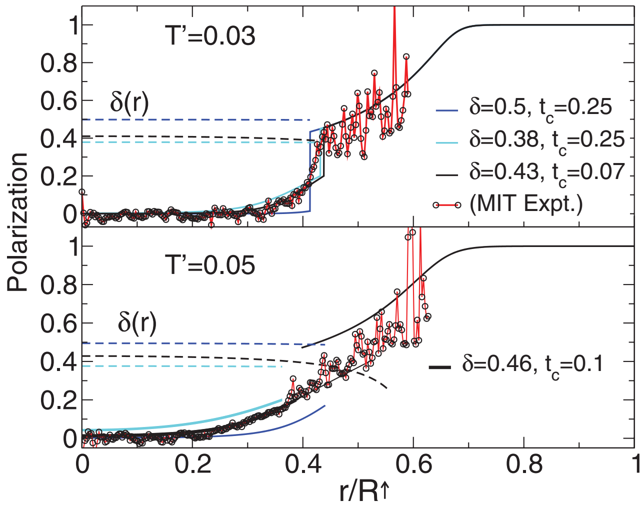

FIG. 3 (color). Polarization versus radius, theory and experiment, for different values of δ and t_c at $T' = 0.03$ and $T' = 0.05$. The dashed curves show the local finite temperature gap. The results indicate that the data provide both an upper and a lower bound on the gap: $0.5 \geq \delta \geq 0.4$.

tween theory and data at $T' = 0.05$ suggests that the superfluid extends further out. Polarization in the superfluid state (thin solid black-line) extrapolated to $p \simeq 0.4$ provides a better description of the data than the normal state. A clear signature of a first-order transition is also absent. In both cases there appears to be evidence for a mild decrease in the gap with increasing T/E_F and polarization.

For a fixed central density and $R_\uparrow$, our analysis predicts that the phase-boundary R_c moves outward in the trap with increasing temperature. This behavior is sensitive to the thermal properties of both phases at low temperature. At small temperature and polarization, the thermal response of the superfluid phase in the vicinity of the transition is stronger than that of the normal phase—driven entirely by the fact that spin-up quasiparticles are easy to excite and have a large density of states.

The comparison in Fig. 3 provides compelling lower and upper bounds for the superfluid gap. Even if the temperature was extracted incorrectly from the experiment, the extracted gap cannot be too small. A gap smaller than $\approx 0.4E_F$ would produce a shell of polarized superfluid before the transition even at zero temperature. Furthermore, the radial dependence of this polarization would be quite different than observed experimentally, rising abruptly from the point, where $\Delta = \delta\mu$ and being concave rather than convex. A gap larger than $\approx 0.5E_F$ would be unable to produce the observed polarization in the superfluid phase. We have also examined the dependence of our results on the universal parameters ξ and χ. Both of these are expected to be uncertain by 0.02. These uncertainties, as well as the uncertainties in the superfluid quasiparticle dispersion relation do not significantly alter the extracted bounds on the superfluid gap.

In summary, we have extracted the pairing gap from measurements of spin-up and spin-down densities in polarized Fermi gases in the unitary regime. These systems have an extremely large gap of almost one-half the Fermi energy—the value extracted in this work is clearly the largest gap measured in any Fermi system. Future more precise experiments extending over the BCS-BEC transition region would allow an experimental determination of the evolution of the pairing gap from the weak-coupling regime of traditional superfluids and superconductors to the strongly interacting regime. This could resolve long-standing issues regarding, for example, the pairing gap in neutron matter and the cooling of neutron stars.

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[1] L. P. Gorkov and T. K. Melik-Barkhudarov, Sov. Phys. JETP **13**, 1018 (1961).
[2] G. B. Partridge *et al.*, Phys. Rev. Lett. **97**, 190407 (2006).
[3] Y. Shin, M. W. Zwierlein, C. H. Schunck, A. Schirotzek, and W. Ketterle, Phys. Rev. Lett. **97**, 030401 (2006).
[4] G. B. Partridge, W. Li, R. I. Kamar, Y. an Liao, and R. G. Hulet, Science **311**, 503 (2006).
[5] M. W. Zwierlein, A. Schirotzek, C. H. Schunck, and W. Ketterle, Science **311**, 492 (2006).
[6] Y. Shin, C. H. Schunck, A. Schirotzek, and W. Ketterle, Phys. Rev. Lett. **99**, 090403 (2007).
[7] J. Carlson, S.-Y. Chang, V. R. Pandharipande, and K. E. Schmidt, Phys. Rev. Lett. **91**, 050401 (2003).
[8] G. E. Astrakharchik, J. Boronat, J. Casulleras, and S. Giorgini, Phys. Rev. Lett. **93**, 200404 (2004).
[9] S. Y. Chang, V. R. Pandharipande, J. Carlson, and K. E. Schmidt, Phys. Rev. A **70**, 043602 (2004).
[10] J. Carlson and S. Reddy, Phys. Rev. Lett. **95**, 060401 (2005).
[11] C. Lobo, A. Recati, S. Giorgini, and S. Stringari, Phys. Rev. Lett. **97**, 200403 (2006).
[12] S. Zhang, K. E. Schmidt, and J. Carlson (private communication).
[13] N. Prokof'ev and B. Svistunov, arXiv:0707.4259v2.
[14] S. Gupta *et al.*, Science **300**, 1723 (2003).
[15] C. Chin *et al.*, Science **305**, 1128 (2004).
[16] Y. Shin, C. H. Schunck, A. Schirotzek, and W. Ketterle, Phys. Rev. Lett. **99**, 090403 (2007).
[17] G. Baym, C. J. Pethick, Z. Yu, and M. W. Zwierlein, Phys. Rev. Lett. **99**, 190407 (2007).
[18] A. Bulgac and M. M. Forbes, Phys. Rev. A **75**, 031605 (2007).
[19] K. B. Gubbels, M. W. J. Romans, and H. T. C. Stoof, Phys. Rev. Lett. **97**, 210402 (2006).