Stochastic Contributions to Global Temperature Changes

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Records of the mean annual global surface temperatures from 1850 to 1970 show annual temperatures that are correlated with temperatures of the previous years as a one-dimensional random walk with a limiting feedback. This description accounts for the variation in those temperatures observed until the present by assuming that the base temperature is proportional to the increase in carbon dioxide concentration over the level in 1890. Climate models that better fit the observed variations are shown to be statistically improbable and thus likely to be artifacts.

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Stochastic properties of global temperature changes.— Annual mean global surface temperatures from 1850 to the present have been compiled by the Climatic Research Unit at the University of East Anglia [1,2] from global temperature measurements. Similar average temperatures have been derived by the Goddard Institute of Space Studies [3] from 1880 to the present. While the two compilations differ somewhat, they are not independent.

I consider the statistical properties of the East Anglia temperatures beginning in 1850, and the Goddard temperatures beginning in 1880, to termination dates of 1940, 1970, and 2006. While the properties that were determined were nearly independent of the termination date, the values shown in Table I show results using the 1970 terminal date which avoids complications from the significant global warming from 1970 to the present.

The root-mean-square values, $R(j)$, of the differences between mean annual temperature measurements, T_k, for times 1 yr apart, two years apart, and three years apart, $j = 1$, 2, and 3, were calculated for both the East Anglia and Goddard studies.

$$R(j) = \sqrt{\sum_{k=k_0+j}^{k=1970} \frac{(T_k - T_{k-j})^2}{1970 - k_0}} \qquad (1)$$

For the East Anglia compilation, the beginning year, $k_0 = 1850$, for the Goddard compilation, $k_0 = 1880$. The values for the three sets of measurements are shown in Table I together with the values of the squares of the ratios, $[R(2)/R(1)]^2$ and $[R(3)/R(1)]^2$. The six sets of differences, $T_k - T_{k-j}$, for $j = 1$, 2, and 3, for the East Anglia and Goddard compilations, were all excellent fits to normal distributions as evaluated using the Jarque-Bera criteria [4].

If the fluctuations in annual temperatures follow largely from stochastic effects that are independent of the previous years' temperatures such as random measurement errors, $[R(3)/R(1)]^2 \approx [R(2)/R(1)]^2 \approx 1$. However, if the temperatures could be described completely in terms of year-to-year differences which varied randomly with differences distributed normally, $[R(3)/R(1)]^2 \approx 3$ and $[R(2)/R(1)]^2 \approx 2$. In this case, the temperature would be expected to vary stochastically over long periods of time rather as a one-dimensional random walk with the most probable temperature excursion increasing with the square root of the time and effectively independent of any base temperature. However, if there is also a negative feedback based on temperatures set by geological factors, these ratios will be smaller, though, as illustrated by the Monte Carlo calculations, still greater than one. Thus, the measured values are interpreted as indicating that global mean temperatures vary stochastically as a random walk but with a feedback constraint that keeps the excursions from a geologically determined base temperature limited.

The origin of such a stochastic behavior of the annual global temperatures might be found in the stochastic character of very many elements of global circulation that contribute to that temperature variation [5].

An expression describing the temperature variations.— Though the salient properties of the mean annual global temperatures are contained in the correlations listed in Table I, those properties can be illuminated by considering the random-walk recipe that reproduces those correlations. That procedure can be used to generate Monte Carlo simulations which can serve to produce estimates of the fundamental stochastic uncertainties in annual temperatures. Thus, the average temperature deviation over the year n, T_n, relates to the temperature deviation the year before, T_{n-1}, as,

$$T_n = (T_{n-1} + Q\sigma)(1 - 1/Y) + (\tau_n - \tau_{n-1}) + Q'\sigma' \quad (2)$$

where T_n and T_{n-1} are measured from a base temperature, τ_n.

The first term describes the correlated relation of T_n to the previous year's temperature, Here, Q is the rms value of the random temperature change over a year without feedback effects, σ is a random number with a Gaussian distribution, a mean of zero, and a standard deviation of one, and Y is a feedback parameter in units of years.

TABLE I. The correlation parameters from the East Anglia and Goddard compilations of annual global temperatures together with Monte Carlo simulations and the standard deviations of values from that calculation which are essentially the same as the uncertainties in the measured indices.

Measurement	R(1)	R(2)	R(3)	$[R(2)/R(1)]^2$	$[R(3)/R(1)]^2$
Goddard	0.0987	0.1212	01301	1.506	1.737
East Anglia	0.1177	0.1454	0.1533	1.527	1.695
Monte Carlo	0.1080	0.1346	0.1479	1.556	1.879
Stand. Dev.	0.0078	0.011	0.013	0.178	0.301

The second term expresses possible changes in the base temperature, τ_n, generated by geological factors such as changes in greenhouse forcings. I take the average of $|\tau_n - \tau_{n-1}| \ll 0.1\ ^\circ\mathrm{C/year}$ over the period from $n = 1850$ to $n = 1910$ to be sufficiently small that no significant error follows from the choice of $(\tau_n - \tau_{n-1}) = 0$ as the base temperature change over that span. The coolings from the volcanic eruptions of Krakatoa in 1883 and Pinatubo in 1991 seem discernible but do not much affect averages over a long period of time.

The third term describes that noise—from both climate variations and measurement errors—which is not correlated with previous years' temperatures. Here, σ' is also a random number with a Gaussian distribution, a mean of zero, and a standard deviation of one, and Q' is the rms value of the uncorrelated noise. Since the uncorrelated noise adds in quadrature to the correlated noise, my choice of $Q' = 0$ in the Monte Carlo calculations—a good approximation if $Q' < 0.05\ ^\circ\mathrm{C/year}$ $(\approx 0.5Q(1 - 1/Y))$—will not likely lead to a significant error.

There are many possible sources of feedback effects. Short term feedbacks probably do not play an important role in annual global temperature differences, and the feedback parameter Y in Eq. (2) primarily reflects effects of long-term feedbacks. There may be several such feedbacks. For the small annual temperature deviations that are observed, the different long-term feedbacks can be expected to be proportional to the temperature deviation and such a set of feedbacks can be represented by one value.

The first term of Eq. (2), with $Y = 2.75$ years, $Q = 0.155\ ^\circ\mathrm{C}$ was used as the basis for Monte Carlo calculations of the temperature variations over 105 years (the mean of the East Anglia and Goddard spans to 1970) that simulated those distributions. The average values of the temperature differences $R(1)$, $R(2)$, and $R(3)$ together with the ratios $[R(3)/R(1)]^2$ and $[R(2)/R(1)]^2$ from 1000 iterations are shown in Table I together with the standard deviations of the distributions. The East Anglia and Goddard compilation values are in accord with each other, and the values from the Monte Carlo calculations when the standard deviations, calculated from the Monte Carlo calculations, are considered.

While the values of Y and Q are significant factors in connecting the measured values of $R(1)$, $R(2)$, and $R(3)$, to general principles, for the most part the applications of the measurements to general climate behaviors follow from the values of the ratios themselves and *not* from the any specific analytic description, such as Eq. (2), which must involve uncertainties in interpretation.

Historic temperatures.—The Monte Carlo procedure was used to calculate year-by-year temperature variations over a one-thousand year span and to construct a running 40-year average of the annual temperatures.

The average high-low temperature difference of the 40-year-mean temperatures was found to be $0.204 \pm 0.0285\ ^\circ\mathrm{C}$. If the Goddard compilations from meteorological stations [3] are considered, which emphasize continental temperatures which vary more than global temperatures moderated by the oceans, those number are increased by about 30%. These projected variations are to be compared with the high-low differences over the 1000 years from 900 to 1900 listed in the report Surface Temperature Reconstructions [6]. Variations of about 0.25 °C are found by Mann and Jones [7]; Moberg *et al.* [8] find differences of about 0.6 °C; and Esper *et al.* [9] find variations of about 1 °C.

This comparison suggests that year-to-year stochastic global temperature variations may account for a significant part of the historic temperature variations over the millennium ending in 1900. However, those random variations cannot account for the large temperature changes associated with a medieval warm period and little ice age.

Recent temperature increases.—The annual temperatures listed in the Goddard [3] and East Anglia [1] compilations are "anomalies" differing from arbitrarily chosen base temperatures. With an addition of 0.377 °C for the East Anglia values and 0.268 °C for the Goddard values, the two temperature sets have the same mean for the range 1880 to 2006 and an approximately null mean value for 1880 to 1920. The adjusted anomalies are presented as year-by-year values of 5-year means in Fig. 1. The maximum high-low differences from the East Anglia 5-year set from 1850 to 2002 is 1.03 °C, and that maximum, from the Goddard compilation from 1880 to 2002 is 0.89 °C. For both 5-year sets, the lowest annual temperatures occurred in 1908 with the higher temperatures in 2002.

Monte Carlo procedures using Eq. (2) with the assumption of no change in the base temperature, τ_n, show that the average difference between the maximum and minimum

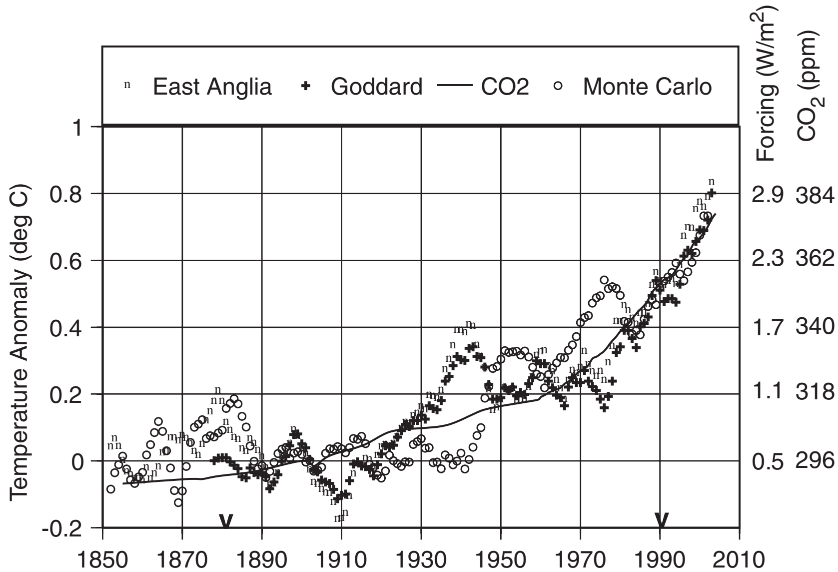

FIG. 1. The solid line shows the variation of the CO_2 concentration since 1850 and base global temperatures that correspond to those concentrations. The solid squares and crosses show the yearly global temperature anomalies from the East Anglia and the Goddard compilations adjusted to have the same overall mean and average values near zero for the preindustrial period. The values plotted are running 5 yr means. The forcing values are taken to be proportional to the added concentration of CO_2 as described in the text. The open circles show the results of a Monte Carlo simulation of temperature anomalies taken from the forcing curve with added random year-by-year fluctuations calculated using Eq. (2) with the annual deviation taken as $Q = 0.155\,^\circ\mathrm{C}$ and the feedback parameter, $Y = -2.75$ years. The eruptions of Krakatoa in 1883 and of Pinatubo on 1991 are marked by **V** in the lower border of the graph.

5-year temperature anomaly means, taken over 150 years, is 0.435 °C with a standard deviation of 0.055 °C. Aside from the improbable coincidence of the temperature increase and the increase in world industrial activity and consequent generation of greenhouse gases, the observed temperature variation is much larger than that which might be expected from noise alone during 155 yr or 125 yr compilation periods. Hence, accepting the description of temperature changes expressed in Eq. (2), the calculations support the commonly held view that the base temperature, τ_n, must have increased significantly over the last century.

According to conventional views, the recent temperature increases were driven by increases in anthropogenic forcings which are, in turn, presumed to be largely proportional to the increase in the atmospheric concentration of the greenhouse gases, carbon dioxide, methane, and nitrous oxide [10,11]. I take that forcing change rate as proportional to the increase in the concentration of CO_2 and consider a climate model such that increases in mean global temperatures are proportional to increases in atmospheric CO_2.

Figure 1 shows annual concentrations of CO_2 from 1850 to 2005. The values prior to 1953 were taken from the Siple ice cores [12] with linear interpretations between ice core dates. After 1953, the values were taken from measurements at Mauna Loa [13]. The increase in CO_2 concentration, C, from 1890 to 2005 was fitted by a least squares evaluation to annual temperatures, T, taken as the mean of the values from the East Anglia and Goddard compilations. From this fit, $dT/dC = 0.00920 \times (1 \pm 0.045)\,^\circ\mathrm{C/ppm}$, where the error is generated almost solely from the stochastic noise in the temperature measurements.

The deviations of the annual measured temperatures from the base temperatures calculated from the CO_2 concentrations are as expected from the stochastic noise in the global temperatures. The expected rms difference between the values of the base temperature derived from the CO_2 concentrations for the 126 years from 1880 to 2005 and the annual mean of the measured values from the two compilations was determined by Monte Carlo calculations to be $0.128 \pm 0.013\,^\circ\mathrm{C}$. The measured rms difference was found to be $\sigma = 0.114\,^\circ\mathrm{C}$. Hence, the fit of the temperature rise to the CO_2 concentration is as good as can be expected. A closer fit, that is a model that generates temperature such that the rms difference between the observed and predicted values is much less than σ, must be suspect as an artifact.

In the fits of the temperature to the incremental CO_2 concentration, I gave no special attention to the sharp increase in global temperature from 1910 to 1942, the small reduction from 1940 to 1975, and the sharp increase from 1975 to 2005. Such deviations from the simple increase proportional to the CO_2 concentration can be expected from the stochastic noise described in Eq. (2). Indeed, the difference between the largest positive deviation from the value predicated by the forcing in 1942 and the largest negative deviation in 1975 is 0.418 °C, which is in accord with the value of $0.415 \pm 0.070\,^\circ\mathrm{C}$ taken from the Monte Carlo calculations for a span of 116 years from

1890 to 2006. Thus, the stochastic noise description of global temperature variations *requires* fluctuations such as those which are observed.

To provide some more subjective insight into stochastic noise effects, I show by the open circles in Fig. 1 the results of a Monte Carlo exposition of that temperature increases defined by the simple quadratic relation of the average of the two recorded temperature sets plus noise. This plot follows from the first calculation made of that specific noise and forcing; the values were not selected from a set of calculations. The large fluctuations that are observed are clearly of the magnitude and kind to account for the well-known structures in the temperature vs time values. (By coincidence, an inversion of the random deviations from this Monte Carlo simulation generate maxima about 1940 and minima about 1975 very much like the measured values).

Physically, I assume that the temperature rise follows from increases in forcing estimated [10,11] as 2.15 $\mathrm{W/m^2}$ in 2000 from a value of about 0.5 $\mathrm{W/m^2}$ in 1890. This immediately leads to a temperature-to-forcing ratio $r = 0.335 \pm 0.0085\,^{\circ}\mathrm{C}/(\mathrm{W/m^2})$ where the error reflects only the uncertainty in the magnitude of the forcing. This response is in accord with the value of $0.3\,^{\circ}\mathrm{C}/(\mathrm{W/m^2})$ stated by an NRC committee [6] for the theoretically expected increase rate without feedbacks.

Future temperatures.—From that ratio of temperature rise to forcing, and estimates of future increases in forcing, we can estimate future temperatures. Here, I take the IPCC A2 scenario [14] of future forcing levels to estimate future temperatures and warming. (That A2 scenario seems to be conservative in presuming limited societal constraints on industrial activities.) From the 2001 IPCC Synthesis Report(14) Fig. 3-1-j, the forcing is presumed to increase by 6.4 $\mathrm{W/cm^2}$ from 2000 to 2100. From the value of the temperature-to-forcing ratio estimate given above, that increase in forcing will lead to a temperature increase of 2.15 °C which is less than the predictions of the general circulation models shown in Fig. 3-1-k, which give values from 2.5 °C to 4.5 °C.

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