Intrinsic Gap of the $\nu = 5/2$ Fractional Quantum Hall State

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The fractional quantum Hall effect is observed at low magnetic field where the cyclotron energy is smaller than the Coulomb interaction energy. The $\nu = \frac{5}{2}$ excitation gap at 2.63 T is measured to be 262 ± 15 mK, similar to values obtained in samples with twice the electronic density. Examining the role of disorder on the $\frac{5}{2}$ state, we find that a large discrepancy remains between theory and experiment for the intrinsic gap extrapolated from the infinite mobility limit. The observation of a $\frac{5}{2}$ state in the low-field regime suggests that inclusion of nonperturbative Landau level mixing may be necessary to fully understand the energetics of half-filled fractional quantum Hall liquids.

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Since the discovery of the fractional quantum Hall effect (FQHE), understanding the role played by electron-electron interactions has been the source of major breakthroughs in our understanding of strongly interacting two-dimensional electron gases (2DEGs). Chief among these is the composite fermion picture of the incompressible FQH liquid [1,2], extremely successful in explaining both the complete series of observed FQH states in the first Landau level (FLL) and the absence of such a liquid at precisely half-filling. In the second Landau level (SLL), however, the situation is more complex where experiments have shown, unambiguously, exact quantization of the Hall resistance at filling factor $\nu = \frac{5}{2}$ [3,4] and $\nu = \frac{7}{2}$. In 1991, Moore and Read [5] proposed an elegant many-body wave function to explain this phenomenon that described the $\frac{5}{2}$ FQH state as a "condensation process" of composite fermions. In recent years, this Moore-Read "Pfaffian" state has received considerable interest owing to built-in quantum statistics that are now predicted to be *non-Abelian*. The non-Abelian composite particles that comprise the $\nu = \frac{5}{2}$ FQH state underlie a paradigm for fault-tolerant topological quantum computation first proposed by Kitaev [6] and recently exploited by Das Sarma, Freedman, and Nayak [7]. Yet, in spite of these many recent theoretical advances, an unequivocal experimental verification of the Moore-Read description is still missing. Furthermore, continued discrepancies between experiment and theory, such as the large difference between the measured and calculated activation energy gap, remain problematic.

In an effort to better understand electron-electron interaction at half-filling, we present in this work a detailed analysis of the $\nu = \frac{5}{2}$ state for a sample with, to our knowledge, the lowest electron density reported to date (by nearly a factor of 2). This allows the study of the FQHE in a regime where the cyclotron energy is smaller than the Coulomb interaction energy. We compare the measured energy gap with neighboring FQH states in the SLL and discuss these results in the context of previous studies allowing us to deduce the intrinsic gap in the zero-disorder limit. Our analysis shows that large discrepancies remain between theory based on a Moore-Read Pfaffian state and experiment at $\nu = \frac{5}{2}$ that cannot be attributed to disorder alone. In contrast, a similar analysis for the $\nu = \frac{1}{3}$ Laughlin state shows much better agreement with current models.

The sample used in this study was a 40 nm wide, modulation-doped, GaAs/AlGaAs quantum well, with a measured density of $1.6(1) \times 10^{11}$ cm^{-2} and mobility of $14(2) \times 10^6$ cm^2/V · s. The sample was cooled in a Janis JDR-100 dilution refrigerator enclosed inside a shielded room, with a base temperature of ~16 mK in continuous mode, and equipped with a 9 T magnet. Treatment with a red light-emitting diode (LED) was used during the cooldown. *In situ* powder filters and RC filters were used on the sample leads to ensure efficient cooling of the 2DEG. Temperatures were monitored with a RuO resistive thermometer and a magnetization thermometer, both calibrated with superconducting fixed points. Transport measurements were performed using a standard lock-in technique at ~6.5 Hz and small excitation current, $I_{\mathrm{exc}} = 2$–10 nA.

Figure 1 shows the magnetoresistance (R_{xx}) and corresponding Hall resistance (R_{xy}), taken around $\nu = \frac{5}{2}$ in the SLL at ~20 mK. A vanishingly small magnetoresistance is observed at $\nu = \frac{5}{2}$ together with a wide plateau in the corresponding Hall trace quantized to within 0.05% of $R_{xy} = 2h/5e^2$. The unambiguous $\frac{5}{2}$ state observed here, occurring at ~2.63 T, represents to our knowledge the lowest magnetic field observation of the $\frac{5}{2}$ to date [3,4,8–20]. Strong FQHE minima are also observed at $\nu = \frac{14}{5}, \frac{8}{3}, \frac{7}{3}, \frac{16}{7}$, and $\frac{11}{5}$, each of which exhibit plateaus in R_{xy}. The four reentrant phases observed in the Hall trace on either side of the $\frac{5}{2}$ plateau (two peaks tending towards $R_{xy} = h/3e^2$ and two tending towards $R_{xy} = h/2e^2$) together with the observation of the $\nu = \frac{16}{7}$ state, and the hint of

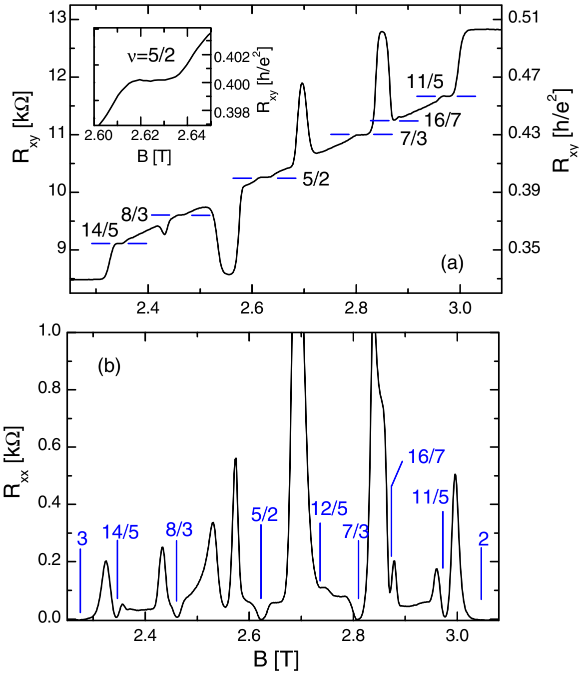

FIG. 1 (color online). (a) Hall resistance and (b) corresponding magnetoresistance in the second Landau level of our low-density, high mobility 2DEG ($T \sim 20$ mK).

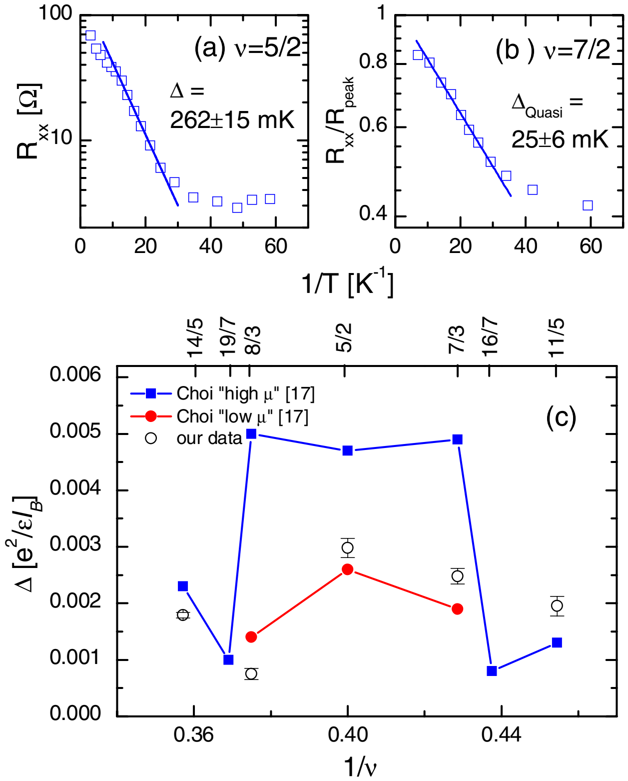

FIG. 2 (color online). (a) Arrhenius plot showing activated temperature behavior at $\nu = \frac{5}{2}$. (b) Quasigap measurement for $\nu = \frac{7}{2}$ (see text). (c) Energy gaps for all FQH states observed in the second Landau level, plotted in Coulomb energy units. Open circles are our data. Solid squares and circles, respectively, refer to the "high mobility" and "low mobility" samples in Ref. [17].

an emerging minimum at $\nu = \frac{12}{5}$, are all signatures of an extremely high quality sample [4,14,15,17]. The deep R_{xx} minima appearing in the reentrant insulating phase at ~2.55 T [Fig. 1(b)] is similar to that observed elsewhere at low temperatures [15,19,20].

The temperature dependence of the FQHE minimum at $\nu = \frac{5}{2}$ is shown in Fig. 2(a), with all data acquired at fixed magnetic field. The corresponding energy gap was determined by a linear fit to the thermally activated transport region, where the resistance is given by the equation $R_{xx} \propto e^{-\Delta/2k_BT}$. The gap error quoted was estimated from the goodness of the linear fit. Examination of weakly formed FQHE states under single shot of the dilution refrigerator down to ~9 mK indicated the electrons continued to cool, suggesting that the low temperature tail-off observed in the data of Fig. 2 does not reflect a saturation in the electronic temperature. Instead, it may indicate a transition from activated conduction to hopping conduction [21], and/or could result from the energy dependent Landau level broadening due to disorder [22]. We also observed a FQH state at $\nu = \frac{7}{2}$, the electron-hole conjugate of $\nu = \frac{5}{2}$, appearing at a magnetic field less than 2 T. However, due to a competition between the weakly formed $\frac{7}{2}$ and rapidly emergent neighboring reentrant states, the R_{xx} minimum did not fall significantly with temperature near base. The "quasigap" was therefore determined by measuring the depth of the $\frac{7}{2}$ minima with respect to the average resistance of the two neighboring peaks (R_{xx}/R_{peak})[8,9,13]. The resulting Arrhenius plot, which clearly shows activated behavior [Fig. 2(b)] gives an estimate for the $\frac{7}{2}$ gap value of ~25 mK.

In Fig. 2(c), the gap values measured in the second Landau level are plotted in Coulomb energy units, $e^2/\epsilon l_B$, where $l_B = \sqrt{\hbar/eB}$ is the magnetic length, and $\epsilon = 12.9$ is the GaAs dielectric constant. Results from recent gap measurements in the SLL by Choi *et al.* are also shown for comparison [17]. The excellent agreement between our data set and that of the Choi *et al.* "low mobility" sample ($\mu = 10.5 \times 10^6$ cm^2/V · s) is surprising given the factor of two difference in electron densities between our sample (1.6×10^{11} cm^{-2}) and theirs (2.8×10^{11} cm^{-2} and 3.2×10^{11} cm^{-2} for the "low mobility" and "high mobility," respectively). Simple dimensional considerations imply that the interaction energy, and hence the FQH gap, should scale as $\sqrt{B}$, which would predict an ~40% enhancement in the gap between the low density (ours) and the high density (Choi *et al.*) samples. Our finding that the gap is almost the same for the two samples with similar mobility (independent of density), while significantly enhanced in samples with higher mobility (Choi *et al.* "high mobility," $\mu = 28.3 \times 10^6$ cm^2/V · s) indicates that disorder more strongly affects the gap than the applied magnetic field. Furthermore, the strong gap measured in a low magnetic field where the cyclotron energy is reduced compared to the Coulomb interaction suggests nonperturbative Landau level coupling may affect the $\nu = \frac{5}{2}$ FQH gap in a way not yet understood theoretically.

In Fig. 3(a), we show a plot of all the $\frac{5}{2}$ gap values found in the literature versus the inverse transport lifetime, $\tau_{\rm tr}^{-1}$, deduced from the reported mobilities [4,9,11,13,14,17,19,20]. In spite of the large spread in the $\frac{5}{2}$ data, owing to wide ranging differences in sample parameters, i.e., dopant, well width, etc., a clearly discernible trend (indicated by the solid curve as a guide to the eye) is observed pointing towards a disorder-free intrinsic gap value in the range of $\Delta^i_{5/2} \sim 0.005$–$0.010 e^2/\epsilon l_B$. This estimate is in good agreement with a similar extrapolation reported very recently by Pan *et al.* [20,23]. Furthermore, from the low-field Shubnikov–de Haas oscillations we measured the quantum lifetime to be $\tau_q = 33(3)$ ps, which gives the level broadening (Landau level FWHM) to be $\Gamma = 0.23(3)$ K. This gives a direct experimental estimate for the intrinsic gap, $\Delta^i = \Delta^{\rm exp} + \Gamma$, of $\sim 0.006 e^2/\epsilon l_B$, also in good agreement with the extrapolated intrinsic gap value in Fig. 3(a). This experimentally measured intrinsic gap, however, remains well below (by a factor of 3 to 5) the theoretically estimated value for a Moore-Read-type Pfaffian wave function ($\sim 0.025 e^2/\epsilon l_B$) [24,25]. By contrast, the intrinsic gap at $\nu = \frac{1}{3}$ using the same procedure [Fig. 3(b)] [21,26–30] is found to be $\Delta^i_{1/3} \sim 0.045 e^2/\epsilon l_B$, which is in good agreement with theory ($\sim 0.055 e^2/\epsilon l_B$) [25,29].

Morf and d'Ambrumenil proposed that since the disorder-induced Landau level broadening is expected to be roughly equal for FQH states corresponding to particle-hole conjugate pairs, then plotting the corresponding gap values as a function of Coulomb energy directly gives a measure for the intrinsic gap (slope of a fitted line to this data) [31]. The inset of Fig. 3 shows the $\frac{5}{2}$ and $\frac{7}{2}$ gap values obtained in our low electron density sample (open squares) together with those from Ref. [14] (open triangles). The dashed line shows the predicted trend for a disorder-free gap. The slope extracted from a linear fit yields a value for the gap in our sample of $\sim 0.018 e^2/\epsilon l_B$, in disagreement with the intrinsic gap estimated both from our sample and from the extrapolation towards the infinite mobility limit. Furthermore, the Landau Level broadening deduced from Fig. 3 implies a rather large value (~ 1.25 K) that is an order of magnitude larger than the value determined experimentally from the Shubnikov–de Haas oscillations (~ 0.23 K).

It is instructive to consider the energetics of our low-density $\frac{5}{2}$ FQH state. At the observed field of 2.63 T the cyclotron energy is 52 K, the interaction energy is 81 K, and the Zeeman energy (assuming the GaAs band g factor) is 0.75 K. The level broadening in our sample was measured to be ~ 0.23 K and the mobility broadening ~ 0.006 K [32]. Also important is the suppression of the ideal two-dimensional FQH excitation gap due to the finite

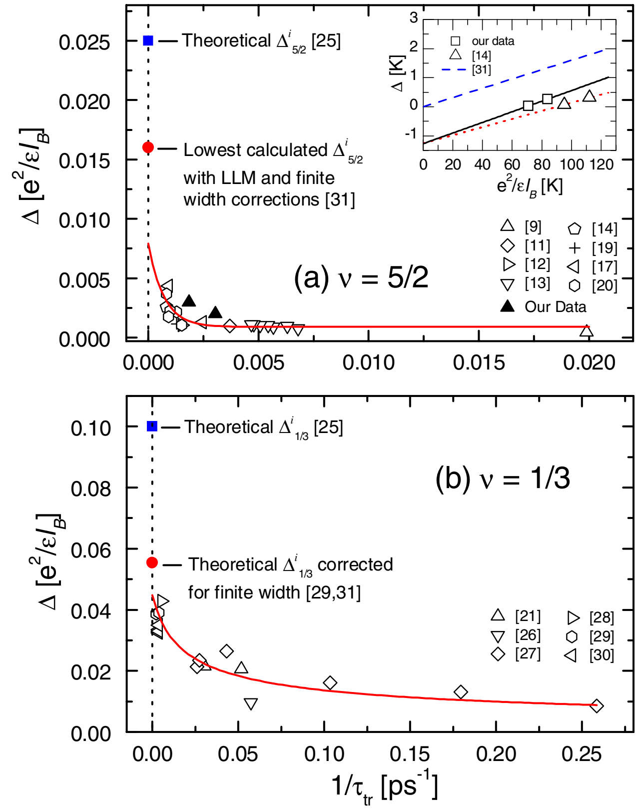

FIG. 3 (color online). (a) Experimental (open symbols) and theoretical (solid square and circle) $\nu = \frac{5}{2}$ gap energy values found in literature. Solid triangles represent our data (the two data points result from two different mobilities measured on separate cooldowns). Inset: Intrinsic gap and disorder at $\frac{5}{2}$ deduced from particle-hole conjugate pairs (see text). (b) Same plot as in (a) but for $\nu = \frac{1}{3}$ gap values reported in the literature.

width $d = 40$ nm of our quasi-2D square well sample. For our $\frac{5}{2}$ FQH gap, this is only about 15% (using our sample parameters in [25]). Taking all of these energies into account we conclude: (i) our measured gap value of 0.262 K is at least a factor of 5 lower than the ideal 2D theoretical $\frac{5}{2}$ excitation gap (~ 2 K at 2.6 T), even if the theoretical gap is corrected for finite width and level broadening suppression (~ 1.5 K); (ii) the cyclotron gap, i.e., the Landau level separation, is smaller than the interaction energy in our system, suggesting considerable nonperturbative inter-Landau level coupling which has not so far been included in the theory and may be important in understanding the $\frac{5}{2}$ FQH state; (iii) the Zeeman energy at 2.6 T is extremely small compared with the Coulomb energy, so the observation of a strong $\frac{5}{2}$ gap at this field might suggest that the $\frac{5}{2}$ FQHE is spin-unpolarized. However, the $\frac{5}{2}$ FQHE has been observed in magnetic fields as large as 12 T [13], i.e., with an increase in Zeeman energy by a factor of 5, where the system is most likely spin-polarized, without affecting much the $\frac{5}{2}$ gap. Therefore, unless a quantum phase transition occurs between a low-field (~ 2.6 T) spin-unpolarized state, and a high-field

($\sim$12 T) spin-polarized $\frac{5}{2}$ FQH state, our experiment rather points towards a spin-polarized state at $\nu = \frac{5}{2}$ even in the zero-field limit, consistent with a Moore-Read Pfaffian wave function.

In conclusion, the $\frac{5}{2}$ energy gap was measured for a sample with an electron density nearly two times smaller than previously observed, and was found to be comparable to samples with higher densities, and similar mobilities. Extrapolating the experimentally measured energy gap values to zero disorder yields an estimate for the intrinsic gap which remains well below the theoretical value. By contrast, a similar extrapolation for the $\frac{1}{3}$ Laughlin state is in much better agreement with theory. Our study suggests that the large discrepancies observed between theory and experiment at $\nu = \frac{5}{2}$ cannot simply be attributed to disorder, but rather may indicate that our knowledge of electron-electron interactions for the $\nu = \frac{5}{2}$ FQH state remains incomplete. Based on the fact that the Coulomb interaction energy scale for our low-density $\frac{5}{2}$ FQH state is larger than the cyclotron energy, we speculate that the nonperturbative aspects of Landau level mixing (as well as disorder), not considered in the theoretical literature, may play an important role in the understanding of the enigmatic $\frac{5}{2}$ FQH state.

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