Generation of Spatially Broadband Twin Beams for Quantum Imaging

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We generate spatially multimode twin beams using 4-wave mixing in a hot atomic vapor in a phase-insensitive traveling-wave amplifier configuration. The far-field coherence area measured at 3.5 MHz is shown to be much smaller than the angular bandwidth of the process and bright twin images with independently quantum-correlated subareas can be generated with little distortion. The available transverse degrees of freedom form a high-dimensional Hilbert space that we use to produce quantum-correlated twin beams with finite orbital angular momentum.

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Quantum imaging is concerned with extending the realm of nonclassical optical effects from the time and frequency domain to the spatial degrees of freedom [1,2]. Spatially multimode squeezed light holds the promise of improved optical resolution [3] and image detection [4], quantum holographic teleportation [5], and parallel quantum information encoding [6]. Recently, it enabled the detection of transverse beam displacements smaller than the standard quantum limit (SQL) [7], and noiseless image amplification has been demonstrated [8,9]. Previous approaches to the generation of nonclassical multimode light have focused on parametric down-conversion in a $\chi^{(2)}$ crystal, following two basic configurations. In the first one, a traveling-wave parametric down-converter is shown to be intrinsically multimode [10]. Pulsed operation is required to compensate for the weakness of the nonlinear coefficient. In the second approach, an optical parametric oscillator operates continuously inside a cavity with degenerate transverse modes [11]. This device is difficult to control above threshold because different transverse modes tend to be strongly coupled, and an alternative is to tune the cavity to be resonant with a single higher order transverse mode. Orthogonal modes separately prepared in a nonclassical state can then be combined with a lossless combiner [12].

Following the recent development of a 4-wave mixing (4WM) amplifier operating deep in the quantum regime [13,14], we present a different approach based on 4WM in an atomic vapor [15]. It combines the advantages of both of the above approaches by exploiting a large single-pass gain. In the absence of a cavity, no mode selection occurs and spatially multimode twin beams can be generated with little optical aberrations. As a result, unprecedented levels of spatial quantum correlations between the twin beam intensities are recorded.

The setup has been described in Ref. [14] and is sketched in Fig. 1. A weak probe beam intersects a strong (420 mW) pump beam inside a 12 mm long ^{85}Rb cell at a small polar angle θ and with an azimuthal angle φ. The pump has a 550 μm waist ($1/e^2$ radius) and the beams overlap over the full length of the cell. The pump and the probe, with angular frequencies ω_0 and $\omega_p < \omega_0$, respectively, are tuned $\approx$ 800 MHz to the blue of the $D1$ line at 795 nm and are resonant with a 2-photon Raman transition between the two electronic ground states $F = 2$ and $F = 3$, which are separated by 3 GHz. The coupling between the light fields and the atomic levels follows a double-lambda configuration [16] and gives birth to a 4WM process that converts two photons from the pump into one probe photon and one conjugate photon at the frequency $\omega_c = 2\omega_0 - \omega_p$ [14]. As a result, the probe beam is amplified and a conjugate beam emerges at an angle θ from the pump, on the other side from the probe (azimuth $\varphi + \pi$). The noise on the intensity difference between the probe and the conjugate is recorded with an amplified balanced photodetector and analyzed with a spectrum analyzer. The total detection efficiency is 0.9, and the noise measurements are corrected only for the detector electronic noise. The estimated uncertainty on the normalized (i.e., with respect to the SQL) noise measurements is less than $\pm$0.2 dB. The conjugate optical path is lengthened by almost 10 ns to compensate for propagation effects in the medium [17]. Distances measured perpendicularly to the propagation direction (z axis) are reported as divergence angles with respect to the center of the cell.

The 4WM is a phase-insensitive amplification process that can be ideally described by the two-photon squeeze operator $\hat{S}_{ab} = \exp(s\hat{a}\,\hat{b} - s\hat{a}^\dagger\hat{b}^\dagger)$ where $\hat{a}$ and $\hat{b}$ are the

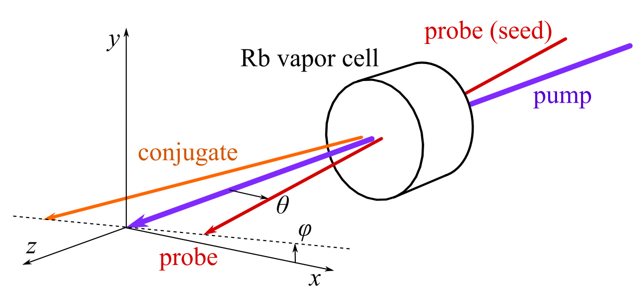

FIG. 1 (color online). Setup geometry.

photon annihilation operators for the optical modes of the probe and the conjugate, respectively. The (chosen to be real) squeeze parameter s is related to the gain G by $G = \cosh^2 s$. The input probe field is a coherent state, and the input conjugate field is the vacuum. For large values of G (4–9 in our experiment), this process leads to substantial quantum intensity correlations between the output beams. For a Gaussian probe beam of waist half the size of the pump waist in the medium, the measured probe-conjugate intensity-difference quantum noise reduction is more than -8 dB at 1 MHz [13], and squeezing is observed at frequencies up to 20 MHz.

The first evidence of the multimode behavior of our system stems from the fact that it can operate for various angles θ and φ of the probe beam. Given the cylindrical symmetry of the setup, the absence of dependence on the azimuthal angle φ is obvious. On the other hand, the angle θ is set by the phase matching, which, in the case of the forward 4WM geometry used here, is usually a stringent condition [15]. In our case, two factors moderate this conclusion. First, the appearance of a strong dispersion of the index of refraction for the probe [17] allows the existence of a pair of frequencies for the pump and the probe such that the 4WM is phase matched for a nonzero θ, around $\theta_0 = 7$ mrad. Second, the large gain exhibited by the atomic medium makes it possible to use a relatively thin medium. Therefore, there should be a sizable range of angles $[\theta_0 - \theta_m, \theta_0 + \theta_m]$ for which the dephasing length is longer than the medium length and for which the beams are quasi-phase-matched.

The maximum mismatch angle, for which the dephasing length is equal to the cell length L, is $\theta_m \approx \sqrt{\lambda/L} = 8$ mrad. Figure 2 shows the gain and the intensity-difference squeezing measured at 1 MHz as a function of θ. The width of the squeezing dip sets the angular bandwidth to be $\Delta\theta \approx 8$ mrad. The angular bandwidth may also be limited by the beam overlap inside the cell.

It should be emphasized that the angular bandwidth is determined here while keeping all the experimental parameters constant. The 4WM is truly multimode and can

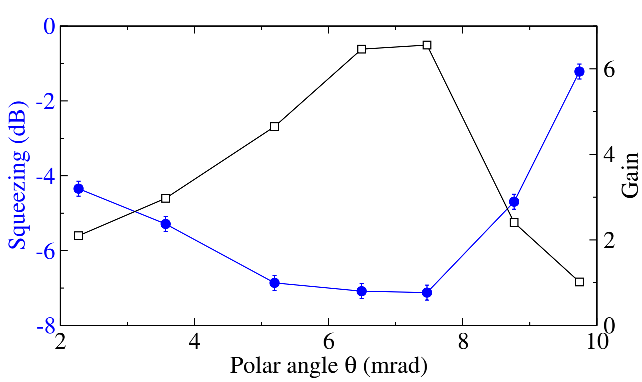

FIG. 2 (color online). Gain (□) and quantum noise reduction on the intensity difference (•) as a function of the angle θ.

generate quantum correlations simultaneously on a collection of modes without the need of the multiplexing described in Ref. [12].

Formally, a multimode state of the light field is a state that cannot be written as a single occupied optical mode with all the other modes in a vacuum state [11]:

$$|\Psi\rangle = |\Psi_0\rangle_0 \otimes |0\rangle_1 \otimes \cdots \otimes |0\rangle_i \otimes \cdots, \tag{1}$$

where i indexes a basis of optical modes coupled to the medium. Even when the probe is seeded with a coherent state, which is a single mode according to the definition above, and the conjugate is seeded with the vacuum, the output of the medium is multimode. Because many transverse optical modes are coupled to the medium, the 4WM is described by a collection of squeeze operators $\hat{S}_{a_i b_i} = \exp(s_i \hat{a}_i \hat{b}_i - s_i \hat{a}_i^\dagger \hat{b}_i^\dagger)$, one for each pair of coupled probe-conjugate modes, described by the photon annihilation operators $(\hat{a}_i, \hat{b}_i)$. These operators transform the probe and conjugate modes from a vacuum state into a two-mode squeezed vacuum of squeeze parameter s_i. The output of the medium when the process is seeded by a coherent state on the probe is thus a pair of bright twin beams surrounded by orthogonal two-mode squeezed vacuum modes. When the entire transverse spatial extent of the bright twin beams is collected by the balanced photodetector, the measured intensity noise is dominated by the two bright modes, and these do not beat with any of the orthogonal vacuum transverse modes. In contrast, when the bright twin beams are partly blocked, some of the transverse vacuum modes are not orthogonal to the detected area of the bright modes and beat against them. This leads to a modification of the noise properties of the beams, which depends on the size and the position of the detection area.

The dependence on the detection area suggests a method to discriminate between single and multimode fields [11]. The Mandel parameter is defined as $Q = (\langle \Delta \hat{N}^2 \rangle / \langle \hat{N} \rangle) - 1$, where $\hat{N}$ is the photon number operator. It is the intensity noise normalized to the noise of a coherent state of the same intensity $\langle \hat{N} \rangle$ (i.e., the SQL) less one. When a light field is attenuated with a beam splitter, Q varies linearly with the transmitted intensity, and in the limit of null transmission tends to 0, the value for a coherent state. As shown in Ref. [18], the Q parameter for a partially detected single-mode beam varies as if the beam was simply attenuated so as to give the same detected intensity. On the contrary, a multimode field has a Q parameter that is not related to the mean value of the transmitted field. For instance, a field composed of independent beams having the same intensity noise will exhibit the same Q whatever number of those beams hit the photodetector.

To confirm the multimode character of an amplified Gaussian probe (coherent state input), we plot in Fig. 3(a) the Q parameter measured at 1 MHz as a function of the intensity, when the output probe is either attenuated

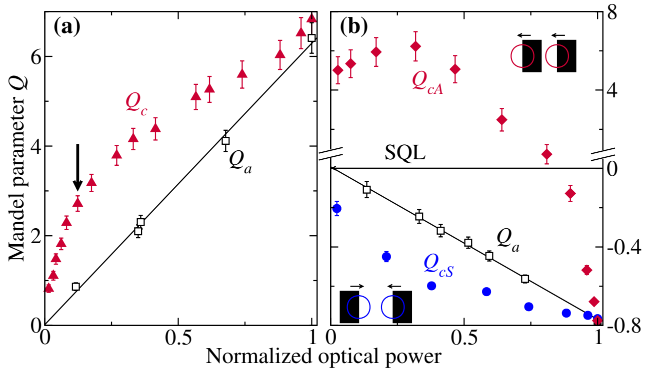

FIG. 3 (color online). Mandel parameter for the probe only (a) and for the intensity difference (b) as a function of the transmission. The beams are either attenuated (□) or clipped with an iris (▲) or with edges placed symmetrically (•) or antisymmetrically (◆) in the far field. Note the change of scale on the rightmost y axis at 0.

(Q_a) or clipped (Q_c) with a centered iris placed in the far field, at 3 times the pump Rayleigh range after the cell [19]. For a transmission of one, the output beam displays some excess noise because of the phase-insensitive amplification process. As expected, Q_a varies linearly with the transmitted intensity, while Q_c, starting from the full transmission and reducing the intensity, decreases more slowly than Q_a. This indicates a certain degree of independence of the various transverse areas with respect to their intensity noise.

We now address the multimode character of the strong quantum correlations between the twin beams. To this end, the Q parameter is now calculated for the difference $\hat{N} = \hat{N}_p - \hat{N}_c$, where $\hat{N}_p$ and $\hat{N}_c$ are the probe and the conjugate photon number operators. Figure 3(b) represents Q when the beams are either attenuated with a beam splitter (Q_a) or clipped along the polar direction with a straight edge in the far field. The edges clip the same fractional amount of light from the probe and the conjugate symmetrically (Q_{cS}) or antisymmetrically (Q_{cA}) with respect to the pump, as shown in Fig. 3(b). The analyzing frequency is now 3.5 MHz (see below), and the measured squeezing at a transmission of 1 is −6.5 dB, corresponding to a gain $G = 4.5$. As in the single beam case, Q_a varies linearly, whereas Q_{cS} and Q_{cA} have nontrivial variations. The fact that Q_{cS} is smaller than Q_a shows that the squeezing is spatially multimode and that the intensity fluctuations are symmetrically correlated with respect to the pump axis, as one would expect from the transverse phase matching condition. Uncorrelated parts of the beam cross sections result in excess noise in the intensity difference, as shown by the behavior of Q_{cA} [note the change of scale in Fig. 3(b)].

The fact that, in both the single beam and the dual beam cases, the Q_c's do not stay constant and eventually tend to zero in the limit of a small detection area hints at the existence of an area scale, called the coherence area [20], under which the light field behaves like a single mode. This is because the finite transverse size of the gain area, given by the near-field pump size, limits the smallest divergence that the coupled probe-conjugate modes can have to roughly the divergence of the diffraction-limited pump. In the single beam experiment, an estimate of the intensity at which Q_c starts converging towards Q_a, indicated in Fig. 3(a) by the arrow, corresponds to an aperture size of the iris of 1.4 mrad. This value is comparable to the size of the pump in the far field ($1/e^2$ diameter of 1.0 mrad).

Although the variations of Q_{cS} contain the effects of the finite coherence area, it is difficult to extract a length scale, as factors like the finite size of the beams or the amount of excess intensity noise come into play. To get a more precise estimate of the length scale that characterizes the spatial intensity correlations between the probe and conjugate, we select a narrow band of the conjugate beam cross section with a slit placed roughly at the center of the beam and we scan another slit with the same orientation across the probe beam. As shown in Fig. 4, the intensity-difference excess noise (in dB), measured at 3.5 MHz, displays a dip whose position indicates the location of the probe area that is correlated with the selected conjugate area. The size of the dip gives an estimate of the size θ_c of the coherence area. After deconvolution from the width of both slits, we find $\theta_c = 1.2$ mrad (full width at $1/e^2$) averaged over both orientations of the slits. These measurements were made with a gain of 4.5, yielding −6.5 dB of intensity-difference squeezing for the full twin beams. At larger gain, two effects may contribute to an observed increase of the size of the coherence area. First, the nonlinear dependence of the gain on the pump intensity leads to an effective narrowing of the gain area [4,10]. Second, cross-phase modulation between the pump and the probe introduces optical aberra-

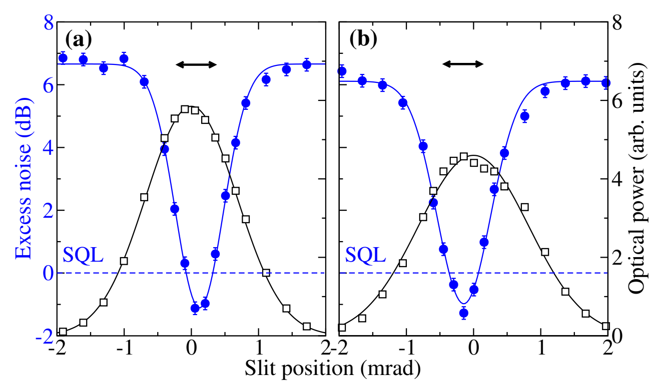

FIG. 4 (color online). Normalized intensity-difference noise (•) and transmitted probe power (□) as a function of the position of a slit placed on the probe while a similar slit is placed at a fixed position on the conjugate. The slits are orientated in the azimuthal (a) or polar (b) direction with respect to the pump. The data are fitted with Gaussians. The size of the slits is indicated by the double arrows.

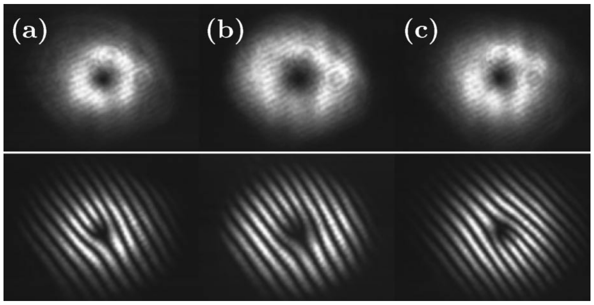

FIG. 5. Intensity profiles of the beam at the input (upper row) and the output (lower row) of the interferometer for (a) the input probe, (b) the output probe, and (c) the conjugate.

tions on the probe. An obvious way of reducing these effects would be to use a larger pump beam waist.

A rough estimate of how many independently quantum-correlated pairs of probe-conjugate optical modes can be generated by the 4WM process can be made by counting the number of coherence areas that can fit in the (solid) angular bandwidth. That number is $\Delta\theta/\theta_c \approx 6$ radially and $\pi\theta_0/\theta_c \approx 18$ in the azimuthal direction, for a total number of modes of about 100.

To demonstrate the multimode capabilities of the system, we generate twin beams out of nontrivial optical modes. In a first experiment, we seed the 4WM process with a probe made out of two spots (two Gaussian beams) having the same polar angle θ and separated by 3 mrad in the azimuthal direction. The intensity-difference squeezing measured when the two spots for both the amplified probe and the conjugate hit the balanced detector is -6.6 dB, and the probe-conjugate spot pairs measured individually yield a quantum noise reduction of -5.9 dB and -5.8 dB (all measured at 3.5 MHz). At frequencies lower than 3 MHz, we found that, while the global squeezing is unchanged, the squeezing for each individual probe-conjugate pair is degraded by the presence of the other (undetected) pair. The origin of this coupling between the beam pairs will be studied elsewhere. The independence of the probe-conjugate spot pairs in terms of the correlations at certain frequencies demonstrates the possibility of using the transverse structure of the twin beams as a quantum resource for parallel continuous-variable (CV) operations on the field quadratures.

In a second experiment, we seed the process with a Laguerre-Gauss (LG) mode of order 1, and we check the phase properties of each beam by interfering it with its mirror image [21]. Figure 5 shows the intensity profiles and the interferograms of the input probe and the output twin beams. The fringe count on the interferograms shows that the process produces twin LG modes of order ± 1. The measured intensity-difference squeezing between the output LG modes is -7.3 dB, which demonstrates the possibility of using this system to encode quantum information in the spatial basis of the orbital angular momentum [22]. More generally, one can generate any pair of correlated images whose spatial extent fits in the angular bandwidth and whose spatial frequencies are smaller than the inverse of the coherence area.

We have shown that 4WM in an atomic vapor can generate highly quantum-correlated twin beams in a large number of transverse modes simultaneously. This opens the door to CV quantum information encoding in a high-dimensional Hilbert space. A natural extension of this work would be to demonstrate the entanglement of these modes, both in the phase (intensity) [23] and in the displacement (tilt) [24] spaces. Finally, direct imaging of the spatial distribution of the fluctuations with a camera, in the spirit of Ref. [8], should give access to the spatial frequency spectrum of squeezing.

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