Quantum Spin Dynamics of Mode-Squeezed Luttinger Liquids in Two-Component Atomic Gases

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We report on the observation of many-body spin dynamics of interacting, one-dimensional (1D) ultracold bosonic gases with two spin states. By controlling the nonlinear atomic interactions close to a Feshbach resonance we are able to induce a phase diffusive many-body spin dynamics of the relative phase between the two components. We monitor this dynamical evolution by Ramsey interferometry, supplemented by a novel, many-body echo technique, which unveils the role of quantum fluctuations in 1D. We find that the time evolution of the system is well described by a Luttinger liquid initially prepared in a multimode squeezed state. Our approach allows us to probe the nonequilibrium evolution of one-dimensional many-body quantum systems.

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Among the applications of ultracold atomic gases, atom interferometry stands out due to its potential for high precision measurements [1]. In atom interferometry, the physical quantity of interest is measured in terms of the relative phase accumulated by the atomic wave function, subsequently mapped onto atomic populations for efficient readout. Because of their intrinsic phase coherence and the possibility to create nonclassical spin states for precision metrology, Bose-Einstein condensates (BEC) seem ideal candidates for such experiments. However, interatomic interactions mitigate this conclusion. For a two-component interacting BEC, it has been shown [2,3] using a single-mode approximation (SMA) that the relative phase between the two components undergoes a complicated evolution [Figs. 1(c)–1(e)], creating quantum correlations [4] while single-particle coherence is suppressed. Therefore, this dynamics is often termed *phase diffusion*.

Here, we investigate such an interaction-induced dynamics in quasi-1D two-component quantum gases by monitoring the loss of coherence in a Ramsey-type interferometer sequence. In order to distinguish different contributions affecting the coherence through the spin or spatial wave functions, we employ a novel many-body spin echo sequence using a Feshbach resonance to adjust sign and magnitude of the atomic interactions. When applied to a single spatial mode BEC, this spin echo would lead to full revivals of coherence, which are not observed in our experiment. In contrast, quantum fluctuations play a key role for 1D interacting systems [5,6], which must necessarily be described as multimode quantum gases, as during the dynamical evolution higher energy modes become populated [7]. The Luttinger liquid (LL) formalism [8,9], which reduces the interacting problem to an effective low-energy model of decoupled harmonic oscillator modes, provides such a description. We show theoretically that our preparation sequence amounts to producing a multimode-squeezed state in the *spin excitation modes* of the LL oscillators, with each oscillator itself prepared in a well-defined mode-squeezed state, and remaining in a squeezed state at all times. Monitoring the phase dynamics of this strongly nonequilibrium state allows one to probe fundamental aspects of 1D physics, namely, the competing dynamics of the (quasi-)condensate fraction (zero momentum mode) and of the low-energy excitations, highly relevant for squeezing experiments in 1D configurations [10]. From our model we find that only the lowest oscillator mode shows the familiar revival dynamics, whereas the full model leads to the partial revivals that we observe experimentally.

We consider a ^{87}Rb BEC of around 2.8×10^5 atoms loaded into a 2D-optical lattice (laser wavelength 843 nm), creating a 2D array of 1D degenerate quantum gases [see

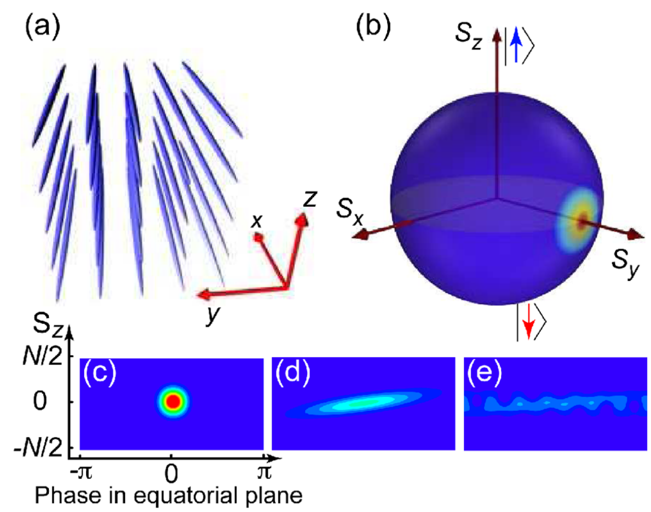

FIG. 1 (color online). (a) Array of quasi-1D spinor systems. (b),(c) Coherent spin state exhibiting Gaussian distributed fluctuations of the mean spin. (c)–(e) Time evolution of the initial CSS under nonlinear interactions redistributing the initial Gaussian fluctuations towards increased phase fluctuations.

Fig. 1(a)]. Within each of the tubelike traps the radial and axial trap frequencies are $\omega_r \approx 2\pi \times 42$ kHz and $\omega_{ax} \approx 2\pi \times 90$ Hz, respectively, from which we calculate a mean atom number per tube of $N = 60$.

In order to extract information about the phase dynamics, we monitor the coherence of the system in a Ramsey-type interferometer. Starting from a spin-polarized ensemble in state $|\downarrow\rangle \equiv |F = 1, m_F = +1\rangle$, we use a two-photon $\pi/2$ pulse combining a microwave and a radio frequency photon to couple this state to the $|\uparrow\rangle = |2, -1\rangle$ state and bring each atom into the single-particle superposition $(|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$. This prepares a coherent spin state (CSS) within each tube with expectation value of the magnetization $\langle \hat{m}_z \rangle = 0$ and variance $\langle \hat{m}_z^2 \rangle = N/2$, with $\hat{m}_z = \hat{n}_\downarrow - \hat{n}_\uparrow$ [see Fig. 1(b); cf. [11]]. In order to observe interaction driven effects, we let the system evolve for a given time at a particular value of the inter-spin-state interaction strength, selected by using a Feshbach resonance around $B = 9.12$ G. Thereby the interspecies scattering length $a_{\uparrow\downarrow}$ can be changed by a few 10% from its background value [12], where a_{ij} is the s-wave scattering length for collisions between atoms in spin states i and j.

After this time evolution, a final $\pi/2$ pulse with phase θ relative to the first pulse is applied, mapping the final relative phase onto populations of spin states $|\uparrow\rangle$ and $|\downarrow\rangle$ that are readout using state-selective absorption imaging. In the absence of interactions and dephasing, such a sequence results in sinusoidal Ramsey fringes in the relative population $N_\uparrow/N_{\rm tot}$ as a function of θ. Experimentally the coherence is quantified through the visibility of the Ramsey fringe $N_\uparrow/N_{\rm tot} = \frac{1}{2}[1 + \mathcal{V}(t)\cos(\theta)]$, which is used to fit the experimental data and extract $\mathcal{V}(t)$ for a specific interaction time. Far from the Feshbach resonance ($B = 8.7$ G), where phase diffusion can be neglected, we measure a e^{-1}-decay time $t_{\rm dec} = 54$ ms, which can be attributed to residual single-particle decoherence effects, e.g., caused by magnetic field fluctuations.

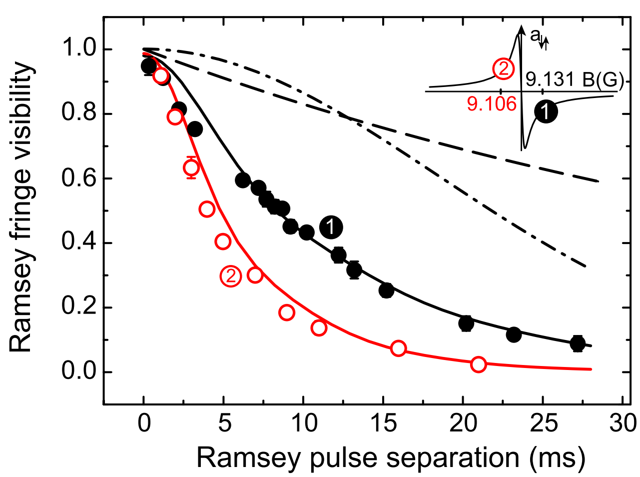

FIG. 2 (color online). Ramsey fringe contrast drop for a time evolution at $B = 9.106$ G (○) and $B = 9.131$ G (●). The dashed line indicates the independently measured decoherence far away from resonance ($t_{\rm dec} = 54$ ms). The solid lines are predictions of our LL model with the phase width $(\Delta\phi_{\rm LL})^2 = (\Delta\phi_0)^2 \approx 0.033$ fixed. The dash-dotted line is a prediction of Eq. (2) based on SMA. For values see text.

Close to the Feshbach resonance, however, we find a markedly faster decay of the Ramsey contrast. In Fig. 2 we monitor such a behavior of the Ramsey fringe over time for two magnetic fields located almost symmetrically around the center of the Feshbach resonance. This can be expected from enhanced phase diffusion due to increased interactions near the resonance. Phase diffusion results from a spread in the distribution of populations that are converted into phase fluctuations by the nonlinear interactions during the evolution; see Figs. 1(c)–1(e). In the simplest case where all atoms occupy the same orbital wave function, the Ramsey fringe contrast decays according to

$$\mathcal{V}_{\rm SMA}(t) \approx \exp\left(-\frac{1}{2}\chi^2\langle \hat{m}_z^2\rangle t^2\right). \quad (2)$$

For the initial state we prepare, the population variance $\langle \hat{m}_z^2 \rangle = N/2$ leads to a phase uncertainty $(\Delta\phi_0)^2 = 2/N \approx 0.033$ of the collective spin vector in the equatorial plane (see Fig. 1) in each tube, and a phase spreading time scale $t_\phi \sim 1/(\chi\sqrt{N})$. The parameter χ, related to the second derivative of the chemical potential [3], is directly proportional to the difference $a_s = (a_{\uparrow\uparrow} + a_{\downarrow\downarrow} - 2a_{\uparrow\downarrow})/2$. Far from the resonance, all three scattering lengths $a_{\uparrow\uparrow}$, $a_{\downarrow\downarrow}$, and $a_{\uparrow\downarrow}$ are approximately equal, so that $\chi \approx 0$ and phase spreading can be neglected. However, near the Feshbach resonance, the change of interspecies scattering length can lead to a significant nonlinear interaction energy. Following Refs. [3,13,14], we estimate $\chi \approx 2\pi \times 4.6$ Hz for $B = 9.131$ G and our trapping parameters with an atom number of $N \approx 60$. Although this value, together with the observed decoherence rate, is roughly on the order of the observed rate at which the coherence is lost in our case, the pure SMA Eq. (2) cannot explain our experimental observation in Fig. 2. Close to the resonance we lose up to 50% of the atoms due to inelastic collisions. However, as these collisions usually remove atoms from both spin states symmetrically, they do not modify the magnetization of the system and thus only weakly influence the dynamical evolution of the coherence for our measurement times [3].

In contrast to the simple model of Eq. (2), which predicts a symmetric decay around the resonance, we systematically observe a faster drop of contrast below the resonance. This results from dynamics of the *spatial* wave function. Below resonance, the interspecies repulsion is stronger than the intraspecies repulsion ($\chi < 0$), and the system becomes dynamically unstable towards demixing of the two species. This reduces the Ramsey fringes' visibility below the resonance, which cannot be distinguished from the effect of the coherent phase diffusion dynamics.

In order to separate the effects of phase diffusion from other mechanisms reducing the Ramsey fringe contrast, we apply a many-body spin echo operation after an initial evolution time T, similar to the one used in cavity quantum electrodynamics experiments [15]. We stress that our echo technique is acting on the *many-body* quantum state, thereby extending previous theoretical work on echo operations neglecting phase diffusion [16]. Our echo operation is performed by first holding the sample for a time $T = 6$ ms at a magnetic field $B_1 = 9.131$ G above the Feshbach resonance ($\chi > 0$), and subsequently jumping below the resonance ($\chi < 0$). This operation effectively changes the sign of χ, while heating or atom loss can be avoided [11]. In a SMA one would expect this sequence to correspond to a perfect time reversal, leading to a full revival of the contrast after another interaction time T according to $\mathcal{V}_{\rm SMA}(t) = \mathcal{V}_0 \exp[-\frac{\chi^2}{2}\langle\hat{m}_z^2\rangle(t-2T)^2]$. This contradicts our observation of only partial revivals, shown for two spin echo sequences in Fig. 3.

In order to explain our observation, we model our system in a LL approach going beyond the usual SMA [2,7]. A drastic simplification follows from the near equality $a_{\downarrow\downarrow} \approx a_{\uparrow\uparrow}$, which results in a decoupling of elementary excitations into almost independent density and spin fluctuations. The latter can be described in terms of two conjugate fields, $\hat{m}_z$ and $\hat{\phi}_s$, describing, respectively, fluctuations of the local

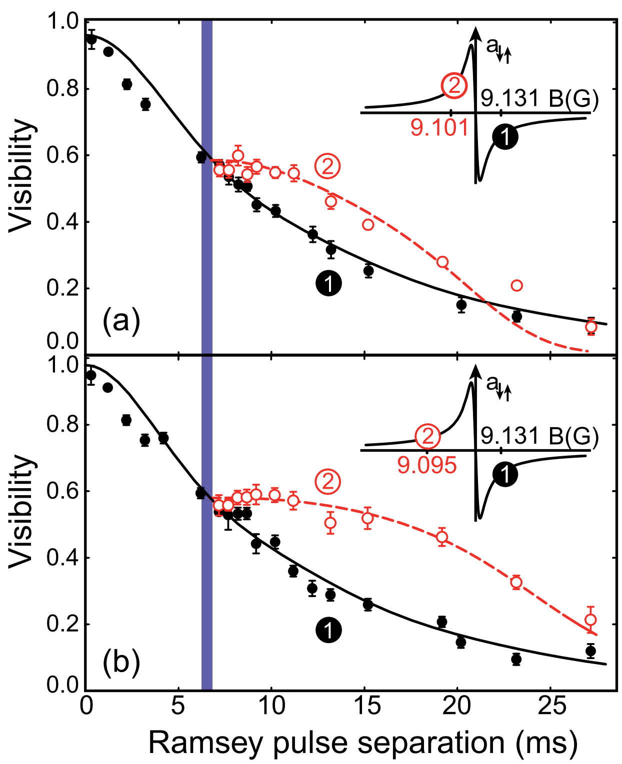

FIG. 3 (color online). Ramsey fringe visibility versus time for holding above the Feshbach resonance, $B = 9.131$ G (○), and with time reversal (spin echo) after 6.7 ms (●) jumping from $B = 9.131$ G to (a) $B = 9.101$ G and (b) $B = 9.090$ G. The solid and dashed lines are calculations from our model with $(\Delta\phi_{\rm LL})^2 = 0.04$ (a) and $(\Delta\phi_{\rm LL})^2 = 0.025$ (b). For $B = 9.131$ G, $a_s = 0.17a_{\uparrow\uparrow}$, and we compute $\gamma = 0.165$ and $K = 8.0$.

magnetization and of the relative phase. At long wavelengths, the spin part of the Hamiltonian reads

$$H_s = \int dx\left[g_s\hat{m}_z^2 + \frac{n_{\rm tot}}{4M}(\nabla\hat{\phi}_s)^2\right], \qquad (4)$$

with $n_{\rm tot} = n_\uparrow + n_\downarrow$ the linear density. For a uniform 1D system of length L, the fields $\hat{m}_z$ and $\hat{\phi}_s$ can be expanded in terms of momentum eigenmodes $\hat{a}_q$, $\hat{a}_q^\dagger$ as [8,9] $\hat{\phi}_s(x) = \phi_0 + \sum_q (2qLK/\pi)^{-1/2} e^{-|q|/2q_c} {\rm sgn}(q)[e^{iqx}\hat{a}_q + {\rm H.c.}]$. Each mode is characterized by the wave vector q and frequency $\omega_q(t) = v_s(t)|q|$, where $v_s(t) = [g_s(t)n_{\rm tot}/M]^{1/2}$ is the spin velocity and $g_s(t)$ denotes the spin coupling constant. The sum over LL modes exclude the zero mode and are restricted to values of q below a cutoff momentum $q_c \sim \xi_h^{-1}$, where ξ_h is the healing length. The interactions are encoded in the LL parameter K, which can take values ranging from 1 (the so-called Tonks-Girardeau limit [14]) to infinity (noninteracting gas). For weakly interacting bosons [9] $K \approx \pi/[\sqrt{\gamma}(1-\sqrt{\gamma}/2\pi)^{1/2}]$ where $\gamma = 2a_sM\omega_r/\hbar n_{\rm tot}$. In our experiments [12] $\gamma \sim 0.1$–0.2 and $K \sim 5$–8 for different data sets. The contribution of the zero energy mode ϕ_0 is identical to that in the SMA, as described before [2]. The dynamics of the low-energy LL excitations on top of the zero mode dynamics is that of a collection of independent harmonic oscillators with (time-dependent) frequencies ω_q.

In order to use the LL decomposition to compute the time evolution, we need to identify how to describe the initial state in terms of those LL modes. Our experimental scheme ideally corresponds to an instantaneous projection of the spin state (initially polarized in $|\downarrow\rangle$) onto a state with zero relative phase directly after the first $\pi/2$ pulse, $\hat{\phi}_s(x)|\psi(0)\rangle \approx 0$. The connection with the LL formalism is done by identifying the initial CSS as a multimode squeezed state for the elementary spin excitations,

$$|\psi(0)\rangle = \prod_q \sqrt{(1-|w_q|^2)}\exp(w_q\hat{a}_q^\dagger\hat{a}_{-q}^\dagger)|0\rangle, \qquad (5)$$

where $w_q = (1-\alpha_q)/(1+\alpha_q)$. Here, $\alpha_q = \Delta\phi_{\rm LL}|q|/|q_c|$ is a mode-squeezing parameter, and the phase variance $(\Delta\phi_{\rm LL})^2 = (\Delta\phi_0)^2 = \frac{2}{N}$. Because of experimental imperfections (e.g., unknown temperature), the exact width of the prepared squeezed state is unknown and cannot be determined independently. We still consider the initial state as a squeezed state of the LL oscillators, with a fitted $\Delta\phi_{\rm LL}$.

The time evolution of a squeezed state under the LL Hamiltonian amounts to the replacement $\hat{a}_q \rightarrow \hat{a}_q\exp(-i\omega_q t)$. In addition, the reversal of the sign of interaction at time $t = T$ amounts to a sign reversal of the spring constant of each LL harmonic oscillator. We are able to compute this time evolution exactly [11], using the formalism of harmonic oscillators with time-dependent frequencies $\omega_q(t)$. Here, we concentrate on the comparison between the predictions of the calculations and the experimental results. From our model we are able to calculate the coherence factor $\mathcal{V}(t) = \mathrm{Re}\{\frac{1}{L}\int_0^L dx \langle\psi(t)|e^{i\hat{\phi}_s}|\psi(t)\rangle\}$, measuring the relative phase $\hat{\phi}_s$ between $|\uparrow\rangle$ and $|\downarrow\rangle$. The LL formalism allows us to write $\mathcal{V} = \mathcal{V}_{\mathrm{SMA}}(t)\mathcal{V}_{q\neq 0}(t)$, where $\mathcal{V}_{\mathrm{SMA}}$ is given by Eq. (2) and the term describing the contribution of the $q \neq 0$ modes to the decay of contrast $\mathcal{V}_{q\neq 0}(t)$ is known explicitly [11]. The combination of these effects leads to the typical behavior of $\mathcal{V}(t)$ illustrated in Fig. 3 where we quantitatively compare the experimental results with the predictions of our LL model. The interaction parameters of the LL model were determined from the microscopic data [12], and in our computations we only consider the density in the central tubes computed in the Thomas-Fermi approximation. Overall, we find very good agreement between the LL model and the experimental data using $\Delta\phi_{\mathrm{LL}}$ as the only fit parameter with values of the same order of magnitude as the initial width. For longer times ($t > 20$ ms), the model deviates from the measured data. This breakdown is due to the phenomenon of demixing discussed above, when the excitations become so strong that density fluctuations are significant [6]. Its time scale can be estimated as the time required for the formation of random magnetization domains, when $\sum_q\langle|\hat{m}_q|^2\rangle \sim N$. For our experimental parameters, it is of the order $1/\chi \sim 25$ ms.

From this analysis we find indeed that although the zero mode evolution is perfectly refocused by changing the sign of g_s, the nonzero modes still undergo dephasing even under the echo sequence. The reason for this is the kinetic energy term in Eq. (4), unaffected by the spin echo. Hence, the reversal is exact for the $q = 0$ mode, significant for low-lying spin waves with $q \sim 1/L$, but increasingly less efficient for higher lying spin wave modes with $L^{-1} \ll q < q_c$. Note that a full revival could in principle be achieved by also reverting the second, kinetic energy term of the Hamiltonian in Eq. (4). This could be realized by inducing a negative effective mass, e.g., through a weak optical lattice along the direction of the tubes [17].

In conclusion, we have studied the spin dynamics of quasi-1D two-component quantum gases with adjustable interaction. For strong interactions we observe an accelerated decay of coherence in the system. By application of a novel many-body spin echo technique, we are able to reverse the interaction driven dynamics, leading to a partial revival of the coherence in the system. We attribute this revival to the dynamics of the ground state mode, reflecting the coherent nature of the phase diffusion dynamics. This is supported by quantitative comparison to our LL model. The missing fraction of coherence in the revival is also quantitatively in agreement with our model and demonstrates the importance of the dynamical evolution of higher lying modes in 1D systems. Our experiment shows that quantum fluctuations are a crucial component in the discussion of phase diffusion dynamics [2] and spin squeezing [4,10] in low dimensional systems. While our work demonstrates that these quantum fluctuations fundamentally limit the performance of atom interferometers in 1D, it also indicates an avenue to overcome such limitations by inverting both interaction and kinetic energy terms simultaneously during the interferometer sequence.

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