Compton Scattering in Ignited Thermonuclear Plasmas

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Inertially confined, ignited thermonuclear D-T plasmas will produce intense blackbody radiation at temperatures $T \gtrsim 20$ keV; it is shown that the injection of GeV electrons into the burning core can efficiently generate high-energy Compton scattering photons. Moreover, the spectrum scattered in a small solid angle can be remarkably monochromatic, due to kinematic pileup; a peak brightness in excess of 10^{30} photons/(mm^2 mrad^2 s 0.1% bandwidth) is predicted. These results are discussed within the context of the Schwinger field and the Sunyaev-Zel'dovich effect.

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From an electromagnetic radiation viewpoint, inertially confined, ignited deuterium-tritium plasmas [1] behave rather like blackbodies, with characteristic temperatures of a few tens of keV. It is shown that the injection of relativistic electrons into the burning core can generate high-energy photons via Compton scattering [2]. In such a system, the blackbody radiation spectral density becomes sufficiently high to alleviate the small Klein-Nishina scattering cross section [3] and yield significant scattering rates. Remarkably, the kinematics of the interaction results in highly monochromatic γ rays within a small scattering solid angle. Finally, it is also speculated that electron focusing could be used to focus the γ rays in a very small spatial region, where the electromagnetic fields may approach or exceed the Schwinger limit [4]. These results are also discussed within the context of the Sunyaev-Zel'dovich effect [5].

It is instructive to first consider the Compton scattering mean free path of an electron, $1/\sigma n_\lambda$, where $\sigma = 8\pi r_0^2/3 = 0.665$ b is the total (Thomson) cross section, defined in terms of the classical electron radius r_0, and n_λ is the blackbody photon density, which can be derived by integrating Planck's distribution [6] over all frequencies:

$$n_\lambda = \int_0^\infty \frac{\omega^2 d\omega}{\pi^2 c^3 (e^{\hbar\omega/kT}-1)} = \frac{1}{\pi^2 c^3}\left(\frac{kT}{\hbar}\right)^3 \int_0^\infty \frac{\chi^2 d\chi}{e^\chi - 1} = \frac{2\zeta(3)}{\pi^2}\left(\frac{kT}{m_0 c^2}\frac{1}{\lambda_c}\right)^3. \quad (1)$$

Here, ζ is the Riemann ζ function, k is Boltzmann's constant, T is the radiation temperature, c is the speed of light in vacuum, $\hbar$ is the reduced value of Planck's constant, and λ_c is the Compton wavelength. Numerically, $n_\lambda \simeq 2.53 \times 10^{32}$ m^{-3} for $kT/e = 20$ keV, approximately one order of magnitude above the photon density of a 1 μm, 1 TW laser focused down to 10 μm. The mean free path is <60 μm, which is similar to the size of the imploded core ($\delta \simeq 50$ μm): injected electrons have a high probability of interacting with a blackbody quantum to scatter a high-energy photon.

This quantity can be compared to the mean free path of the next dominant energy loss mechanism for relativistic electrons interacting with a high density target, namely, bremsstrahlung; in this case, the cross section can be approximated by [7]

$$\varsigma \simeq 4\alpha Z^2 r_0^2\left[\ln\left(\frac{183}{\sqrt[3]{Z}}\right) + \frac{1}{18}\right], \quad (2)$$

provided that the electron relativistic factor $\gamma \gg 1/\alpha\sqrt[3]{Z}$, where α is the fine structure constant. For GeV electrons and a target made of H isotopes ($Z = 1$), this condition holds, and $\varsigma \approx 12.2$ mb. Next, the target density must be taken into account; it is of the order of 350 g cm^{-3}, which yields a mean free path of the order of 1.6 cm $\gg \delta$. Other interaction mechanisms, including ionization and Møller scattering, are strongly suppressed at 1 GeV.

Returning to Compton scattering, a more precise calculation can be performed by determining the scattering rate:

$$\frac{dN}{d\tau} = \int_0^\infty d\omega \int_0^\pi \sin\theta d\theta \int_0^{2\pi} \frac{\sigma c^2}{\omega} u_\mu(\tau) k^\mu \frac{d^3 n_\lambda}{d\omega \sin\theta d\theta d\varphi} \times [r_\nu(\tau)] d\varphi. \quad (3)$$

Here, τ is the proper time along the electron trajectory $r_\nu(\tau)$, $u_\mu(\tau) = dr_\mu/cd\tau = (\gamma, \mathbf{u})$ is the four-velocity, and $k_\mu = \omega(1, \sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)/c$ is the four-wavenumber of the blackbody radiation. The photon phase space density of the isotropic blackbody radiation is

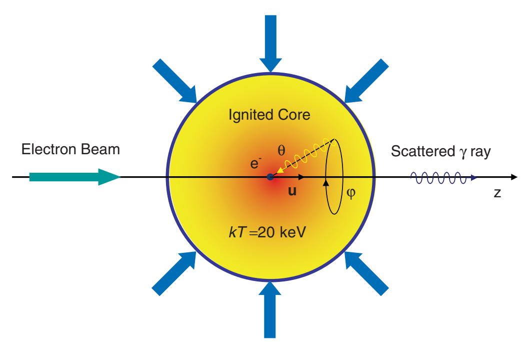

FIG. 1 (color). Schematic of the interaction geometry.

$$\frac{d^3 n_\lambda}{d\omega \sin\theta d\theta d\varphi} = \frac{\omega^2}{4\pi^3 c^3 (e^{\hbar\omega/kT} - 1)}. \tag{4}$$

Integrating over frequency and solid angle for a ballistic electron trajectory defining the z axis, as shown in Fig. 1, and using the laboratory time,

$$\begin{aligned}\frac{dN}{dt} &= \frac{\sigma}{4\pi^3 c^2}\int_0^\infty d\omega \int_0^\pi \sin\theta d\theta \int_0^{2\pi} (1-\beta\cos\theta) \\ &\quad\times \frac{\omega^2}{e^{\hbar\omega/kT}-1} d\varphi \\ &= \frac{2\sigma}{\pi^2 c^2}\zeta(3)\left(\frac{kT}{\hbar}\right)^3 = \sigma n_\lambda c. \end{aligned}\tag{5}$$

This result is independent of the electron energy, indicating that fast ignition electrons [8] can Compton radiate significantly.

It is also worth noting that the normalized vector potential, which is related to the photon density by $A_0^2 = 2n_\lambda \lambda \lambda_c r_0$, where λ is the radiation wavelength, remains small: $A_0^2 = 3.41 \times 10^{-5} \ll 1$. As a result, the radiation process can be analyzed within the framework of linear (single photon) Compton scattering.

To derive the spectral brightness QED units are used: charge, mass, length, and time are measured in units of e, m_0, $\lambda_c = \hbar/m_0 c$, λ_c/c. The fundamental quantity of interest is the differential brightness,

$$\begin{aligned}\frac{d^{12}N}{d\Omega dq d^4 x_\nu d^3 u_i d^3 k_i} &= \frac{d\sigma}{d\Omega}\delta(q_\mu(u^\mu + k^\mu) - k_\mu u^\mu) \\ &\quad\times j_\mu(x_\nu, u_\nu)\Phi^\mu(x_\nu, k_\nu).\end{aligned}\tag{6}$$

$d\sigma/d\Omega$ is the differential scattering cross section, the argument of the Dirac δ distribution represents the Compton formula, where k_μ and q_μ are the incident and scattered four-wave-numbers, $j_\mu = u_\mu(d^3 n_e/d^3 u_i)/\gamma$ is the electron four-current phase space density, and $\Phi_\mu = k_\mu(d^3 n_\lambda/d^3 k_i)/\omega$ is the incident photon four-flux phase space density, related to the Wigner distribution function [9]. For a single electron, the four-current is $j_\mu = u_\mu \delta(\mathbf{x} - \mathbf{r}(\tau))/\gamma$:

$$\begin{aligned}\frac{d^6 N}{d\Omega dq d\tau d^3 k_i} &= \frac{d\sigma}{d\Omega}\delta(q_\mu(u^\mu + k^\mu) - k_\mu u^\mu) \\ &\quad\times u_\mu \Phi^\mu(r_\nu(\tau), k_\nu).\end{aligned}\tag{7}$$

The differential Klein-Nishina scattering cross section is [10]

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \frac{\alpha^2}{2}\left(\frac{q}{\kappa}\right)^2\left\{\frac{1}{2}\left(\frac{\kappa}{\lambda}+\frac{\lambda}{\kappa}\right) - 1 \right. \\ &\quad \left. + 2\left[\varepsilon_\mu \pi^\mu - \frac{(\varepsilon_\mu u^\mu)(\pi_\mu v^\mu)}{\kappa} + \frac{(\varepsilon_\mu v^\mu)(\pi_\mu u^\mu)}{\lambda}\right]^2\right\}.\end{aligned}\tag{8}$$

$\kappa = u_\mu k^\mu$, and $\lambda = u_\mu q^\mu$ are the incident and scattered light-cone variables [11], $v_\mu = u_\mu + k_\mu - q_\mu$ is the electron four-momentum after the interaction, ε_μ and π_μ are the incident and scattered four-polarizations. Blackbody radiation is randomly polarized; Eq. (8) must be averaged over both ε_μ and π_μ.

Using spherical coordinates, and introducing $x = -\cos\theta$, with $dx = \sin\theta d\theta$ so that $\mathbf{k} = \omega(\sqrt{1-x^2}\cos\varphi, \sqrt{1-x^2}\sin\varphi, -x)$, defining the z axis to coincide with the electron velocity, with $u_\mu = (\cosh\rho, 0, 0, \sinh\rho)$, where ρ is the rapidity, and considering on-axis radiation, with $q_\mu = q(1, 0, 0, 1)$, the polarization-averaged differential cross section is

$$\left\langle\frac{d\sigma}{d\Omega}\right\rangle = \frac{\alpha^2 q^2[x^2 - 1 + \frac{qe^{-\rho}}{\omega}(\cosh\rho + x\sinh\rho) + \frac{\omega e^\rho}{q}(\cosh\rho + x\sinh\rho)^3]}{4\omega^2(\cosh\rho + x\sinh\rho)^4}. \tag{9}$$

The Compton formula, $\kappa - \lambda = k_\mu q^\mu$, reduces to $q = \omega(\cosh\rho + x\sinh\rho)/[\omega(1+x) + e^{-\rho}]$.

The integral over the incident photon phase space is

$$\frac{d^3 N}{dt dq d\Omega} = \int_0^\infty \int_{-1}^1 \int_0^{2\pi} \left\langle\frac{d\sigma}{d\Omega}\right\rangle \delta\left(q - \frac{\omega(\cosh\rho + x\sinh\rho)}{\omega(1+x) + e^{-\rho}}\right)\frac{u_\mu k^\mu}{\gamma\omega}\frac{\omega^2}{4\pi^3(e^{\omega/kT} - 1)} d\omega dx d\varphi. \tag{10}$$

The integral over φ reduces to a multiplication by 2π; the integral over ω can be performed by using the following property of the δ distribution [12]: $\delta(g(\omega)) = \sum_n \delta(\omega - \omega_n)/|g'(\omega_n)|$, where the ω_n are the roots of $g(\omega) = 0$. We then find

$$\frac{d^3 N}{dt dq d\Omega} = \frac{\alpha^2 q^2 e^{-\rho}\operatorname{sech}(\rho)}{8\pi^2}\int_{-1}^1 \frac{\frac{1}{(\cosh\rho + x\sinh\rho)[\cosh\rho - q + x(\sinh\rho - q)]} + \frac{x^2 - 1}{(\cosh\rho + x\sinh\rho)^2[\cosh\rho - q + x(\sinh\rho - q)]^2} + \frac{\cosh\rho + x\sinh\rho}{[\cosh\rho - q + x(\sinh\rho - q)]^3}}{\exp\{\frac{qe^{-\rho}}{kT[\cosh\rho - q + x(\sinh\rho - q)]}\} - 1} dx. \tag{11}$$

While the polar integral cannot be resolved analytically, it can be performed computationally; the results are shown in Fig. 2, for $kT = 20$ keV, and different values of the rapidity. For electrons at rest, the scattered radiation follows Planck's blackbody spectrum; at higher rapidity, the spectrum undergoes kinematic pileup and becomes monochromatic. This is readily explained by considering energy conservation: the maximum scattered photon energy corresponds to an event where the electron is left at rest, and is equal to $\gamma + \omega$, in our units. For GeV electrons, $\gamma \gg kT$, and $\gamma + \omega \simeq \gamma$; this corresponds to stopping the beam into the core, and producing an extremely bright GeV photon beam.

An approximate analytical expression can be derived by considering the dominant term in the cross section for highly relativistic electrons ($\cosh\rho \simeq \sinh\rho \simeq e^{\rho}/2$):

$$\frac{d^3N}{dtdqd\Omega} \simeq \frac{\alpha^2 q^2 e^{-\rho}}{2\pi^2(e^{\rho}-2q)^3}\left\{1 - kT\frac{e^{\rho}}{q}(e^{\rho}-2q) \times \ln\left[e^{[qe^{-\rho}/kT(e^{\rho}-2q)]}-1\right]\right\}. \tag{12}$$

The source brightness is then derived by summing incoherently over the electron beam phase space; for Gaussians and N_e electrons:

$$\left\langle\frac{d^3N}{dtdqd\Omega}\right\rangle = \frac{\alpha^2 q^2 N_e}{16\pi^3}\int_0^{\infty} e^{-(\rho-\rho_0/\Delta\rho)^2}\frac{d\rho}{\sqrt{\pi}\Delta\rho}\int_0^{\pi} e^{-(\delta/\Delta\delta)^2}\frac{2\pi\delta d\delta}{\pi\Delta\delta^2}\int_{-1}^{1}dx\int_0^{2\pi}S(\rho,\delta,x,\psi,q)d\psi. \tag{13}$$

Here, $\rho_0 = \mathrm{argcosh}\gamma_0$, the rapidity spread $\Delta\rho = \Delta\gamma/\sinh\rho_0 = \Delta\gamma/\sqrt{\gamma_0^2 - 1}$, the angular spread $\Delta\delta = \varepsilon_n/\gamma_0\sigma_b$, where ε_n is the electron beam normalized emittance, and σ_b its focal size, $\psi = \varphi + \xi$, where φ and ξ are the azimuthal angles for the incident photons and electrons. The function S is obtained by generalizing Eqs. (9)–(11) to arbitrary scattering geometries. For $\varepsilon_n/\gamma_0\sigma_b \ll 1$, a Taylor expansion can be performed to integrate over the electron beam angular spread. The linear term integrates out over ψ; the quadratic term leads to a correction proportional to $(\varepsilon_n/\gamma_0\sigma_b)^2$: $\Delta\delta^2 S_{2\varepsilon}(q,\rho)$. This term can then be compared to the unperturbed on-axis spectral density $S_0(q,\rho)$. To illustrate, for $\gamma_0 = 1.5\times10^3$, if $\varepsilon_n = 1$ mm mrad, and $\sigma_b = 50\ \mu$m, one finds that $\sqrt{|S_{\varepsilon 2}/S_0|} \gtrsim 0.5$ mrad $\gg \Delta\delta = 13.3\ \mu$rad; this condition relaxes further at higher energy.

Energy spread is analyzed in Fig. 3, clearly indicating a degradation of the brightness as the energy spread increases.

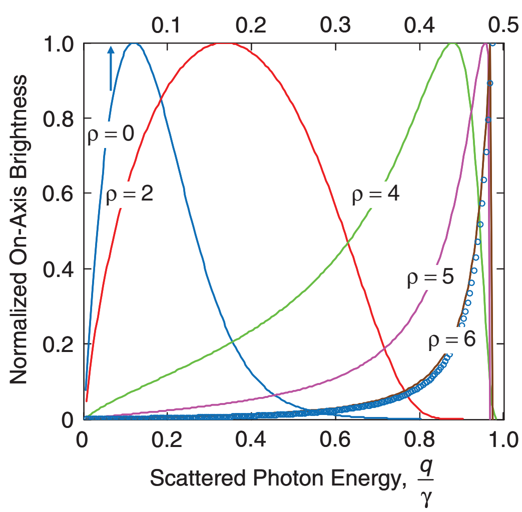

FIG. 2 (color). On-axis spectrum for different values of the rapidity. For $\rho = 6$, the exact (brown line) and dominant (blue circles) terms are compared. The blackbody temperature is 20 keV, and the scattered photon energy is normalized to the electron energy.

The γ-ray brightness on axis is remarkable: it exceeds all other sources, expect for the planned Linac Coherent Light Source (LCLS), and reaches GeV photon energies, 5 orders of magnitude beyond the LCLS range, as shown in Fig. 4.

The core blackbody photon density is so high that each injected electron can radiate away its kinetic energy via Compton scattering; for GeV electrons, the γ^{-1} cone angle may allow one to focus the γ rays to a minimum spot size given by ε_n/γ_0, using the mechanism described for the proposed γ-γ collider [13]. Since the γ-ray pulse duration is equal to the bunch length, the electromagnetic power density and maximum electric field at focus are very high:

$$E \sim \sqrt{\frac{1}{\varepsilon_0}\frac{q}{e}\gamma_0 m_0 c^2\frac{1}{c\Delta t}\left(\frac{\gamma_0}{\varepsilon_n}\right)^2}. \tag{14}$$

For 1 GeV, $q = 1$ nC, $\Delta t = 10$ fs, and $\varepsilon_n = 1$ mm mrad [14], $\Upsilon = eE\lambda_c/m_0c^2 = 0.287$; the electric field approaches the Schwinger critical field for pair creation in vacuum.

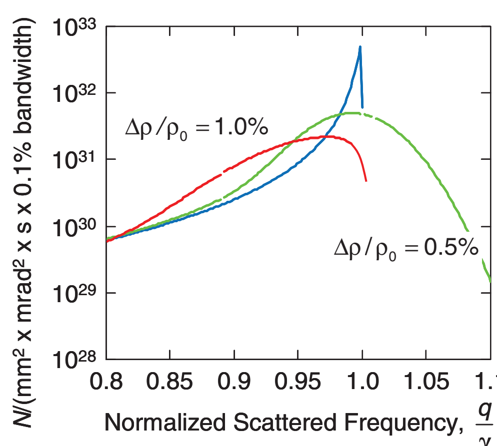

FIG. 3 (color). On-axis spectral brightness predicted for a 1 GeV, 1 nC, 10 fs, 1 mm mrad electron beam that could be produced by laser wakefield acceleration in the "cavitation" regime [14], for 3 values of the energy spread: 0% (blue curve), 0.5% (green curve), and 1% (red curve).

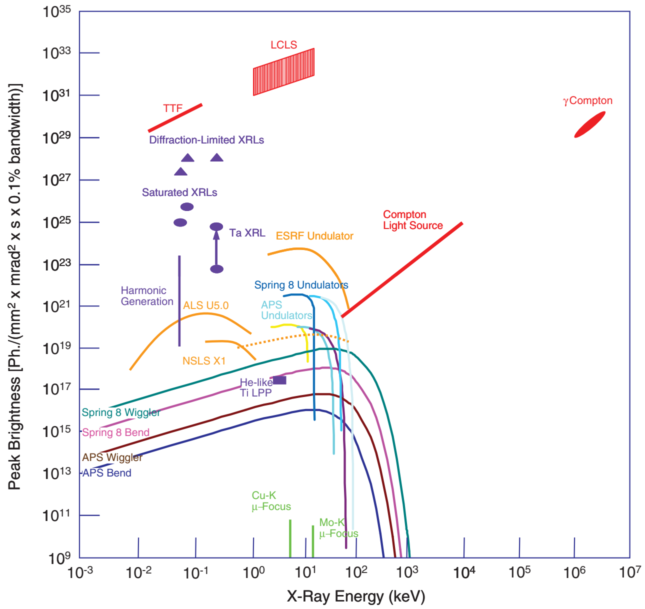

FIG. 4 (color). Peak brightness of various light sources, including the Compton γ-ray source based on the interaction between a relativistic (GeV) electron beam and a 20 keV ignited core.

Important considerations that may prevent attaining such energy densities include the fact that the megagauss magnetic fields produced in the core could prevent focusing down to the minimum size by effectively heating the electron beam. The angular deviations due to such fields can be estimated by using the relativistic gyrofrequency: $e\delta/\gamma_0 m_0 c \approx 15\ \mu\text{rad}\,\text{T}^{-1}$. Penetration into the core by GeV electron should not be problematic, as we have shown that their range is quite long; on the other hand, the ignited core is a much more complex physical system than a blackbody; advanced three-dimensional simulations, coupled to MHD, nuclear physics, and radiation transfer codes are required to fully model γ-ray production via Compton scattering.

It is also interesting to consider this radiation mechanism in relation with the Sunyaev-Zel'dovich effect [5], where relativistic electrons interacting with the cosmic microwave background effectively Comptonize the spectrum and induce deviations and fluctuations from a pure blackbody spectrum. Within this context, one of the main differences is the electron frame temperature of the blackbody, measured in QED units, $\gamma kT/m_0 c^2$. Recoil plays a major role for GeV electrons interacting with a 20 keV thermonuclear blackbody, leading to kinematic pileup, while only extremely high energy electrons ($\gamma m_0 c^2 \simeq 8.53 \times 10^{16}$ eV) would experience similar recoil from a 2.725 K blackbody.

In conclusion, a novel type of light source has been described, whereby a relativistic electron beam is injected into an ignited thermonuclear core, where Planckian blackbody radiation is sufficiently intense to interact very strongly via Compton scattering and generate extremely bright γ rays; kinematic pileup is shown to result in remarkably narrow spectra at high energy. This particular effect may have interesting ties with astrophysical phenomena [4,15], and the interaction of relativistic electrons with blackbody radiation in general is also connected with relativistic statistical mechanics [16] and QED in gravitational fields [17]. Though clearly highly challenging, an experimental realization could be performed at the National Ignition Facility.

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[1] J. Lindl, Phys. Plasmas **2**, 3933 (1995).
[2] A. H. Compton, Phys. Rev. **21**, 483 (1923); G. R. Blumenthal and R. J. Gould, Rev. Mod. Phys. **42**, 237 (1970).
[3] O. Klein and Y. Nishina, Z. Phys. **52**, 853 (1929).
[4] J. Schwinger, Phys. Rev. **82**, 664 (1951).
[5] Ya. B. Zeldovich and R. A. Sunyaev, Astrophys. Space Sci. **4**, 301 (1969).
[6] M. Planck, Ann. Phys. (Leipzig) **312**, 390 (1902).
[7] H. L. Anderson, *A Physicist's Desk Reference* (American Institute of Physics, New York, NY, 1989), 2nd ed., Chap. 16.
[8] M. Tabak, J. Hammer, M. E. Glinsky, W. L. Kruer, S. C. Wilks, J. Woodworth, E. M. Campbell, M. D. Perry, and R. J. Mason, Phys. Plasmas **1**, 1626 (1994).
[9] E. Wigner, Phys. Rev. **40**, 749 (1932); W. E. Brittin and W. R. Chappell, Rev. Mod. Phys. **34**, 620 (1962).
[10] G. Bhatt, H. Grotch, E. Kazes, and D. A. Owen, Phys. Rev. A **28**, 2195 (1983).
[11] L. M. Brown and R. P. Feynman, Phys. Rev. **85**, 231 (1952).
[12] G. B. Arfken and H. J. Weber, *Mathematical Methods for Physicists* (Academic, San Diego, CA, 1995), Chap. 1.15.
[13] B. Badelek *et al.*, Int. J. Mod. Phys. A **19**, 5097 (2004).
[14] W. P. Leemans, B. Nagler, A. J. Gonsalves, Cs. Tóth, K. Nakamura, C. G. R. Geddes, E. Esarey, C. B. Schroeder, and S. M. Hooker, Nature Phys. **2**, 696 (2006); J. Faure, C. Rechatin, A. Norlin, A. Lifschitz, Y. Glinec, and V. Malka, Nature (London) **444**, 737 (2006).
[15] D. Fargion and A. Salis, Phys. Usp. **41**, 823 (1998); B. A. Remington, D. Arnett, R. P. Drake, and H. Takabe, Science **284**, 1488 (1999).
[16] J. H. Eberly and A. Kujawski, Phys. Rev. **155**, 10 (1967).
[17] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975); W. G. Unruh and R. M. Wald, Phys. Rev. D **25**, 942 (1982).