Nonfactorized Calculation of the Process ^{3}He$(e, e'p)^2$H at Medium Energies

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The exclusive process ^{3}He$(e, e'p)^2$H has been analyzed by a nonfactorized and parameter-free approach based upon realistic few-body wave functions corresponding to the AV18 interaction and treating the rescattering of the struck nucleon within a generalized eikonal approximation. The results of our calculations, compared with recent JLab experimental data, show that the left-right asymmetry exhibits a clear dependence upon the final state interaction demonstrating the breaking down of the factorization approximation at "negative" and large ($\gtrsim$300 MeV/c) values of the missing momentum.

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Recent experimental data obtained at JLab on exclusive electro-disintegration of ^{3}He [1] are at present the object of intense theoretical activity (see [2–4] and references therein). In Refs. [2], the 2-body, ^{3}He$(e, e'p)^2$H, and 3-body, ^{3}He$(e, e'p)np$, breakup channels have been calculated within the following parameter-free approach: (i) initial state correlations (ISC) in the target nucleus ^{3}He have been taken care of by using state-of-the-art few-body wave functions obtained [5] by a variational solution of the Schrödinger equation containing realistic nucleon-nucleon interactions [6]; (ii) final state interactions (FSI) have been treated by a Generalized Eikonal Approximation (GEA) [7], which represents an extended Glauber approach (GA) based upon the evaluation of the relevant Feynman diagrams which describe the rescattering of the struck nucleon in the final state, in analogy with the Feynman diagrammatic approach developed for the treatment of elastic hadron-nucleus scattering [8,9]. In Ref. [2], the results of calculations of the two-body breakup (2BBU) channel have been compared with preliminary JLab data covering the region of "positive" (or "left") values of the missing momentum, i.e., the ones corresponding to $\phi = \pi$, where ϕ is the angle between the scattering and reaction planes. Subsequent published experimental data [1] covered, however, also the region of "negative" (or "right") values of the missing momentum, i.e., the ones corresponding to $\phi = 0$, which have not been considered in [2]. It is the aim of this Letter to analyze the process in the entire experimental kinematical range and, at the same time, to improve our theoretical approach. As a matter of fact, our previous calculations [2] were based upon the so called factorization approximation (FA) in which the cross section is written as a product of the free electron-nucleon cross sections and the nuclear spectral function. Whereas the FA is valid within the Plane Wave Impulse Approximation (PWIA), it has been shown that it is violated when FSI is taken into account (see, e.g., [10–16]). Within the FA, the ϕ-dependence of the cross section is only due to the ϕ-dependence of the elementary cross section for electron scattering off a moving nucleon [17]. Such a dependence is in clear contradiction with the experimental data of Ref. [1] at $p_m \gtrsim 0.35$ GeV/c, which is clear evidence of the breaking down of the FA. Performing nonfactorized calculations within relativistic and nonrelativistic models for complex nuclei is nowadays a common practice. Nonfactorized calculations for the process ^{3}He$(e, e'p)^2$H using realistic nonrelativistic three-body wave functions have only recently been performed [3,4], deriving the cross section in configuration space, with different prescriptions for the nonrelativistic reduction of the current operator. We have extended our previous approach [2] by relaxing the FA and avoiding the problem of nonrelativistic reductions by directly performing calculations in momentum space. We do not consider, for the time being, Meson Exchange Currents (MEC), Isobar Configurations, and similar effects, which have been the object of intensive theoretical studies; we fully concentrate on the effects of the FSI, treating initial and final state correlations, FSI, and the current operator within a parameter-free approach. By considering the general case of a target nucleus A, the relevant quantities which characterize the 2BBU process $A(e, e'p)(A-1)$ are the energy and momentum transfer ν and Q^2, respectively, the missing momentum $\mathbf{p}_m = \boldsymbol{q} - \boldsymbol{p}_1$ (i.e., the momentum of the recoiling system $A-1$), and the *missing energy* $E_m = \sqrt{P_{A-1}^2} + m_N - M_A = \nu - T_{A-1} - T_{\boldsymbol{p}_1} = |E_A| - |E_{A-1}| + E_{A-1}^*$. Here, $\boldsymbol{p}_1$ is the momentum of the detected proton and $E_A(E_{A-1})$ the (negative) ground state energy of the target (recoiling) nucleus and E_{A-1}^* the intrinsic excitation energy of the latter ($E_{A-1}^* = 0$ in the 2BBU channel we are going to consider). The cross section of the process has form

$$\frac{d^6\sigma}{d\Omega' dE' d^3\mathbf{p}_m} = \sigma_{\mathrm{Mott}} \sum_i V_i W_i^A(\nu, Q^2, \mathbf{p}_m, E_m) \quad (1)$$

where $i \equiv \{L, T, TL, TT\}$, V_i are kinematical factors [11], and the nuclear structure functions W_i^A result from proper combinations of the polarization vector of the virtual photon, ε_λ^μ, and the hadronic tensor, $W_{\mu\nu}^A$, the latter depending upon the nuclear current operators $\hat{J}_\mu^A(0)$. In the present Letter, we describe the two- and three-body ground states in terms of realistic wave functions generated by modern two-body interactions [5], and treat the final state interaction by a diagrammatic approach of the elastic rescattering of the struck nucleon with the nucleons of the $A-1$ system. We consider the interaction of the incoming virtual photon γ^* with a bound nucleon (the active nucleon) of low virtuality ($p^2 \sim m_N^2$) in the quasielastic kinematics, i.e., corresponding to $x \equiv Q^2/2m_N\nu \sim 1$. In quasielastic kinematics, the virtuality of the struck nucleon after γ^*-absorption is also rather low and, provided $\mathbf{p}_1$ is sufficiently high, nucleon rescattering with the "spectator" $A-1$ can be described to a large extent in terms of multiple elastic scattering processes in the eikonal approximation. In coordinate representation, the initial three- body wave function is $\Phi_{1/2,\mathcal{M}_3}(\boldsymbol{\rho},\boldsymbol{r})$, whereas the wave function of the final state is

$$\Psi_f = \hat{\mathcal{A}} S_\Delta^{\rm FSI}(\boldsymbol{\rho},\boldsymbol{r}) e^{i\mathbf{p}_1\mathbf{r}_1} e^{i\mathbf{P}_D\mathbf{R}_D} \Phi_{1,\mathcal{M}_2}(\boldsymbol{r})\chi_{1/2,s_f} \quad (2)$$

where $\boldsymbol{\rho}$, $\boldsymbol{r}$, and $\mathbf{R}$ are usual Jacobi coordinates, $\hat{\mathcal{A}}$ denotes a proper antisymmetrization operator, $\Phi^*_{1,\mathcal{M}_2}(\boldsymbol{r})$ is the deuteron wave function, and $\chi_{1/2,s_f}$ the spin wave function of the struck nucleon. The quantity

$$S_\Delta^{\rm FSI}(\boldsymbol{\rho},\boldsymbol{r}) = 1 + T_{(1)}^{\rm FSI}(\boldsymbol{\rho},\boldsymbol{r}) + T_{(2)}^{\rm FSI}(\boldsymbol{\rho},\boldsymbol{r}) \quad (3)$$

is the GEA S matrix, which describes both the case of no FSI, as well as single and double rescattering of the struck nucleon [2,7]. For ease of presentation, we will consider, from now on, the single rescattering contribution only, which has the form $T_{(1)}^{\rm FSI}(\boldsymbol{\rho},\boldsymbol{r}) = -\sum_{i=2}^{3}\theta(z_i - z_1)e^{i\Delta_z(z_i-z_1)}\Gamma(\mathbf{b}_1-\mathbf{b}_i)$, where $\boldsymbol{r}_i \equiv (z_i, \mathbf{b}_i)$. It can be seen that, unlike the usual GA, the GEA S matrix gets also a contribution from a parallel momentum Δ_z of pure nuclear origin and depending upon the external kinematics and the removal energy of the struck proton (within the "frozen approximation" $\Delta_z = 0$, and the usual Glauber profile is recovered). By assuming that the nuclear current operator is a sum of nucleonic currents $\hat{j}_\mu(i)$, its momentum space representation resulting from Feynman diagrams within GEA, can be written as follows

$$J_\mu^3 = \sum_\lambda \int \frac{d\mathbf{p}}{(2\pi)^3}\frac{d\boldsymbol{\kappa}}{(2\pi)^3} S_\Delta^{\rm FSI}(\boldsymbol{p},\boldsymbol{\kappa})\langle s_f|j_\mu(\boldsymbol{\kappa}-\boldsymbol{p}_m;\boldsymbol{q})|\lambda\rangle \mathcal{O}(\boldsymbol{p}_m-\boldsymbol{\kappa},\boldsymbol{p};\mathcal{M}_3,\mathcal{M}_2,\lambda) = J_\mu^{3({\rm PWIA})} + J_\mu^{3(1)} + J_\mu^{3(2)}, \quad (4)$$

where $S_\Delta^{\rm FSI}(\boldsymbol{p},\boldsymbol{\kappa}) = \int d\boldsymbol{r} d\boldsymbol{\rho} e^{-i\boldsymbol{p}\boldsymbol{r}} e^{i\boldsymbol{\kappa}\boldsymbol{\rho}} S_\Delta^{\rm FSI}(\boldsymbol{\rho},\boldsymbol{r})$, is the Fourier transform of the GEA S matrix [Eq. (3)], $\mathcal{O}$ the nuclear overlap in momentum space

$$\mathcal{O}(-\boldsymbol{k}_1 = \boldsymbol{p}_m - \boldsymbol{\kappa}, \boldsymbol{p}; \mathcal{M}_3, \mathcal{M}_2, \lambda) = \int d\boldsymbol{\rho} d\boldsymbol{r} e^{i(\boldsymbol{p}_m-\boldsymbol{\kappa})\boldsymbol{\rho}} e^{i\boldsymbol{p}\boldsymbol{r}} \Phi_{1/2,\mathcal{M}_3}(\boldsymbol{\rho},\boldsymbol{r})\Phi^*_{1,\mathcal{M}_2}(\boldsymbol{r})\chi^+_{1/2\lambda}, \quad (5)$$

and $\langle s_f|j_\mu(\boldsymbol{\kappa}-\boldsymbol{p}_m;\boldsymbol{q})|\lambda\rangle$ the nucleon current operator, λ and s_f being the spin projections of the struck proton before and after γ^* absorption. In absence of any FSI, the momentum space S matrix is $S_\Delta^{\rm FSI}(\boldsymbol{\kappa},\mathbf{p}) = (2\pi)^6\delta(\boldsymbol{\kappa})\delta(\mathbf{p})$ and only the PWIA contribution $J_\mu^{3({\rm PWIA})}$ survives in Eq. (4). When FSI is active, the contributions from single and double rescattering have to be taken into account; let us consider the former: it results from the single scattering momentum space term of $S_\Delta^{\rm FSI}(\boldsymbol{p},\boldsymbol{\kappa})$, which has the following form

$$T_{(1)}^{\rm FSI}(\boldsymbol{p},\boldsymbol{\kappa}) = -\frac{(2\pi)^4}{k^*}\frac{f_{NN}(\boldsymbol{\kappa}_\perp)}{\boldsymbol{\kappa}_\parallel + \Delta_z - i\varepsilon}\left[\delta\left(\boldsymbol{p}-\frac{\boldsymbol{\kappa}}{2}\right) + \delta\left(\boldsymbol{p}+\frac{\boldsymbol{\kappa}}{2}\right)\right] \quad (6)$$

where $f_{NN}(\boldsymbol{\kappa}_\perp)$, the Fourier transform of the profile function $\Gamma(\mathbf{b})$, represents the elastic scattering amplitude of two nucleons with center-of-mass momentum k^*. Placing Eq. (6) in Eq. (4), one obtains the single scattering contribution $J_\mu^{3(1)}$ to the nuclear current, *viz.*

$$J_\mu^{3(1)} = \sum_\lambda \int \frac{d\boldsymbol{\kappa}}{(2\pi)^2 k^*}\langle s_f|j_\mu(\boldsymbol{k}_1;\boldsymbol{q})|\lambda\rangle \frac{f_{NN}(\boldsymbol{\kappa}_\perp)}{\boldsymbol{\kappa}_\parallel + \Delta_\parallel - i\varepsilon}\left[\mathcal{O}\left(-\boldsymbol{k}_1, \frac{\boldsymbol{\kappa}_\Delta}{2}; \mathcal{M}_3, \mathcal{M}_2, \lambda\right) + \mathcal{O}\left(-\boldsymbol{k}_1, -\frac{\boldsymbol{\kappa}_\Delta}{2}; \mathcal{M}_3, \mathcal{M}_2, \lambda\right)\right]; \quad (7)$$

here, $\boldsymbol{k}_1$ is the momentum of the proton before γ^* absorption and $\boldsymbol{\kappa} = \boldsymbol{k}_1 + \boldsymbol{q} - \boldsymbol{p}_1 = \boldsymbol{k}_1 + \boldsymbol{p}_m$ the momentum transfer in the NN rescattering. The longitudinal part of the nucleon propagators has been computed using the relation $[\boldsymbol{\kappa}_\parallel + \Delta_z \pm i\varepsilon]^{-1} = \mp i\pi\delta(\boldsymbol{\kappa}_\parallel + \Delta_z) + PV[\boldsymbol{\kappa}_\parallel + \Delta_z]^{-1}$. Note that in the eikonal approximation, the trajectory of the fast nucleon is a straight line so that all "longitudinal" and "perpendicular" components are defined with respect to this trajectory, which means that the z axis has to be directed along the momentum of the detected fast proton. Looking at the structure of Eq. (7), it can be seen that due to the coupling of the nucleonic current operator $\langle s_f|j_\mu(\boldsymbol{k}_1;\boldsymbol{q})|\lambda\rangle$ with the nuclear overlap integral, a factorized form for Eq. (7) cannot be obtained; the same holds for the double-scattering contribution and, consequently, for the cross section. However, as shown in Ref. [2], if the longitudinal part of the nucleonic current can be disregarded, the factorization form can approximately be recovered. Using the above formalism, and including the contribution from double rescattering $J_\mu^{3(2)}$, the cross section [Eq. (1)] and the left-right asymmetry defined by

$$A_{TL} = \frac{d\sigma(\phi = 0^\circ) - d\sigma(\phi = 180^\circ)}{d\sigma(\phi = 0^\circ) + d\sigma(\phi = 180^\circ)} \quad (8)$$

have been calculated. Following de Forest's prescription [17], the "CC1" form of the nucleon current operator has been adopted; the elastic amplitude f_{NN} has been chosen in the usual form $f_{NN}(\boldsymbol{\kappa}_\perp) = k^* \frac{\sigma^{\rm tot}(i+\alpha)}{4\pi} e^{-b^2\boldsymbol{\kappa}_\perp^2/2}$, where the slope parameter b, the total nucleon-nucleon cross section $\sigma^{\rm tot}$ and the ratio α of the real to the imaginary parts of the forward scattering amplitude, were taken from world's experimental data.

The results of our calculations are shown in Figs. 1–3. In Fig. 1, the full nonfactorized cross section is compared with the PWIA result; Fig. 2 shows the nonfactorized and factorized cross sections; in Fig. 3, the asymmetry $A_{\rm TL}$ is exhibited. As in previous calculations of ours, no approximations have been made in the evaluation of the single and double-scattering contributions: intrinsic coordinates have been used and the energy dependence of the profile function has been taken into account in the properly chosen CM system of the interacting pair. In PWIA, the cross section is directly proportional to the two-body spectral function of ^{3}He. It can be seen from Fig. 1 that up to $|\mathbf{p}_m| \sim 400$ MeV/c, the PWIA and FSI results practically coincide and, moreover, fairly well agree with the experimental data; this means that the 2BBU process ^{3}He$(e, e'p)^2$H does provide information on the two-body spectral function; on the contrary, at larger values of $|\boldsymbol{p}_m| \gtrsim 400$ MeV/c, the PWIA appreciably underestimates the experimental data. It is however very gratifying to see that when the FSI is taken into account, the disagreement is strongly reduced and an overall good agreement between theoretical predictions and experimental data is obtained. It should be pointed out that at large values of the missing momentum, the experimental data correspond to a kinematics in which the deuteron momentum is always almost perpendicular to the momentum of the final proton; in this case, the effects of the FSI are maximized; it appears therefore that our approach is a correct one in describing FSI effects. The results presented in Fig. 2 clearly show that treating FSI within the FA in the region $\phi = 0$ and $|\boldsymbol{p}_m| \gtrsim 300$ MeV/c is a poor approximation, unlike what happens at $\phi = \pi$. It should be pointed out that in spite of the good agreement provided by the nonfactorized calculation, in some regions, quantitative disagreements with experimental data still persist. Particularly worth being mentioned is the disagreement in the region around $|\mathbf{p}_m| \simeq 0.6$–$0.65$ GeV/c at $\phi = 0$. The origin of such a disagreement can be better visualized by analyzing the left-right asymmetry. It is well known that when the explicit expres-

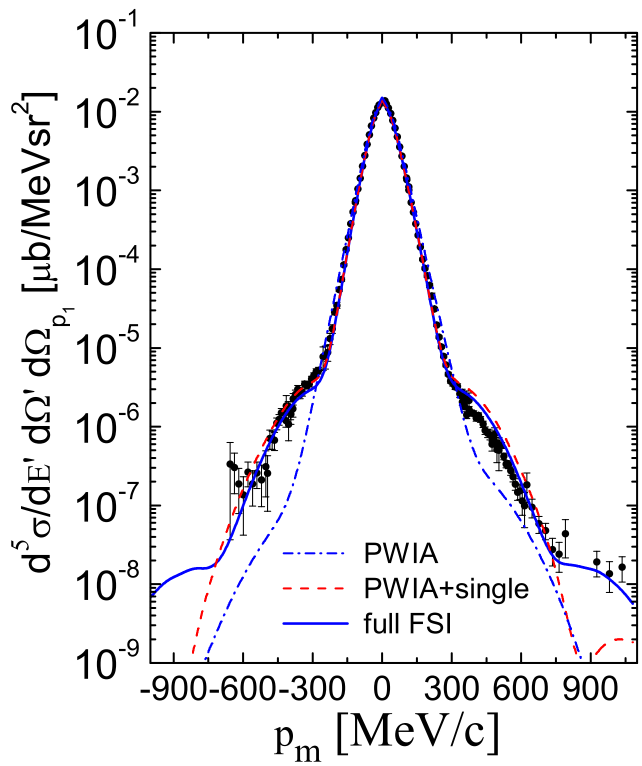

FIG. 1 (color online). The differential cross section for the process ^{3}He$(e, e'p)^2$H calculated within the nonfactorized approach compared with the PWIA result. *Dot-dashed line*: PWIA; *dashed line*: PWIA plus single rescattering ; *solid line*: PWIA plus single and double rescattering. Experimental data from Ref. [1].

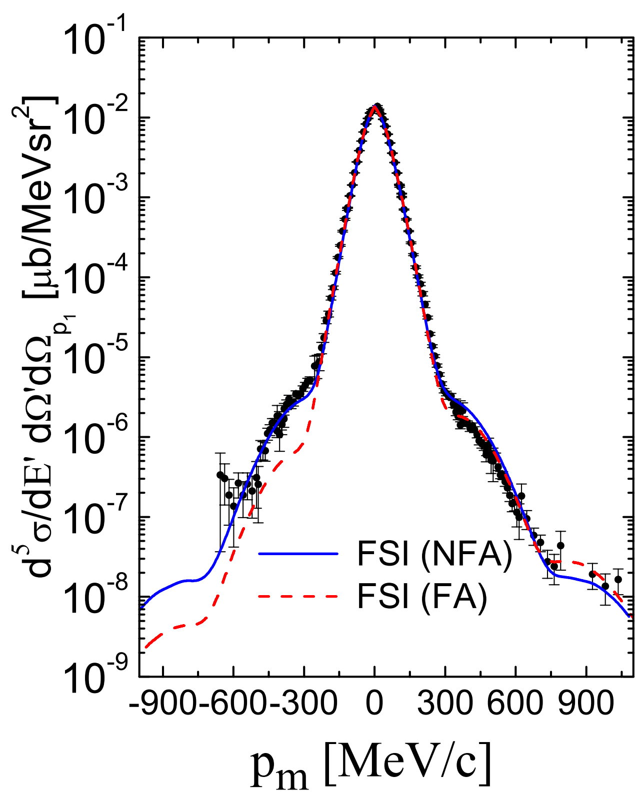

FIG. 2 (color online). The differential cross section for the process ^{3}He$(e, e'p)^2$H calculated taking into account FSI within the nonfactorized approach [FSI(NFA)] compared with the results which include FSI within the factorized approximation [FSI(FA)]. Experimental data from Ref. [1].

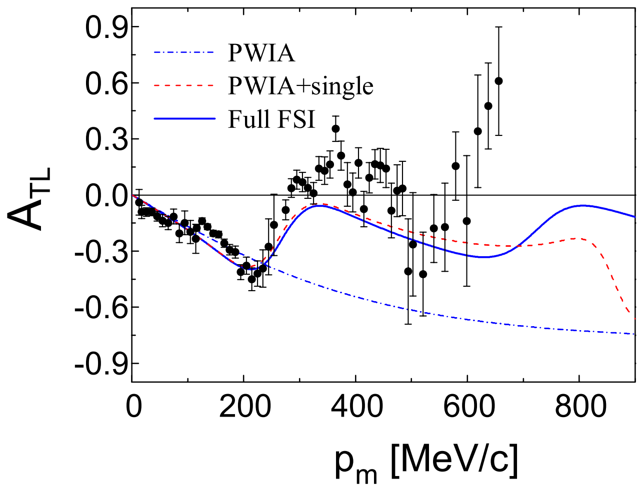

FIG. 3 (color online). The transverse-longitudinal asymmetry for the process ^{3}He$(e, e'p)^2$H. *Dot-dashed line*: PWIA; *dashed line*: PWIA plus single rescattering FSI; *solid line*: PWIA plus single and double rescattering FSI. Experimental data from Ref. [1].

sions of V_i and W_i^A are used in Eq. (8), the numerator is proportional to the transverse-longitudinal response $W_{\rm TL}$, whereas the denominator does not contain $W_{\rm TL}$ at all, which means that $A_{\rm TL}$ is a measure of the relevance of the transverse-longitudinal response relative to the other responses. In the eN cross section, the behavior of the asymmetry $A_{\rm TL}$ is known to be a negative and decreasing function of the missing momentum [17], and the same behavior should be expected in eA scattering within the PWIA. The results presented in Fig. 3 clearly show that at $p_m \leq 250$ MeV$/c$, the PWIA result is in reasonable agreement with the experimental data; however, with increasing p_m, the disagreement between the PWIA and the experimental data appreciably increases, which seems to be a common feature of all calculations so far performed using microscopic three-body nonrelativistic wave functions [1,3]; the origin of such a disagreement is at the moment unclear (Meson Exchange currents considered in [1,3] do not seem to solve this problem). In conclusion, we have calculated in momentum space the cross section of the processes ^{3}He$(e, e'p)^2$H, using realistic ground state two- and three-body wave functions and treating the FSI of the struck nucleon with the spectator nucleon pair within the generalized eikonal approximation. The method we have used is a very transparent and parameter-free one: it only requires the knowledge of the nuclear wave functions, since the FSI factor is fixed directly by NN scattering data. At the same time, calculations are very involved mainly because of the complex structure of the wave function of Ref. [5], which has to be first transformed to momentum space and then used in calculations of multidimensional integrals, including also the computation of principal values. The main results of our calculations can be summarized as follows: (i) the violation of the factorization approximation is appreciable at negative values ($\phi = 0$) of $|\boldsymbol{p}_m| \gtrsim 300$ MeV$/c$, whereas the nonfactorized and factorized results are in much better agreement in the whole range of positive values ($\phi = \pi$) of $|\boldsymbol{p}_m|$; (ii) the left-right asymmetry can reasonably be reproduced at low values of the missing momentum, but a substantial discrepancy between theoretical calculations and experimental data remains to be explained at high values of $|\boldsymbol{p}_m|$.

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[1] M. M. Rvachev *et al.*, Phys. Rev. Lett. **94**, 192302 (2005).
[2] C. Ciofi degli Atti and L. P. Kaptari, Phys. Rev. C **71**, 024005 (2005); Phys. Rev. Lett. **95**, 052502 (2005).
[3] R. Schiavilla, O. Benhar, A. Kievsky, L. E. Marcucci, and M. Viviani, Phys. Rev. C **72**, 064003 (2005).
[4] J.-M. Laget, Phys. Rev. C **72**, 024001 (2005).
[5] A. Kievsky, S. Rosati, and M. Viviani, Nucl. Phys. A **551**, 241 (1993); (private communication).
[6] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995).
[7] L. L. Frankfurt, M. M. Sargsian, and M. I. Strikman, Phys. Rev. C **56**, 1124 (1997); M. M. Sargsian, Int. J. Mod. Phys. E **10**, 405 (2001).
[8] V. N. Gribov, Sov. Phys. JETP **30**, 709 (1970).
[9] L. Bertocchi, Nuovo Cimento A **11**, 45 (1972).
[10] J. M. Udias, Phys. Rev. C **48**, 2731 (1993).
[11] S. Boffi, C. Giusti, and F. D. Pacati, Phys. Rep. **226**, 1 (1993).
[12] J. J. Kelly, Adv. Nucl. Phys. **23**, 75 (1996).
[13] S. Jeschonnek and T. W. Donelly, Phys. Rev. C **57**, 2438 (1998).
[14] S. Janssen, J. Ryckebusch, W. Van Nespen, and D. Debruyne, Nucl. Phys. A **672**, 285 (2000).
[15] S. Jeschonnek, Phys. Rev. C **63**, 034609 (2001).
[16] J. R. Vignote *et al.*, Phys. Rev. C **70**, 044608 (2004); J. R. Vignote *et al.*, in *Nuclear Theory '23*, S. Dimitrova (Heron Press, Sofia, 2004), p. 84.
[17] T. de Forest Jr., Nucl. Phys. A **392**, 232 (1983).