# Entanglement on Demand through Time Reordering

J. E. Avron, G. Bisker, D. Gershoni, N. H. Lindner, and E. A. Meirom
*Department of Physics, Technion-Israel Institute of Technology, 32000 Haifa, Israel*

R. J. Warburton
*School of Engineering and Physical Sciences, Heriot-Watt University, Edinburgh EH14 4AS, United Kingdom*
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Entangled photons can be generated ''on demand'' in a novel scheme involving unitary time reordering of the photons emitted in a radiative decay cascade. The scheme yields polarization entangled photon pairs, even though prior to reordering the emitted photons carry significant ''which path information'' and their polarizations are unentangled. This shows that quantum chronology can be manipulated in a way that is lossless and deterministic (unitary). The theory can, in principle, be tested and applied to the biexciton cascade in semiconductor quantum dots.

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Entangled quantum states are an important resource in quantum information and communication [1]. Entangled photons are particularly attractive for applications due to their non interacting nature and the ease by which they can be manipulated. There is, therefore, considerable interest in the development of sources for reliable (nonrandom) polarization entangled photon pairs. Currently, the most important and practical source of polarization entangled pairs is down conversion [2,3] which has a large random component. Furthermore, since the entanglement is created by a coincidence detection of the pair, the entangled state becomes unavailable for further manipulations.

Quantum dots are a source of single photons ''on demand'' [4–6]. Recently, it has been shown [7,8] that they can be used as sources of polarization entangled photon pairs. The entangled photon pair is obtained from the decay cascade of a biexciton in a quantum dot. The biexciton has two decay channels, each emitting a photon pair with a polarization characteristic to the channel. Perfect ''which path ambiguity'' requires that the intermediate exciton state is doubly degenerate as illustrated in Fig. 1(a). In this idealized setting, the first generation photons have identical colors (energies) and the second generation photons also have identical, though in general different, colors. With perfect ''which path ambiguity'' the state of polarization of the two photons is maximally entangled, and each pair can, in principle, be produced ''on demand'' [9].

Quantum dots do not have perfect cylindrical symmetry and this lifts the degeneracy of the intermediate exciton states [10]. We shall refer to this splitting as ''detuning.'' Since the detuning is large (compared with the radiative width), the two decay paths are effectively distinguished by the distinct colors of the emitted photons. This adversely affects the ''raw'' entanglement which is then negligible [7].

In principle, the detuning can be manipulated by Stark and Zeeman effects, by stress, etc., and much experimental effort has been devoted into reducing it to small values (below the radiative width) [11]. This has been an elusive goal so far for both practical and fundamental reasons. The fundamental reason is that quantum mechanics has the principle of level repulsion: In quantum dots the scale of energy responsible for the detuning (exchange) dominates the scale of the radiative width which is the smallest energy scale in the problem [10,12]. This puts an ''in principle'' obstacle to substantial ''which path ambiguity'' in quantum dots.

Entangled photons from quantum dots have been obtained by selectively filtering the photons that conform to the which path ambiguity [7]. The entanglement then comes at the price that a substantial fraction of the photon pairs are lost and the quantum dot does not furnish entangled pairs ''on demand.''

An alternate strategy proposed in [13,14] is to tune the dot spectrum to have coincidence of colors *across* generations, rather than *within* generations. This is illustrated in Fig. 1(b) where the colors of the photons in the first generation match, in pairs, the colors of the photons in the second generation. Since a coincidence of colors in different generations does not require degeneracy, there is no *fundamental* obstacle to tuning the level diagram to a

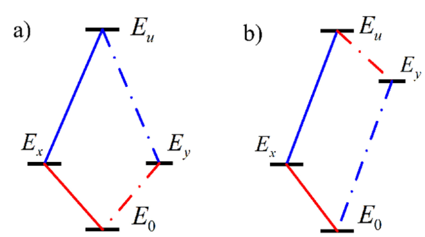


FIG. 1 (color online). Two alternative level schemes that can be used to generate entangled pairs. The solid (dashed-dotted) line represents the decay channel that yields two $x$ ($y$) polarized photons. The energies correspond to the colors of the emitted photons. (a) Represents the situation where photon colors match in the first generation, and has the geometry of a kite (deltoid). (b) Represents the case where colors in different generations match, and is geometrically a parallelogram.

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precise coincidence of colors [12–14]. When this is the case the two decay channels are identical up to time ordering. The different time ordering of the two decay paths of the raw state betray the path which, as it turns out, completely kills the entanglement. In order to regain entanglement one needs to manipulate the time ordering. The theory of reordering the quantum chronology is developed in this Letter. It allows us to derive the measure of entanglement of the reordered state and its dependence on the spectral properties of the radiative cascade. Perhaps the most important result is that it shows that the reordering can be made in a way that conforms with the requirement of on demand. The theory then also allows an optimization of the entanglement and it leads to a proposal of a practical experimental realization.

We denote by $\Delta$ the detuning, the (dimensionless) measure of the color matching of the photons in a given generation. Perfect matching within generation is represented by $\Delta = 0$. Similarly, we denote by $\beta$ the (dimensionless) spectral control which measures the matching of the colors across generations (the biexciton binding energy in a quantum dot). Perfect cross color matching in this case is represented by $\beta = 0$. More precisely

$$\Delta = \frac{E_y - E_x}{2\Gamma}, \qquad \beta = \frac{E_u - E_0 - E_x - E_y}{2\Gamma}. \quad (1)$$

$E_x$, $E_y$, $E_u$ are as in Fig. 1 and $\Gamma$ is the half width of the intermediate levels. As we have explained, in dots there are fundamental reasons that force $|\Delta| \geq 1$, while $\beta$ can, in principle, be tuned to zero. The issue is, can one generate entangled photon pairs by tuning $\beta = 0$ even when $|\Delta| \gg 1$? As we show, the answer is yes.

Suppose that the two photons are emitted along the $z$ axis and have two decay modes with equal amplitudes. Then, the (possibly un-normalized) photon wave function has the form [15]:

$$|\psi\rangle = \sum_{j=x,y} |\alpha_j\rangle \otimes |jj\rangle, \quad (2)$$

where $|\alpha_j\rangle$ describes the photons' wave packet and $|jj\rangle$ their state of polarization.

For reasons that will become clear below we need to allow for a unitary post processing of the raw state emerging from the cascade. The manipulation, $U_j$, is described by a unitary operation that depends on the polarization state (i.e., the decay channel) and is then formally of the form

$$|\alpha_j\rangle \to U_j|\alpha_j\rangle. \quad (3)$$

In the language of quantum information this corresponds to applying single qubit unitary gates on each of the two polarization states (this operation can be made by whomever prepares the state, but can also be made later and so falls under the class of local operations [16]).

As a measure of the entanglement we take the absolute value of the negative eigenvalue in the Peres test (negativity) [17,18]

$$\gamma(\Delta, \beta; W) = \frac{|\langle\alpha_x|W|\alpha_y\rangle|}{\langle\alpha_x|\alpha_x\rangle + \langle\alpha_y|\alpha_y\rangle}, \qquad W = U_x^* U_y. \quad (4)$$

The maximal value of $\gamma$ is $\frac{1}{2}$ corresponding to maximally entangled (Bell) states. Note that the denominator is just the normalization of the state $|\psi\rangle$.

The $|\alpha_j\rangle$ are fully determined by the (complex) energies of the level diagram, $Z_a = E_a - i\Gamma_a$. In the limit that the dipole approximation holds, the (normalized) wave packets are given by [15,19]:

$$\langle k_1, k_2|\alpha_j\rangle = A(|k_1| + |k_2|, Z_u)[A(|k_1|, Z_j) + A(|k_2|, Z_j)], \quad (5)$$

where,

$$A(k, Z) = \sqrt{\frac{\Gamma}{\pi}}\frac{1}{k - Z}. \quad (6)$$

(We use units where $\hbar = c = 1$.) The photon of the first generation has energy near $E_u - E_j$ while the photon of the second generation has energy near $E_j$ and relative time delay of order $1/\Gamma_j$. Note that positive (negative) delay is associated with $Z_j$ in the lower (upper) half complex energy plane.

In practice, the smallest energy scale in the problem is the radiative width, $\Gamma$. We treat it as a small parameter in the theory and thus can safely drop the absolute value in Eq. (5). This allows for an analytic calculation of some of the integrals that arise. In particular, when the two emitted photons have different colors, i.e., when $||k_1| - |k_2|| \gg \Gamma$, one has

$$\langle\alpha_j|\alpha_j\rangle = 2 \quad (7)$$

to leading order in $\Gamma$.

The numerator in Eq. (4), $\langle\alpha_x|W|\alpha_y\rangle$ can now be written as a sum of the two integrals:

$$y_1 = \frac{2\Gamma\Gamma_u}{\pi^2}\int\frac{W(k_1, k_2)dk_1 dk_2}{|k_1 + k_2 - Z_u|^2(k_1 - Z_x^*)(k_2 - Z_y)} \quad (8)$$

and

$$y_2 = \frac{2\Gamma\Gamma_u}{\pi^2}\int\frac{W(k_1, k_2)dk_1 dk_2}{|k_1 + k_2 - Z_u|^2(k_1 - Z_x^*)(k_1 - Z_y)}, \quad (9)$$

where $\Gamma_x = \Gamma_y = \Gamma$. The first term, $y_1$, may be thought of as the contribution to the entanglement from the coincidence of colors across generations while the second term, $y_2$, as the contribution to entanglement from the matching of colors within a generation.

Consider first the raw outgoing state where $W = 1$. One expects that entanglement can arise only from matching colors within a generation. By first shifting the integration variables, $k_1 - E_x \to k_1$ and $k_2 - E_y \to k_2$ in Eq. (8), and using the residue theorem to compute the integrals, one indeed finds that the cross-generations contribution vanishes:

$$y_1(\beta; W = 1) = 0. \tag{10}$$

For the second integral, shifting $k_j \to k_j + \frac{E_x+E_y}{2}$ first, and using similar elementary manipulations, one finds

$$y_2(\Delta; W = 1) = \frac{-2i}{\Delta - i}. \tag{11}$$

In particular, combining with Eqs. (4) and (7), we find

$$\gamma^2(\Delta, \beta; 1) = \frac{1}{4(\Delta^2 + 1)}. \tag{12}$$

The raw entanglement is *independent* of $\beta$ (to leading order in $\Gamma$) and does not benefit from matching the colors of photons in different generations (tuning to $\beta = 0$). Since typically $|\Delta| \gg 1$, the raw entanglement is small.

Fortunately, a suitable choice of $W$ can yield an entangled state of polarization even when $|\Delta| \gg 1$. Since $W$ is unitary, $W(k_1, k_2)$ is a phase. The optimal choice of phase is one that would make $y_1$ as large as possible. This can be achieved when the integrand in Eq. (8) has a fixed phase [so that the oscillations leading to the cancellation in Eq. (10) are eliminated], e.g.,

$$W_{\rm opt} = -\frac{(k_1 - Z_x^*)(k_2 - Z_y)}{|k_1 - Z_x^*||k_2 - Z_y|}. \tag{13}$$

$W_{\rm opt}(E_x, k_2 - E_y)$ is plotted in Fig. 2(a).

By inserting Eq. (13) into Eq. (8) and shifting the integration variables, this choice for $W_{\rm opt}$ gives for $y_1$

$$y_1 = \frac{2}{\pi^2}\int \frac{g}{|k_1 + k_2 - \beta - ig|^2}\frac{dk_1 dk_2}{|k_1 - i||k_2 + i|}, \tag{14}$$

where $g = \Gamma_u/\Gamma$. Let us study the case of $\beta = 0$, where this function (which is even in $\beta$) achieves its maximum. Combining this with Eq. (4) and (7), one finds for the optimal $W$

$$\gamma(g; |\Delta| \gg 1, \beta = 0; W_{\rm opt}) = \tfrac{1}{2} + f(g), \tag{15}$$

which is plotted, as a function of $g$ for perfect color matching, $\beta = 0$, in Fig. 2(b). The function $f(g)$ is negative, monotonically decreasing and vanishes for $g = 0$, where maximal entanglement, $\gamma = \frac{1}{2}$, is achieved. For systems such as the biexciton radiative cascades, one cannot get maximal entanglement even when the colors perfectly match, since $g \approx 1.5$–2; however, one does get substantial entanglement, $\gamma \approx 0.4$.

The entanglement $\gamma$ is, of course, sensitive to the matching of colors across generations, so that when $\beta \gg 1$ the entanglement becomes small. This is illustrated in Fig. 2(c) which shows $\gamma$ as a function of $\beta$ for $g = 2$, the value relevant to biexciton decay.

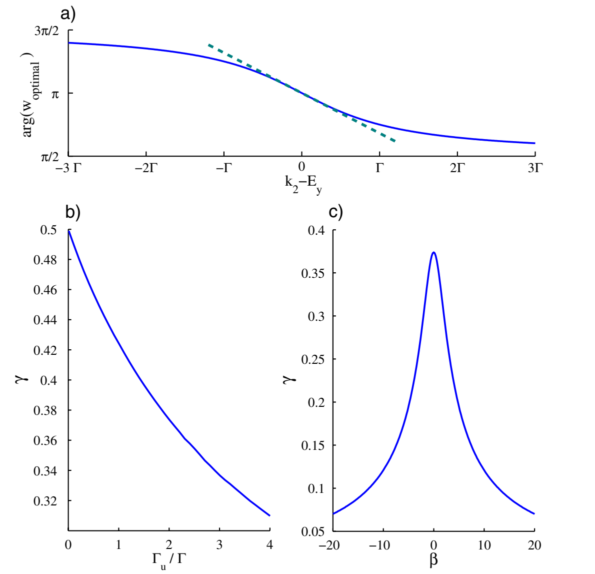


FIG. 2 (color online). (a) The argument of $W_{\rm opt}$ as a function of $k_2 - E_y$ for $k_1 = E_x$. The dashed line represents a feasible linear approximation, generated by an optical delay. (b) The off diagonal matrix element $\gamma$ for $W_{\rm opt}$, as a function of the ratio $\Gamma_u/\Gamma$. For typical biexciton decays, $\Gamma_u/\Gamma \approx 2$. (c) $\gamma$ as a function of the color matching dimensionless parameter, $\beta$, for $\Gamma_u/\Gamma = 2$. The maximal value occurs when $\beta = 0$. The width of the peak is of order $\Gamma$.

Physically, choosing $W$ may be thought of as letting the two polarizations go through different gates that introduce different, but fixed time delays on the two colors. To see this, we note first that each of the two factors of Eq. (13) can indeed be interpreted as a time shift. This follows from the fact that

$$\frac{k - Z}{|k - Z|} \approx ie^{-ik/\Gamma}e^{iE/\Gamma} \tag{16}$$

for $k \approx E$; see Fig. 2(a). This represents a shift of the wave function in coordinate space by $1/\Gamma$ which can also be interpreted as a (nonrandom) shift in time by $1/\Gamma$. Therefore, the two factors in $W_{\rm opt}$ may be implemented, approximately, by manipulating the optical paths.

It is important to understand that the manipulation of the quantum chronology proposed here is a fixed unitary manipulation of the wave function, which is a nonrandom object. It is not a manipulation of the individual detection events which are random and uncontrollable. However, the probability distribution for these random events is determined by the wave function according to the rules of quantum mechanics.

From Eq. (13), we see that $W_{\rm opt}$ has two factors, affecting the two photons. Figure 3 explains why both photons must be manipulated: to yield entangled photons, all the properties distinguishing the $x$ and $y$ polarized photons must be erased. This requires that: the arrival times at the detector $D$ of photons with energies $E_x$ (the red photons in

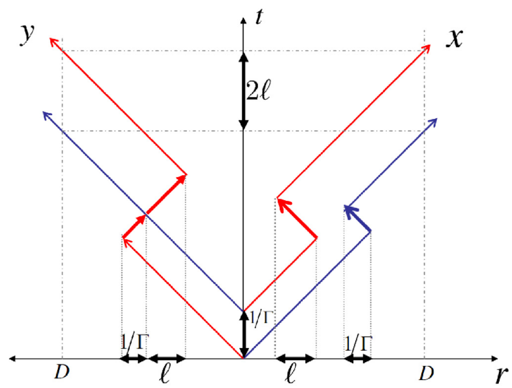


FIG. 3 (color online). A space-time diagram representing the path of the photons. For clarity, $x$ and $y$ polarized photons are drawn as propagating to the left and right, respectively. A delay is represented by reflecting a photon back in space. An observer located a distance $D$ from the origin cannot distinguish between the $x$ and $y$ polarized pairs, neither by their arrival times nor by their energies (for $\beta = 0$). $\ell$ is an arbitrarily chosen distance, which determines the *average* time difference between the actual detection of the first and second photons.

Fig. 3) must be independent of polarization, and likewise for the photons with energy $E_y$ (the blue photons in the figure).

Suppose that we let the $y$-polarized photon emitted *first* (red arrow on left side of the figure), travel a distance longer by $\ell + 1/\Gamma$ from the photon emitted *second* (blue on the left) thereby reversing their order ($\ell \geq 0$ is arbitrary). Now the time order (chronology) of both polarizations agrees, the first photon is blue and the second red. However, the which path information is not yet erased. For the red photon to arrive at the same time, the $x$-polarized photon emitted *second* must be delayed by a distance $\ell$. For the blue photon to arrive at the same time, the $x$-polarized photon emitted *first* must be delayed by a distance $1/\Gamma$. The average delay between the time of arrival of the blue and red photon is then $2\ell$.

The extra optical paths can be represented as unitary gates acting on the photon states by $e^{ikL}$, where $L$ is the path length. The implementation of the delays described above is given by

$$U_x = e^{ik_2/\Gamma} e^{ik_1\ell}, \qquad U_y = e^{ik_1(\ell + 1/\Gamma)}. \tag{17}$$

It follows that $W = U_x^* U_y = e^{ik_1/\Gamma} e^{-ik_2/\Gamma}$ which, by Eq. (16), approximates $W_{\rm opt}$.

From the above discussion, we see that although an exact $W_{\rm opt}$ transformation, Eq. (13), may be an experimental challenge, it should be possible to implement suitable approximations. A possible optical setting is depicted in Fig. 4.

In summary, entanglement can be created by a noninvasive (unitary) manipulation of the quantum chronology. This provides a possible and practical avenue for creating entangled photon pairs on demand.

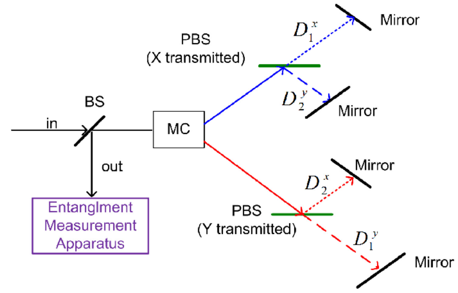


FIG. 4 (color online). An optical setup which introduces the appropriate delays to each of the photons. BS (PBS) stands for a (polarizing) beam splitter, and MC for a monochromator. The MC (approximately linear) dispersion can be set to obtain an approximation to $W_{\rm opt}$ of Eq. (13). The photons pass the first beam splitter and MC, are reflected back by the mirrors and after passing through the MC again, they are measured. The optical path are chosen as $2D_1^y = D + 1/\Gamma + \ell$, $2D_2^y = D$, $2D_1^x = D + 1/\Gamma$, and $2D_2^x = D + \ell$, where $D$ is arbitrary.

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