

Multifrequency Optical Invisibility Cloak with Layered Plasmonic Shells

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Here, we theoretically suggest the possibility of employing a multilayered plasmonic shell as a cloak for reducing the total scattering cross section of a particle, simultaneously at different frequencies in the optical domain. By exploiting the frequency dispersion of plasmonic materials and their inherent negative polarizability, it is shown, theoretically and with numerical simulations, how covering a dielectric or conducting object of a certain size with this multilayered cloak may reduce its “visibility” by several orders of magnitude simultaneously at multiple frequencies.

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Exploiting metamaterials and plasmonic media for cloaking and invisibility applications has been recently investigated by several groups, introducing novel transparency and cloaking techniques [1–8] that may have significant impact in various technological applications. In particular, our original idea [1] for using plasmonic and metamaterial cloaks to drastically reduce the total scattering cross section of a given object by relying on an inherently nonresonant scattering cancellation phenomenon [2] is fairly robust to geometry and frequency variations, and it may be envisioned at optical frequencies by using conventional plasmonic materials [9], or at lower frequencies by utilizing metamaterial technology [3].

One of the peculiar properties of plasmonic materials and metamaterials resides in their negative local polarizability, which is the main mechanism behind the cloaking phenomenon for reduction of total scattering described in [1–3]. In a certain range of frequencies, the polarization vector in a plasmonic material may indeed be oppositely oriented (i.e., antiparallel) with respect to that in a dielectric, implying that a dipole moment of opposite phase may be induced in a properly designed plasmonic shell around a dielectric (or even conducting [2]) object. This allows the possibility of suppressing a large fraction of the scattered wave from a moderately sized object, and therefore dramatically reducing its overall visibility, which may be of interest in many different fields and applications.

Other different proposals for achieving cloaking with the use of metamaterials have been recently suggested, relying on very distinct physical mechanisms, like coordinate transformation techniques [4–7] or on anomalous localized metamaterial resonances [8]. In particular, their functionality implies a rerouting of the impinging light around a given region of space in the cloak volume, effectively avoiding penetration of waves into the cloaked region on one hand and reducing scattering in the outside background on the other. These techniques are usually designed to work at a single resonant frequency and for specific polarizations, and they usually require the use of complicated inhomogeneity and anisotropy profiles and relatively bulky covers.

In comparison, the scattering cancellation technique proposed in [1–3] is inherently nonresonant [2], and therefore its bandwidth of operation may be less narrow, being essentially limited just by the dispersion of plasmonic materials. Still, the functionality of the setup proposed in [1–3] is limited to a single frequency range since, to satisfy the energy consideration, causality and Kramers-Kronig relations, passive plasmonic materials and metamaterials with low or negative values of permittivity are indeed required to be dispersive [10]. What is interesting, however, is that, as we show in the following, such frequency dispersion may be exploited to our advantage, if we wish to design multilayered cover shells that may cloak the object of interest at different frequencies.

Our earlier results [1,2] may be extended to a multilayered scenario by applying the well known Mie theory to a multilayered spherical object. Following an analytical approach similar to that of Ref. [11], the Mie scattering coefficient of order n for the case at hand, relative to the amplitude of the n th multipolar scattering of the system, may be written as

$$c_n^{\text{TM}} = -\frac{U_n^{\text{TM}}}{U_n^{\text{TM}} + iV_n^{\text{TM}}}, \quad (1)$$

for the TM polarization (and in an analogous way for TE spherical harmonics), where an $e^{-i\omega t}$ time convention is assumed. The coefficients U_n and V_n are real in the limit of negligible losses, and they may be evaluated by extending the results of [11] to an N -layer cover, as the determinants of $(2N+2) \times (2N+2)$ matrices resulting from fulfilling the boundary conditions of a multilayered spherical geometry [12] (their expressions are not reported here for the sake of brevity). The total scattering from the cloaked system, which determines its overall visibility, is determined by all the coefficients given in Eq. (1) for the different multipolar orders, all with proper weights.

As an example, consider a spherical object of permittivity ϵ and radius a covered by a two-layered spherical cloak with radii $a_{\text{out}} > a_{\text{in}} > a$ and permittivities ϵ_{in} , ϵ_{out} , as depicted in the inset of Fig. 1. In the limit of relatively small objects, consistent with Refs. [1,2], the condition

$U_n^{\text{TM}} = 0$, which ensures $c_n^{\text{TM}} = 0$, i.e., a zero scattering contribution from the n th spherical harmonic, may be obtained under the generalized approximate condition:

$$\eta_{\text{out}} \simeq \eta_{\text{in}} \sqrt{\frac{[(n+1)\epsilon_{\text{out}} + n\epsilon_0]\{\eta_{\text{in}}^{2n+1}(\epsilon_{\text{out}} - \epsilon_{\text{in}})[(n+1)\epsilon_{\text{in}} + n\epsilon] + (\epsilon_{\text{in}} - \epsilon)[(n+1)\epsilon_{\text{in}} + n\epsilon_{\text{out}}]\}}{(\epsilon_{\text{out}} - \epsilon_0)\{n(n+1)(\epsilon_{\text{in}} - \epsilon)(\epsilon_{\text{out}} - \epsilon_{\text{in}}) + \eta_{\text{in}}^{2n+1}[(n+1)\epsilon_{\text{in}} + n\epsilon][(n+1)\epsilon_{\text{out}} + n\epsilon_{\text{in}}]\}}}. \quad (2)$$

In this small-radii limit the dipolar scattering order is dominant, and therefore Eq. (2) with $n = 1$ ensures a generalized transparency condition for small three-layered spherical objects. For relatively larger objects, the full-wave formulas like Eq. (1) should be employed, leading to the possibility of obtaining a drastic reduction of scattering from objects with size comparable with the wavelength of operation. Even for larger objects, a proper design of the parameters for canceling the dominant scattering orders may provide a drastic reduction in the total scattering of the object.

The extra degrees of freedom available here, thanks to the addition of the extra layer, may allow designing the cloak for different frequencies of operation: by properly tailoring the plasma frequencies of the two plasmonic materials composing the cloak, it may indeed be possible to design a proper cover that suppresses the overall scattering from the object simultaneously at distinct frequencies. The idea behind this possibility resides in the fact that each one of the layers is designed to show its plasmonic behavior in a different frequency window. Extension of this technique to multiple layers is straightforward, and it

would allow an even larger flexibility in the design of the cloak for multiple frequencies of operation and possibly larger frequency bandwidths. The main limitation in the total number of layers may come from the technological realization of thin concentric layers, combined with the fact that the size of the total cloaked system should be kept sufficiently small in order to avoid significant excitation of higher-order scattering harmonics. Within the limitations of current nanotechnology, cloaking simultaneously at several different frequencies may indeed be a realistic possibility.

As an example of the potentials of this technique, here we have designed a two-layered cloak capable of making a nanoparticle with permittivity $\epsilon = 3\epsilon_0$ and diameter $2a = 200$ nm “invisible” simultaneously at the free-space wavelengths of $\lambda_0 = 500$ nm and $\lambda_0 = 625$ nm. It should be underlined here that the possibility of realizing multilayered nanoparticles is indeed within the realms of current nanotechnology [13]. Because of the various additional degrees of freedom offered by the extra cloaking layer, in the design of this cover we assume to have two Drude-type plasmonic (meta)materials with plasma frequencies near the frequency of interest, such that $\text{Re}[\epsilon_{\text{in}}] = 0.2$ and $\text{Re}[\epsilon_{\text{out}}] = 0.2$ for the two wavelengths of interest $\lambda_0 = 500$ nm and $\lambda_0 = 625$ nm, respectively. Here, the subscripts “in” and “out” refer to the inner and outer shells of the cloak, respectively. Both materials have been supposed to follow a classic Drude model of the type $\epsilon_{\text{in}} = \epsilon_0[1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}]$, which takes into account the frequency dispersion and material absorption. A suitable design for the shell thicknesses has been carried out by imposing the full-wave condition $U_1^{\text{TM}} = 0$ at the two wavelengths of interest, resulting in $a_{\text{in}} = 107.5$ nm and $a_{\text{out}} = 131.5$ nm, whose thicknesses are feasible with current nanotechnology [13]. Needless to say, the design may be varied depending on the properties of the available materials and may be scaled in frequency at will.

Figure 1 reports the variation of the total scattering efficiency Q_{sca} , defined as the ratio of the total scattering cross section σ_{sca} to the physical cross section of the particle, for the original isolated spherical particle (red line), for the covered spherical particle in the two cases of small Ohmic losses ($\gamma = 10^{-4}\omega_p$, black line) and reasonable Ohmic losses ($\gamma = 10^{-2}\omega_p$, green line), and for the case in which the cover material is replaced with the same dielectric material of the particle, i.e., with $\epsilon = 3\epsilon_0$. The curves have been evaluated analytically by using the rigorous Mie theory and validated numerically with finite-

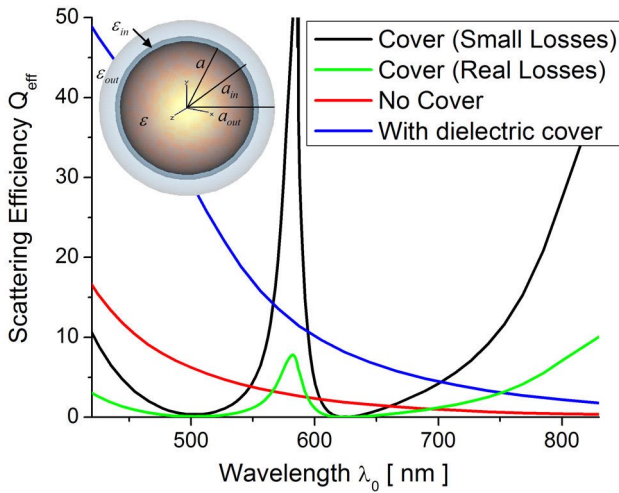


FIG. 1 (color online). Total scattering efficiencies for the geometry depicted in the inset, i.e., a dielectric particle of radius $a = 100$ nm and permittivity $\epsilon = 3\epsilon_0$, cloaked by a 2-layered shell designed for cloaking at $\lambda_0 = 500$ nm and $\lambda_0 = 625$ nm. The four curves refer to the following: (a) covered particle with small losses in the plasmonic materials (black), covered particle with reasonable losses (green), the original particle (red), and the same particle with a dielectric material replacing the shell region (blue).

integration-technique commercial software [14] to take into account any possible small variations from the ideal spherical shapes of the particles, which have been shown in our earlier work not to affect the results achievable with this technique [2].

It is clear how the presence of the layered shell allows a drastic reduction of the overall 3D scattering of the system at the two distinct design frequencies, independent of the angle of observation, form of excitation, and incident polarization. The total scattering extinctions have been reduced by about 30-fold (at $\lambda_0 = 500$ nm) and 120-fold (at $\lambda_0 = 625$ nm) with respect to the uncloaked scenario, and by about 142-fold and 512-fold with respect to the case of a particle of the same size as the covered case, but all made with a dielectric material with permittivity ε . It can be seen that around the two design frequencies the results are affected in a very minor way by the presence of reasonable absorption in the material parameters, which is consistent with our results in [2]. It is also evident that the bandwidths over which the cloaking effect is dominant are not too narrow, not being limited to just the design frequencies. This is consistent with the inherently non-resonant features of this phenomenon, a clear advantage over the other metamaterial cloaking methodologies recently proposed in the literature [4–8].

In Fig. 1, we also see a sharp peak in the scattering efficiency for the covered case around $\lambda_0 = 580$ nm, due to the close presence of a dipolar and quadrupolar plasmonic resonance, since V_1^{TM} and V_2^{TM} get close to zero around that frequency. The presence of similar resonant peaks is always expected at the wavelength between the two cloaking wavelengths, independent of their specific position. This is inherently dictated by the passivity conditions for materials employed here. It is evident, however, that the amplitude of this scattering peak is drastically reduced by the presence of realistic losses, due to the high level of the fields induced in the plasmonic shells at this resonant configuration.

Figures 2 and 3 report the magnitude (row a) and phase (row b) of the near-zone electric field distribution on the H plane of the system, i.e., in the plane orthogonal to the impinging electric field vector, in the three cases of cloaked particle (with reasonable losses included, corresponding to the green line in Fig. 1), original uncovered particle (corresponding to the red line), and a dielectric particle with the same size as the covered particle (corresponding to the blue line), for $\lambda_0 = 625$ nm and $\lambda_0 = 500$ nm, respectively. In all the cases, the incident plane wave is arriving from the bottom of the figure. It may be clearly seen how the presence of the bilayered cloak restores at the two distinct frequencies a uniform field amplitude outside the sphere, even in the very near field of the structure, thus drastically reducing the “disturbance” of the sphere on the impinging uniform plane-wave distribution. Also the plane-wave phase fronts are fully restored at both frequencies of inter-

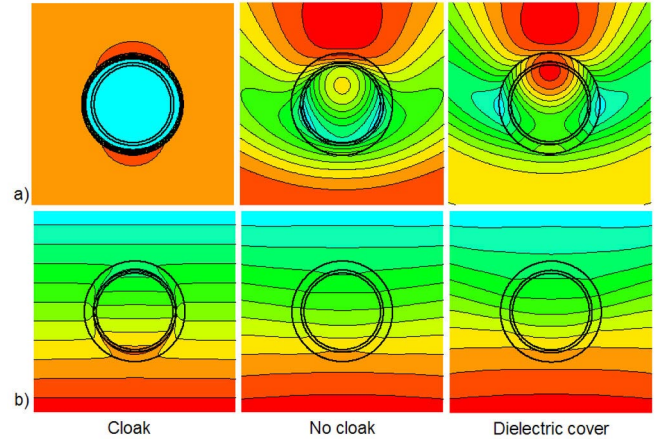


FIG. 2 (color online). Distribution of the total electric field orthogonal to the H plane in the three cases corresponding to the covered particle with losses, original particle, and particle covered by dielectric of Fig. 1 for $\lambda_0 = 625$ nm. Brighter colors correspond to higher values of the field.

est, and the difference with the uncloaked cases is evident. This confirms that the invisibility phenomenon achieved by the multilayered cover is not limited to the far field, as ensured by Fig. 1, but it also arises in the very near field of the object at both frequencies, consistent with our previous findings [1–3]. We also note that the cloaking shell performs better at larger wavelengths, since the original particle disturbance is more dominated by the electric dipole moment, which is the one mainly canceled by the shell. At the larger frequency (Fig. 3), even though the cloaking still works very well (and the visibility of the particle has been reduced by more than 30-fold despite the increase in its volume), some small perturbation of the field in the surrounding of the cloak is present, due to the relatively larger electrical size of the system at this wavelength and thus a minor contribution from higher-order scattering orders, which are not fully canceled by the cloak. Another interesting feature in the phase plots is how the cloaking shell

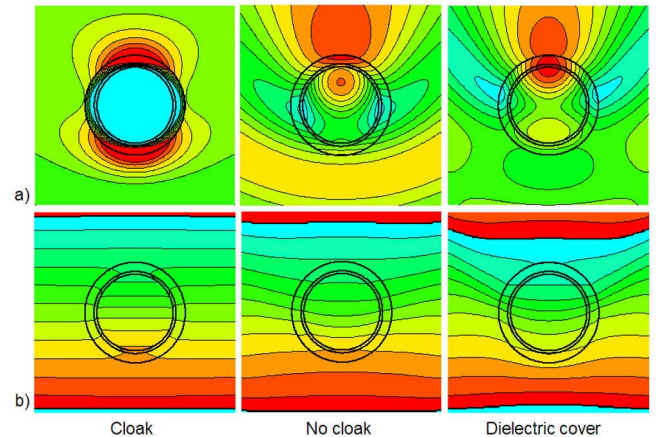


FIG. 3 (color online). Similar to Fig. 2, but for $\lambda_0 = 500$ nm.

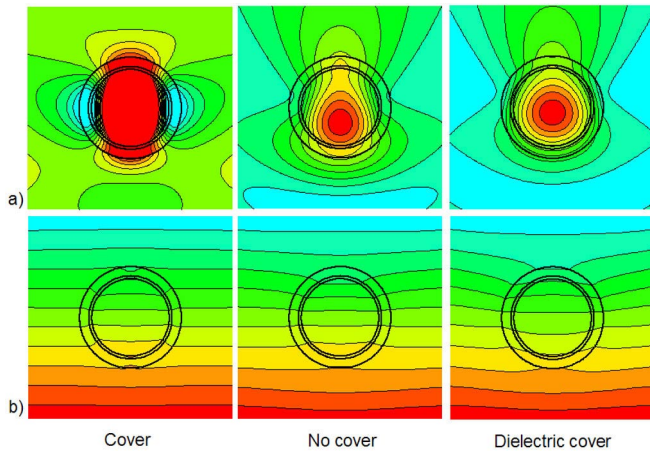


FIG. 4 (color online). Distribution of the total magnetic field orthogonal to the E plane in the three cases of Figs. 2 and 3 for $\lambda_0 = 625$ nm. Brighter colors correspond to higher values of the field.

effectively restores planar phase fronts both inside the dielectric particle and in the outside region, even though the two regions have different phase velocities (due to the different index of refraction), and therefore cannot be matched without the proper plasmonic material.

Figures 4 and 5 show analogous results in the orthogonal plane of polarization, which is dominated by higher-order multipoles. It is evident how the higher-order moments are also reduced in amplitude by the presence of the multilayer cloak, even though the cloak has been originally designed to cancel just the electric dipole moment (whose contribution is zero on this plane), thus ensuring an overall invisibility all around the particle even in the very near field. The polarization independence of the present method, evident in these plots, is another advantage of this method over

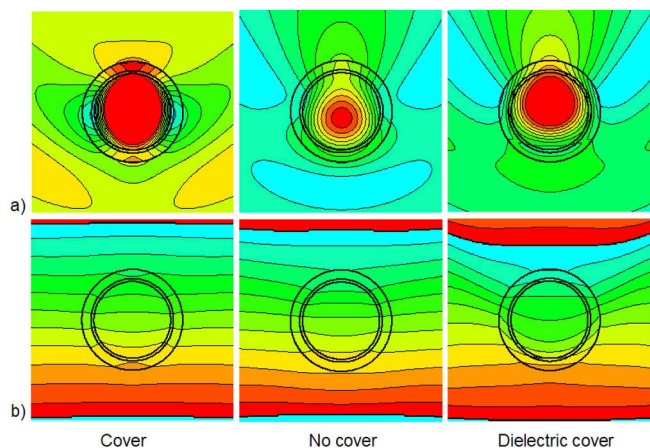


FIG. 5 (color online). Similar to Fig. 4, but for $\lambda_0 = 500$ nm.

other recently proposed techniques for metamaterial cloaking.

The reported theoretical results in this Letter suggest that by using multilayered plasmonic shells it is possible to drastically reduce the total scattering of a dielectric nanoparticle at different optical frequencies simultaneously. Extension to a larger number of different layers, in order to achieve invisibility simultaneously at a larger number of frequencies, is feasible following a similar design. Different from the transformation-based cloaking methods, here each cloaking layer adds only a few percent to the thickness of the object, and therefore additional layers would not make the electrical size of the whole system too large and bulky to affect the feasibility of the method. Moreover, these results may be directly extended to lower frequencies by realizing metamaterial plasmonic covers, as those envisioned in Ref. [3]. Because of the near-field cloaking, they may also be directly extended to collections of nanoparticles [15], whose total dimensions may be substantially larger than the wavelength of operation, for possible experimental verification of this phenomenon. Applications of these concepts may be envisioned in different scientific fields, such as camouflaging and noninvasive probing in nano-optics.

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