Nuclear Structure Relevant to Neutrinoless Double β Decay: ^{76}Ge and ^{76}Se

J. P. Schiffer,[1,*] S. J. Freeman,[2] J. A. Clark,[3] C. Deibel,[3] C. R. Fitzpatrick,[2] S. Gros,[1] A. Heinz,[3] D. Hirata,[4,5] C. L. Jiang,[1] B. P. Kay,[2] A. Parikh,[3] P. D. Parker,[3] K. E. Rehm,[1] A. C. C. Villari,[4] V. Werner,[3] and C. Wrede[3]

[1]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[2]*University of Manchester, Manchester M13 9PL, United Kingdom*
[3]*Yale University, New Haven, Connecticut 06520, USA*
[4]*GANIL (IN2P3/CNRS -DSM/CEA), B.P. 55027 14076 Caen Cedex 5, France*
[5]*Dept. of Physics and Astronomy, The Open University, Milton Keynes, MK7 6AA, United Kingdom*

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The possibility of observing neutrinoless double β decay offers the opportunity of determining the *effective* neutrino mass *if* the nuclear matrix element were known. Theoretical calculations are uncertain, and measurements of the occupations of valence orbits by nucleons active in the decay can be important. The occupation of valence neutron orbits in the ground states of ^{76}Ge (a candidate for such decay) and ^{76}Se (the daughter nucleus) were determined by precisely measuring cross sections for both neutron-adding and removing transfer reactions. Our results indicate that the Fermi surface is much more diffuse than in theoretical calculations. We find that the populations of at least three orbits change significantly between these two ground states while in the calculations, the changes are confined primarily to one orbit.

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An essential step in studying the nature of the neutrino is the attempt to observe neutrinoless double β decay [1] and major efforts are being undertaken with this objective in mind. Observation of such a process would immediately show that neutrinos are their own antiparticles, and its rate may well give the first direct measure of the neutrino mass *if* the corresponding nuclear matrix element can be reliably calculated. As an example, for one of the likely candidates (^{76}Ge), theoretical calculations have yielded answers that are spread over more than an order of magnitude. This prompted the statement by Bahcall *et al.* [2], "The uncertainty in the calculated nuclear matrix elements for neutrinoless double beta decay will constitute the principal obstacle to answering some basic questions about neutrinos." There have been suggestions that relate the matrix elements for neutrinoless double β decay to those for ordinary single β decay, or to the "normal" two-neutrino modes which have been observed experimentally [3]. However, neutrinoless decay proceeds by the virtual excitation of states in the intermediate nucleus with a momentum transfer much larger than that for these other processes. It will thus involve all possible virtual intermediate states (up to about 100 MeV of excitation), and so will include giant resonances. There is no other experimentally accessible process that could directly determine the matrix element.

Although there is still considerable discussion regarding the best theoretical approach, what unquestionably matters is knowing the population of the valence orbits for the nucleons that switch from neutrons to protons. We have therefore undertaken a set of measurements to determine this quantity experimentally, and report here on the valence neutron populations and the differences in these populations for ^{76}Ge and ^{76}Se. In a previous experiment, the neutron pair correlations in these two nuclei were found to be quantitatively very similar, and there is no significant mixing of excited 0^+ states with the ground states in pair transfer [4].

The Macfarlane-French [5] sum rules for nucleon transfer state that the summed spectroscopic strength for neutron-*adding* reactions with a given set of quantum numbers is equal to the vacancies in that target orbital, while the sum over states for neutron-*removing* reactions will determine the occupancy. Here, we have measured the cross sections and extracted spectroscopic factors of significantly populated states, for both neutron-adding and neutron-removing reactions. The summed spectroscopic factors for *both* reactions can be added and used to provide a normalization, allowing occupation numbers for orbitals to be extracted.

The nucleon transfers reported here have been measured previously [6,7], but not with the same experimental methods, and using different parameters in each DWBA (distorted wave Born approximation) analysis for extracting spectroscopic factors. The aim of the present measurement is to analyze all the results in a consistent manner to permit the extraction of more accurate occupation numbers with a common experimental approach.

The nuclei ^{76}Ge and ^{76}Se have 42 and 44 neutrons, between the closed shells at $N = 28$ and 50. The valence nucleons which may participate in double β decay are therefore in the $1p_{3/2}$, $0f_{5/2}$, $1p_{1/2}$, and $0g_{9/2}$ orbits, listed in order of their binding energies. We have made systematic measurements to obtain accurate cross sections for the neutron-adding (d,p) and $(\alpha,{}^3\mathrm{He})$ reactions, as well as for neutron-removing (p,d) and $({}^3\mathrm{He},\alpha)$ reactions. The momentum matching in (d,p) reactions for transitions with $\ell = 3$ and 4 at energies near the Coulomb barrier is not optimal and thus the cross sections are rather weak. Therefore, helium-induced reactions were used to obtain data with improved momentum matching and larger cross sections for the higher-ℓ transitions. This selectivity is illustrated in Fig. 1.

Deuteron, proton, ^{4}He, and ^{3}He beams from the Yale tandem accelerator were used to bombard isotopically enriched Ge and Se targets of about 200–300 μg/cm^2 evaporated on thin, 50 μg/cm^2 C foils. The momenta of the reaction products were determined and the particles identified with the Yale Enge spectrograph and gas-filled focal-plane detector backed by a scintillator.

The product of target thickness and spectrometer solid angle was found by measuring elastic scattering in the Coulomb regime at 30° for each target used. The beam energies used for this calibration were 6-MeV protons and 10-MeV α particles. For the transfer reactions, the same spectrometer aperture and beam integrator settings were used to minimize potential systematic errors. The beam energies chosen were 15 MeV for the (d,p) reaction and 23 MeV for the (p,d) to keep the energies in each channel comparable. Similarly, $(\alpha,^3\mathrm{He})$ was studied at 40 MeV and $(^3\mathrm{He},\alpha)$ at 26 MeV. Measurements were also carried out on targets of ^{74}Ge and ^{78}Se to provide an additional check. The energy resolution obtained was ~40 keV for the deuteron and proton-induced reactions, and ~70 keV for the ^{3,4}He reactions.

The (d,p) angular distributions have been studied previously and ℓ values were assigned [6,7]. In the current work, the yields were therefore measured only at the angles that correspond to the peaks in the angular distributions for the ℓ values of interest: 11°, 28°, and 37° for $\ell = 1$, 3 and 4, respectively. The helium-induced reactions are forward peaked, and so the most practical forwardmost angles were chosen: 8° for $(\alpha,^3\mathrm{He})$ and 5° for its inverse. The previous

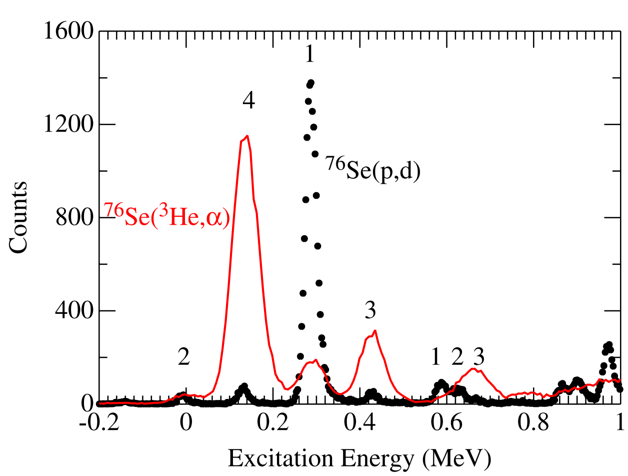

FIG. 1 (color online). Energy spectra for the neutron-removal reactions for ^{76}Se to ^{75}Se. The $\ell = 1$ transitions appear strongly in the 11° (p,d) spectrum (points) while the $\ell = 3$ and in particular $\ell = 4$ are most prominent in $(^3\mathrm{He},\alpha)$ (line) where the resolution is worse because of the higher energy. The ℓ values are indicated by numbers above the peaks.

ℓ-value assignments [6,7] were confirmed, as may be seen in Fig. 2. Our results also agree approximately with the previous relative spectroscopic factors for states populated with a particular target.

We used the finite-range code PTOLEMY [8] for the DWBA calculations. The normalization depends on the choice of the distorting potentials and the bound-state parameters. The extracted relative spectroscopic factors also vary with these choices, but by a smaller amount, and this is a source of some of the uncertainty at the level of a few percent. For the projectile bound-state wave function, the Reid potential was used for the deuteron and a Woods-Saxon one for the α particle and for the various target bound states.

Absolute spectroscopic factors are notoriously difficult to obtain. The values of spectroscopic factors for "good" single-particle states in doubly-magic nuclei are usually

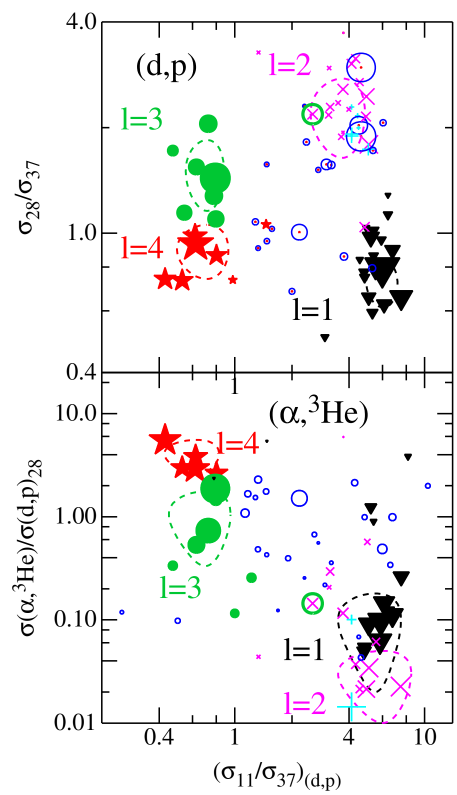

FIG. 2 (color online). Ratios of cross sections, $\sigma_{d,p}(28°)/\sigma_{d,p}(37°)$ *vs.* $\sigma_{d,p}(11°)/\sigma_{d,p}(37°)$ on top and $\sigma_{\alpha,^3\mathrm{He}}/\sigma_{d,p}(28°)$ *vs.* $\sigma_{d,p}(11°)/\sigma_{d,p}(37°)$ below, are shown for different ℓ values and reactions. The symbols, one for each state, indicate the ℓ-value assignments from previous work: triangles (black) are $\ell = 1$, circles (green online) are $\ell = 3$, and stars (red online) are $\ell = 4$. In addition, states not included in the analysis are $\ell = 2$ transitions indicated by × and $\ell = 0$ by + signs. States with unknown ℓ values are indicated by hollow circles. The size of the symbols is a rough measure of the cross sections. The dashed lines indicate the loci of the ratios for well-established ℓ values. The × surrounded by a circle, between the $\ell = 2$ and 3 islands in the lower box, is the 500-keV $5/2^+$-$5/2^-$ doublet in ^{77}Ge discussed in the text.

around 0.5–0.6 because of short-range correlations [9]. Such correlations are expected to be a uniform property of nuclei, not changing between nearby nuclei or configurations. Therefore, the strength can be renormalized and the sum rules applied. Since the sums of the strengths for neutron adding or removing are proportional to the vacancies or occupancies, together they should add up to the $(2J+1)$ degeneracy of the orbits and this determined the normalization. A check is provided, in that the summed spectroscopic factors for a given orbit should add up to the *same* value for each of the targets. The normalization factors were averaged for the two optical potential sets used yielding values of 0.53, 0.56, and 0.57 for $\ell = 1$, 3, and 4 transitions, respectively. The corresponding rms fluctuations among the targets were 2, 12, and 7%, respectively, indicating that the procedure is reasonable. The normalization values are fortuitously close to the depletion that should be expected for "absolute" spectroscopic factors; different potentials or bound-state parameters would yield different normalization values by as much as 30%. However, the variation in spectroscopic factors is much smaller, at the few % level, and has been included in the estimate of uncertainties.

Several points are to be noted in the above sums. Since not all the spins of the states seen in $\ell = 1$ transitions are known, we summed all $\ell = 1$ transitions combining the $j = 1/2$ and 3/2 states. For the neutron-removal reactions, a small correction was made for the unobserved $T_>$ isobaric analog states, corresponding to proton removal. Also for neutron removal in the (p,d) reaction, some previously determined $\ell = 1$ transitions at high excitation energy were beyond the energy range measured here, and a correction was made. There were no known missed states for the neutron-adding measurements or for the other ℓ values. The magnitudes of these corrections for $T_>$ and missed states are less than 5% for $\ell = 1$ and 3, and less than 1% for $\ell = 4$.

Finally, for the $f_{5/2}$ states, no $5/2^-$ state was known in ^{77}Ge, while all other nuclei in this region have such a state well below 1 MeV in excitation energy. In attempting to find such a state in the $(\alpha,^3\mathrm{He})$ reaction, the intensity of the peak around 500 keV excitation was stronger than expected for a known $\ell = 2$ transition to a $5/2^+$ state at 504.8 keV, and the centroid of this peak was 492 keV, somewhat lower than expected. In fact, a tentative state is reported in the compilations [10] at 491.9 keV from unpublished work with the $(^{13}\mathrm{C},^{12}\mathrm{C})$ reaction, and we have assumed that this is the missing $5/2^-$ state. Its strength was included in the sums.

Our data are available online in the XUNDL database [11]. The vacancies and occupancies from the summed normalized spectroscopic factors are shown in Table I. Listed in the Table are the numbers of holes and particles from neutron adding and removing, their sum, and the best average value of the occupancy, all computed with the same normalization for all targets, one normalization for (d,p) and (p,d) and another for $(\alpha,^3\mathrm{He})$ and $(^3\mathrm{He},\alpha)$. The $\ell = 1$ strength is best determined in the (d,p) and (p,d) reactions and the $\ell = 3$ and 4 transitions from the helium-induced reactions. As was noted, the sums of holes and particles for both $\ell = 1$ and 4 transfers are constant to better than 5% across the targets studied. For $\ell = 3$, the situation is somewhat worse, partly because these transitions are relatively weak in both reactions. As a result, components of strength could have been missed. Additionally, there is some ambiguity about the $\ell = 3$ strength in the neutron removal reactions since some of the transitions could be to $7/2^-$ states at higher excitation energies. For cases where there is no evidence on the spins of higher $\ell = 3$ hole states, all $\ell = 3$ transitions above 1 MeV were, somewhat arbitrarily, excluded. The summed strengths for $\ell = 3$ fluctuate more than the others, probably as a result of these arbitrary assignments of weaker transitions. The adopted occupancies for ^{76}Ge and ^{76}Se are shown in bold.

TABLE I. Summed spectroscopic strengths.

Target	Holes	Particles	Holes + Particles	Adopted Occupancy
^{74}Ge $\ell = 1$	1.15			4.85
^{76}Ge	1.12	4.83	5.95	**4.87 ± 0.20**
^{76}Se	1.63	4.49	6.12	**4.41 ± 0.20**
^{78}Se	0.94			5.06
^{74}Ge $\ell = 3$	1.90	4.38	6.28	4.19
^{76}Ge	1.14	3.92	5.06	**4.56 ± 0.40**
^{76}Se	2.10	3.71	5.81	**3.83 ± 0.40**
^{78}Se	2.34	4.63	6.97	3.98
^{74}Ge $\ell = 4$	4.37	5.83	10.20	5.69
^{76}Ge	3.41	6.27	9.68	**6.48 ± 0.30**
^{76}Se	4.36	6.13	10.49	**5.80 ± 0.30**
^{78}Se	2.80	7.31	10.11	7.24

Our measurements provide two determinations of the valence-orbit occupancies in the ^{76}Ge and ^{76}Se ground states, one from the neutron-adding data, and one from the neutron removal. We average these to obtain a best value, weighting the occupancy from neutron addition higher than that obtained from neutron removal in line with its smaller fractional uncertainty. These values are in the last column of the Table.

The uncertainties in the final mean occupancy values are difficult to estimate. Statistical errors in the summed strength are less than 1% and relative systematic errors between targets are believed to be less than 3%. The biggest uncertainties stem from possible missed states, especially for the $\ell = 3$ transitions, and from uncertainties in the DWBA calculations. We estimate that the occupancy is determined to about 0.2 nucleons for the $1p$, 0.3 for the $0g_{9/2}$ orbits, and slightly worse, 0.4, for the $0f_{5/2}$ orbit. These estimates of uncertainties are rather crude. However, we have some confidence in these estimates since, to

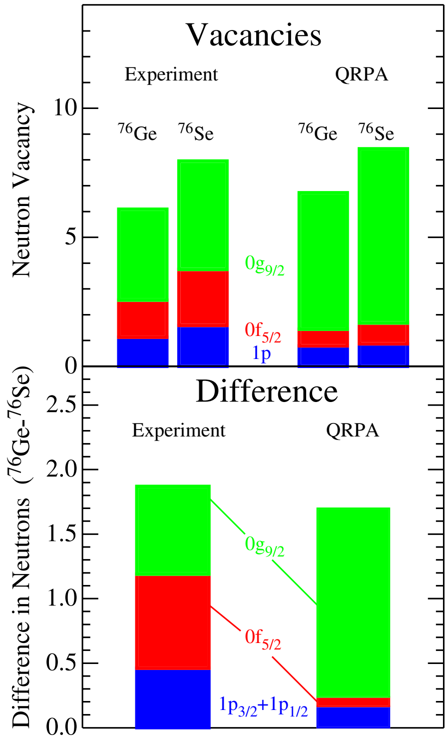

FIG. 3 (color online). The deduced neutron vacancies for ^{76}Ge and ^{76}Se are shown in the three active valence orbits and compared to those from the QRPA calculations of Reference [12]. The naive shell closure should give 6 and 8 vacancies for these two nuclei. The lower part of the figure shows the differences in these occupations (expected to be 2.0), again compared to the QRPA calculation.

within the quoted uncertainties: First, the normalization factors obtained for each target separately are similar. Second, the mean normalizations for each ℓ value and reaction type are also similar. Third, the summed removing and adding strengths for ^{74,76}Ge and ^{76,78}Se, 22.5, 20.7, 22.4, and 23.1, respectively, are consistent with the expected value of 22.0. Fourth, the neutron vacancies obtained for the four nuclei (from the adopted occupancies in the Table) are 7.3, 6.1, 7.9, and 5.7, in good agreement with the expected values of 8, 6, 8, and 6.

Beyond the valence $1p$, $0f_{5/2}$, and $0g_{9/2}$ orbitals, neutron removal from ^{76}Se suggests that approximately 0.2 neutrons are in the $1d_{5/2}$ orbit. The weak $5/2^+$ state in ^{75}Ge is not resolved in our work, but using the results in [10], we obtain a roughly similar value.

The values of vacancies are shown in Fig. 3 along with the QRPA results [12]. There is little question that the vacancies in the $1p$ and, especially, in the $0f_{5/2}$ orbits are significantly larger in the data than in the calculations. For the neutrinoless double β decay, it is the changes in occupancy that are important, and so in the lower part of Fig. 3, we show the differences between ^{76}Ge and ^{76}Se: 0.46 ± 0.20 in $1p$, 0.73 ± 0.40 in $0f_{5/2}$, and 0.68 ± 0.30 in $0g_{9/2}$.

While the QRPA results predict changes between the two nuclei to be mostly in the $0g_{9/2}$ orbit, the experiment shows quite clearly that the changes in the $1p$ and $0f_{5/2}$ orbits are much larger than predicted. The qualitative feature that, in disagreement with QRPA, there are still large vacancies in $1p$ and $0f_{5/2}$ is quite robust. It follows from the relatively large cross sections in the neutron-adding reactions, and it cannot depend on the details of the analysis or the assumptions.

What the consequences may be, of this disagreement in neutron occupancy between QRPA and experiment, on the matrix element for neutrinoless double β decay are not clear at present and will need to be investigated in more detail. Proton occupancies are similarly important, and experiments to determine them are planned.

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*Correspondence to: schiffer@anl.gov

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