Does the Planck Mass Run on the Cosmological-Horizon Scale?

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Einstein's theory of general relativity contains a universal value of the Planck mass. However, one may envisage that in alternative theories of gravity the effective value of the Planck mass (or Newton's constant), which quantifies the coupling of matter to metric perturbations, can run on the cosmological-horizon scale. In this Letter, we study the consequences of a glitch in the Planck mass from subhorizon to superhorizon scales. We show that current cosmological observations severely constrain this glitch to less than 1.2%.

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The Einstein theory of gravity (or general relativity) is among the most successful theories in physics. Despite its simple mathematical structure, and having only a single constant, it has been successful in explaining the cosmological observations on the horizon scale ($\sim 10^{25-28}$ cm), down to the planetary or lunar dynamics on solar system scales ($\sim 10^{9-15}$ cm), and even laboratory tests of the inverse square law on the submillimeter scales (see [1] for an overview).

Such tests severely constrain alternatives to the Einstein theory. Nevertheless, deviations from Einstein gravity are expected on theoretical grounds, because it is a classical theory. While renormalization group studies suggest that it remains a good approximation at high energies [2], the Planck mass becomes scale dependent. Further corrections might be introduced in more complete theories of gravity, such as string theory.

Deviations from Einstein gravity have also been suggested on purely phenomenological grounds, in particular, to explain the rotation curves of galaxies (as a replacement for dark matter) [3–5], or the discovery of the apparent acceleration of cosmic expansion (as a replacement for dark energy or cosmological constant) [6,7]. However, the evidence for any such deviation (rather than simply exotic matter or energy components), is far from conclusive.

A customary way to quantify deviations from Einstein gravity on small scales and in the weak field limit is through introducing a Yukawa fifth force modification to the inverse square law, where the gravitational potential energy takes the form

$$V(r) = -G\frac{m_1 m_2}{r}(1 + \alpha e^{-r/\lambda}), \tag{1}$$

where G is Newton's constant, m_1 and m_2 are the masses of (pointlike) gravitating objects, while α and λ quantify the strength and the scale of the new interaction, respectively. In this model, the effective Newton's constant smoothly goes from G on large scales ($r \gg \lambda$) to $G(1 + \alpha)$ on small scales ($r \ll \lambda$). Current experimental and observational constraints severely limit α in the range $10^{-1} < \lambda < 10^{16}$ cm (see [1] for an overview), and even on scales of $\lambda \sim 10^{25}$ cm [8].

In this Letter, we investigate the possibility of a similar glitch in Newton's constant (or Planck mass) on the scale of the cosmological horizon, or the Hubble radius ($\lambda \sim c/H \sim 10^{28}$ cm). In our case, the scale λ will not be a physical constant of the theory, but rather an emergent scale in the theory, as a consequence of an effective change in the background geometry, from flat Minkowski space on small scales, to the expanding Friedmann-Robertson-Walker background on large scales.

We start by defining an effective Planck mass, and then give a few examples of theories which contain a glitch in their effective Planck masses on the horizon scale. We will then investigate the cosmological consequences of such a glitch for structure formation, and the cosmic microwave background, and provide a limit based on current cosmological observations.

The Planck mass M_p quantifies the strength of coupling between the space-time metric and the energy-momentum of matter in the Universe. In terms of the Einstein equation:

$$G^{\mu}_{\nu} = M_p^{-2} T^{\mu}_{\nu}, \tag{2}$$

where G^{μ}_{ν} and T^{μ}_{ν} are the Einstein and the total energy-momentum tensors, respectively. Notice that in our notation, $M_p = (8\pi G)^{-1/2} \simeq 2.44 \times 10^{18}$ GeV, where G is Newton's gravitational constant, and we have used natural units ($\hbar = c = 1$).

While G (and M_p) are true constants in Einstein's theory, other theories of gravity may predict a time- and/or space-dependent G, as do, e.g., scalar-tensor theories [9–12]. In other words, possible deviations from Einstein gravity, or alternatively, other energy components that are not accounted for in the total energy-momentum tensor $T_{\mu\nu}$, may lead to an effective (or dressed) Planck mass that could run with time and/or the energy or length scale of the interactions. A possible definition for an effective Planck mass may come by perturbing the Einstein constraint (or G^0_0) equation:

$$M_{p,\mathrm{eff}}^{-2} \equiv \delta G^0_0 / \delta T^0_0, \tag{3}$$

which reduces to the Poisson equation around a Minkowski background (or on subhorizon scales). However, Eq. (3) can mix different scales, as it involves the ratio of two variable functions. Moreover, this definition may become ill defined if δT^0_0 crosses zero. Instead, we are going to adopt a more practical definition:

$$M_{p,\mathrm{eff}}^{-2}(|\mathbf{k}|) \equiv \frac{\langle \delta G^0_{0,\mathbf{k}} \delta T^{0*}_{0,\mathbf{k}} \rangle}{\langle \delta T^0_{0,\mathbf{k}} \delta T^{0*}_{0,\mathbf{k}} \rangle}, \tag{4}$$

where $\delta T^0_{0,\mathbf{k}}$ and $\delta G^0_{0,\mathbf{k}}$ are the spatial Fourier transforms of δT^0_0 and δG^0_0 on a given spatial hypersurface, and "$\langle\,\rangle$" represent ensemble averages.

While this definition has the benefit of separating different physical scales, we have introduced an explicit gauge dependence through the choice of a particular spatial hypersurface. On small (subhorizon) scales ($k \gg H$), this gauge dependence is not important, as Eq. (3) reduces to the Poisson equation, and we recover Newtonian gravity. In other words, the difference between $M_{p,\mathrm{eff}}$ in different (physical) gauges is $\sim(k/H)^{-2}$, on subhorizon scales.

On superhorizon scales ($k \ll H$), gauge transformations can change the effective Planck mass only if the Planck mass associated with the background expansion is different from the one associated with the perturbations, i.e., as long as

$$M_{p,\mathrm{IR}}^{-2} = \frac{\dot{G}^0_0}{\dot{T}^0_0} = \frac{\delta G^0_0}{\delta T^0_0}, \tag{5}$$

the effective Planck mass is gauge invariant on superhorizon scales. This condition can naturally result from the assumption of adiabatic initial conditions, which asserts that, up to a time shift, causally disconnected patches of the Universe experience identical histories. Therefore, for *adiabatic* initial conditions, the gauge dependence of our definition of the effective Planck mass may only become important as modes cross the horizon.

From here on, we will refer to the cosmological subhorizon ($k \gg H$) and superhorizon ($k \ll H$) scales as the UV and IR scales, respectively, which have their respective values of the effective Planck mass, $M_{p,\mathrm{UV}}$ and $M_{p,\mathrm{IR}}$. In the language of Eq. (1), the UV-IR mismatch can be parametrized by the dimensionless α parameter:

$$M_{p,\mathrm{IR}}^2 = M_{p,\mathrm{UV}}^2(1+\alpha). \tag{6}$$

As an example, let us consider the quadratic Cuscuton action [13]:

$$S_Q = \int d^4x \sqrt{-g}\left(\mu^2\sqrt{|\partial^\mu\varphi\partial_\mu\varphi|} - \frac{1}{2}m^2\varphi^2\right), \tag{7}$$

where φ is a scalar field, and μ and m are constants of the theory with the dimensions of energy.

If we consider Cuscuton as a part of the gravitational action (and so do not include it in the energy-momentum tensor), the effective Planck mass takes the form

$$M_{p,\mathrm{eff}}^{-2}(k) = \frac{\delta\rho_Q + \delta\rho_m}{\delta\rho_m}, \tag{8}$$

where $\delta\rho_Q$ and $\delta\rho_m$ are the energy density perturbations of Cuscuton and ordinary matter, respectively. Using the solution to the field equation in the longitudinal gauge, obtained in [14], we find that

$$M_{p,\mathrm{eff}}^2 \simeq M_{p,\mathrm{UV}}^2 - \frac{3\mu^4}{2m^2}\left(1+\frac{k^2}{3H^2}\right)^{-1}\left(1-\frac{k^2}{3\dot{H}}\right)^{-1}, \tag{9}$$

to the lowest order in μ, in a flat matter-dominated universe. As we noted above, the exact k dependence of $M_{p,\mathrm{eff}}$ will depend on the choice of gauge, but the IR limit of the effective Planck mass,

$$M_{p,\mathrm{IR}}^2 = M_{p,\mathrm{UV}}^2 - \frac{3\mu^4}{2m^2}, \tag{10}$$

is set by the Friedmann equation [13], and is thus gauge invariant [15].

A very similar behavior can be seen in the dynamics of a canonical scalar (or quintessence) field with a simple exponential potential [16]: $V(\varphi) = M_p^4 \exp(-\kappa\varphi/M_p)$. For a fixed background equation of state, the energy density of the field, asymptotically, reaches a constant fraction of the energy density of the Universe. In particular, for a flat matter-dominated cosmology, this fraction is $\Omega_\varphi = 3\kappa^{-2}$, which translates to a glitch in the effective Planck mass on the horizon scale:

$$M_{p,\mathrm{IR}}^2 = M_{p,\mathrm{UV}}^2(1-3\kappa^{-2}), \tag{11}$$

as quintessence does not cluster on subhorizon scales, and so $M_{p,\mathrm{eff}}$ reaches its fundamental value on small scales.

A third example for models that lead to a similar mismatch between the IR and UV effective Planck masses is "Einstein-aether" theory, where the "aether" is a Lorentz-violating, fixed-norm, timelike vector field [17,18]. The rescaling of Newton's constant induced by such a vector field has been studied, e.g., by Carroll and Lim [19]. In their notation, the Lagrangian for the vector field u^μ is given by

$$\mathcal{L}_u = -\beta_1\nabla^\mu u^\sigma\nabla_\mu u_\sigma - \beta_2(\nabla_\mu u^\mu)^2 - \beta_3\nabla^\sigma u^\mu\nabla_\mu u_\sigma + \lambda_u(u^\mu u_\mu + m_u^2), \tag{12}$$

where m_u is the norm of the vector field, λ_u is a Lagrange multiplier, and β_i's are dimensionless constants of the theory. The coupling of u^μ to the metric then renormalizes both the IR and UV values of the effective Planck mass, so that

$$M_{p,\mathrm{IR}}^2 = M_{p,\mathrm{UV}}^2 + (2\beta_1 + 3\beta_2 + \beta_3)m_u^2. \qquad (13)$$

Unlike the two scalar field models discussed before, the gravitational force is suppressed on large (superhorizon) scales by the Lorentz-violating vector field [19].

We will next look at the observational consequences of a possible glitch between the UV and IR effective Planck masses. The Hubble expansion rate in a flat homogenous cosmology is set by the Friedmann equation:

$$H^2 = \rho_{\mathrm{tot}}/3M_{p,\mathrm{IR}}^2. \qquad (14)$$

As the present-day cosmic density ρ_{tot} is dominated by dark matter and dark energy, which are only seen through their gravitational effects, there is no way to find $M_{p,\mathrm{IR}}$ through measuring the present-day Hubble constant. However, the energy density in the radiation era is dominated by photons and neutrinos and therefore fixed by the cosmic microwave background (CMB) temperature ($T = 2.728 \pm 0.004$ K [20]). Constraints on the expansion rate during the radiation era (at $T \sim 0.1$ MeV) then come from comparing the big bang nucleosynthesis predictions with the cosmological observations of the light element abundances, which correspondingly constrain the running of the Planck mass: $\alpha = 0.0 \pm 0.2$ (95% C.L.) [21].

More interesting constraints can come from the study of cosmological perturbations on small scales. Combining the continuity and Poisson equations with Newton's 2nd law yields

$$\ddot{\delta} + 2H\dot{\delta} = \frac{\bar{\rho}_m}{2M_{p,\mathrm{UV}}^2}\delta, \qquad (15)$$

for the linear matter overdensity perturbations δ ($=\delta\rho_m/\bar{\rho}_m$) on small scales. Substituting the definition of α [Eq. (6)] for $M_{p,\mathrm{UV}}^2$ and using the Friedmann equation [Eq. (14)] in the matter-dominated era (when $H = 2/3t$), this can be easily solved. For $\alpha \ll 1$, the growing mode behaves as $\delta \propto t^{2/3+(2/5)\alpha} \Rightarrow \Phi \propto t^{(2/5)\alpha}$, where Φ is the Newtonian (or longitudinal metric) potential.

Perturbation modes that are inside the horizon at the redshift of matter-radiation equality $z_{\mathrm{eq}} \simeq 3400$ will then all experience the same amount of suppression or enhancement during the matter era. Since the scale factor grows as $t^{2/3}$, this suppression or enhancement is roughly by a factor of $z_{\mathrm{eq}}^{3\alpha/5} \simeq 1 + 5\alpha$. For the angular spectrum of CMB anisotropies, this will result in a small change in the power on small scales, and also in a change of the contribution from the integrated Sachs-Wolfe effect, as a result of the decaying or growing Newtonian potential [22]. For $\alpha > 0$, there will be less power on the large scale CMB power spectrum, whereas a negative α will lead to an increase of power. Correspondingly, the acoustic peak of the CMB power spectrum will be slightly shifted, due to the change in the cosmic expansion history (see Fig. 2 in [14], in which $\Omega_Q = -\alpha$). We compute the constraints on α from the CMB using a modified version of CMBEASY [23] for the quadratic Cuscuton model (see [14] for details). We find that the 3-yr CMB power spectrum of the Wilkinson Microwave Anisotropy Probe (WMAP) [24] constrains α to -0.005 ± 0.040 (at 95% C.L.).

The amplitude of a UV/IR glitch can also be constrained through its impact on structure formation, and, in particular, through the change in the amplitude of the matter power spectrum (in comparison to the CMB power) on small scales at late times. In addition, the factor of suppression or enhancement (depending on the sign of α) of different modes depends on the time when they enter the horizon, as we have seen above. As a result, the cold dark matter power spectrum will also be tilted between the equality and present-day horizon scales.

To quantify these effects, we use the latest data from the distribution of luminous red galaxies from the Sloan Digital Sky Survey [25] (marginalizing over bias). We also include constraints on the cold dark matter power spectrum from observations of the Lyman-α forest [26]. Even though these data extend into the mildly nonlinear regime of the power spectrum, we expect nonlinear effects (of a nonvanishing α) to be of little importance here, in particular, because the bounds on α are already rather tight from CMB alone. Adding the results from supernovae Ia

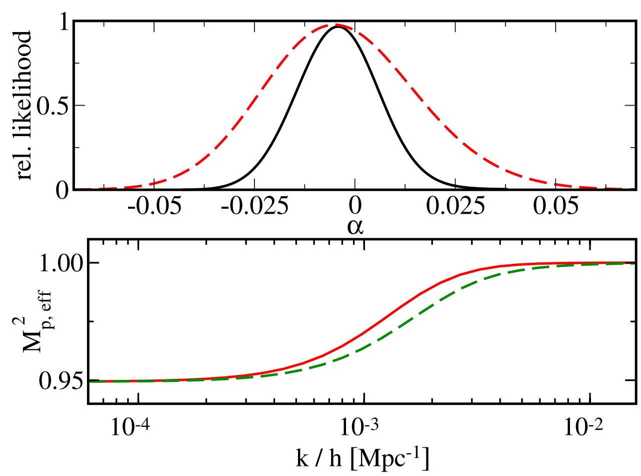

FIG. 1 (color online). Top: Observational constraints on the UV/IR Planck mass mismatch parameter α from 3 years of WMAP data alone [24] (red, dashed line), and our compilation (see the text) of cosmological observations (black straight line). Bottom: Transition between the IR and UV regimes for the effective Planck mass (in units of $M_{p,\mathrm{UV}}$) defined in Eq. (8) for the quadratic Cuscuton with $\alpha = -0.05$ (red, solid line). The transition for a canonical scalar field model ($c_s^2 = 1$) is depicted in green (dashed line). For $c_s^2 \gtrsim 10$, the transitions virtually coincide with the quadratic Cuscuton (for which $c_s^2 = \infty$).

observations [27] as well as the observation of the baryon acoustic oscillations [28], and the data from 3 years of WMAP [24], we find the UV/IR mismatch parameter α to be tightly constrained to -0.004 ± 0.021 (95% C.L.) by our complete set of current observational data (see Fig. 1).

One may wonder if our constraints on α may depend on the specific model that yields the running of the effective Planck mass. Figure 1 compares the UV-IR transition of $M^2_{p,\mathrm{eff}}$ (in longitudinal gauge) for the quadratic Cuscuton and the exponential scalar field models in the matter era. The transition for the canonical scalar field model with $c_s^2 = 1$ is shifted to slightly smaller scales by $\sim$30%. However, the bounds on α are only slightly relaxed, namely, to -0.004 ± 0.024 at the 95% C.L. Therefore, we conclude that the constraints on α are insensitive to the details of UV/IR transition, since most of the observable consequences of the mismatch occur on small subhorizon scales.

What about an arbitrary redshift evolution of the running factor $\alpha(z)$? While this is in principle possible, it would require introducing an *ad hoc* macroscopic scale (coincident with the present-day horizon) into the theory. In lieu of any such scale, the cosmological horizon is the only macroscopic scale in the problem that could control the running of gravitational coupling constants. Therefore, a constant α is the only natural result of a nontrivial *microscopic* physics in the gravity theory.

To summarize, in this Letter, we have studied the running of the Planck mass (or Newton's constant) on the cosmological-horizon scale, as a possible modification of Einstein gravity. We considered observable consequences of this running and found out that any mismatch between UV and IR Planck masses (Newton's constants) is severely constrained to less than 1.2% (2.4%) at the 95% C.L. While future cosmological observations are likely to strengthen this bound by an order of magnitude over the next decade, the expected magnitude of such a running or glitch in well-motivated extensions to Einstein gravity is yet to be determined.

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