Search for Lorentz and *CPT* Violation Effects in Muon Spin Precession

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The spin precession frequency of muons stored in the $(g-2)$ storage ring has been analyzed for evidence of Lorentz and *CPT* violation. Two Lorentz and *CPT* violation signatures were searched for a nonzero $\Delta\omega_a (= \omega_a^{\mu^+} - \omega_a^{\mu^-})$ and a sidereal variation of $\omega_a^{\mu^\pm}$. No significant effect is found, and the following limits on the standard-model extension parameters are obtained: $b_Z = -(1.0 \pm 1.1) \times 10^{-23}$ GeV; $(m_\mu d_{Z0} + H_{XY}) = (1.8 \pm 6.0) \times 10^{-23}$ GeV; and the 95% confidence level limits $\check{b}_\perp^{\mu^+} < 1.4 \times 10^{-24}$ GeV and $\check{b}_\perp^{\mu^-} < 2.6 \times 10^{-24}$ GeV.

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The minimal standard model of particle physics is Lorentz and *CPT* invariant. Since the standard model is expected to be the low-energy limit of a more fundamental theory such as string theory that incorporates gravity, Lorentz and *CPT* invariance might be broken spontaneously in the underlying theory [1]. At low energies, the Lorentz and *CPT* violation signals are expected to be small but perhaps observable in precision experiments.

To describe the effects of spontaneous breaking of Lorentz and *CPT* invariance, Colladay and Kostelecký [2] proposed a general standard-model extension that can be viewed as the low-energy limit of a Lorentz covariant theory. Lorentz and *CPT* violating terms are introduced into the Lagrangian as a way of modeling the effect of spontaneous symmetry breaking in the underlying fundamental theory. Other conventional properties of quantum field theory such as gauge invariance, renormalizability, and energy conservation are maintained, and the effective theory can be quantized by the conventional approach. In a subsequent paper, Bluhm, Kostelecký, and Lane discussed specific precision experiments with muons that could be sensitive to the *CPT* and Lorentz-violating interactions [3].

In this Letter we present our analysis for *CPT* and Lorentz-violating interactions in the anomalous spin precession frequency, ω_a, of the muon moving in a magnetic field. In experiment E821 [4] at the Brookhaven National Laboratory Alternating Gradient Synchrotron, muons are stored in a magnetic storage ring that uses electrostatic quadrupoles for vertical focusing. The storage ring has a highly uniform magnetic field with a central value of $B_0 = 1.45$ T, and a central radius of $\rho = 7.112$ m. Polarized muons are injected into the storage ring, and the positrons (electrons) from the parity-violating decay $\mu^{+(-)} \to e^{+(-)}\bar{\nu}_\mu(\nu_\mu)\nu_e(\bar{\nu}_e)$ carry average information on the muon spin direction at the time of the decay. Twenty-four electromagnetic calorimeters around the ring provide the arrival time and energy of the decay positrons. As the muon spin precesses relative to the momentum with the frequency ω_a, which is the difference between the spin precession frequency ω_S and the momentum (cyclotron) frequency ω_C [4], the number of high-energy positrons is modulated by ω_a. In the approximation that $\vec{\beta} \cdot \vec{B} = 0$,

$$\vec{\omega}_a = -\frac{q}{m_\mu}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta} \times \vec{E}\right], \quad (1)$$

where $\gamma = 1/\sqrt{1-\beta^2}$. The anomaly a_μ is related to the spin g factor by $a_\mu = (g_\mu - 2)/2$, with the magnetic moment given by $\vec{\mu} = g(q/2m_\mu)\vec{s}$, and $q = \pm e$. The "magic" momentum of $p_m = 3.09$ GeV/c ($\gamma_m = 29.3$) was used in E821 so that the second term in Eq. (1) vanishes, and the electric field does not contribute to ω_a.

The magnetic field is measured using nuclear magnetic resonance (NMR) techniques [4]. The NMR frequency, averaged over both the muon distribution and the data collection period, is tied through calibration to the Larmor frequency for a *free* proton and is denoted by ω_p [4]. Thus two frequencies are measured, ω_a and ω_p.

In the analysis presented here, the muon frequency ω_a is obtained from a fit of the positron (electron) arrival-time spectrum $N(t)$ to the 5-parameter function

$$N(t) = N_0 e^{-(t/\gamma\tau)}[1 + A\cos(\omega_a t + \phi)]. \quad (2)$$

The normalization N_0, asymmetry A, and phase ϕ depend on the chosen energy threshold E. While more complicated fitting functions are used in the analysis for the muon anomaly [4], they represent small deviations from Eq. (2) and are not necessary for the CPT analysis. The anomalous magnetic moment a_μ is calculated from $a_\mu = \mathcal{R}/(\lambda - \mathcal{R})$, where $\mathcal{R} \equiv \omega_a/\omega_p$ and $\lambda \equiv \mu_\mu/\mu_p = 3.183\,345\,39(10)$ ($\pm$ 30 ppb) is the muon-proton magnetic moment ratio [5].

For the muon, the Lorentz and CPT violating terms in the Lagrangian are [3]

$$L' = -a_\kappa \bar{\psi}\gamma^\kappa\psi - b_\kappa\bar{\psi}\gamma_5\gamma^\kappa\psi - \tfrac{1}{2}H_{\kappa\lambda}\bar{\psi}\sigma^{\kappa\lambda}\psi + \tfrac{1}{2}ic_{\kappa\lambda}\bar{\psi}\gamma^\kappa \overleftrightarrow{D}^\lambda\psi + \tfrac{1}{2}id_{\kappa\lambda}\bar{\psi}\gamma_5\gamma^\kappa\overleftrightarrow{D}^\lambda\psi, \quad (3)$$

where $iD_\lambda \equiv i\partial_\lambda - qA_\lambda$, and the small parameters a_κ, b_κ, $H_{\kappa\lambda}$, $c_{\kappa\lambda}$, and $d_{\kappa\lambda}$ represent the Lorentz and CPT violation. All terms violate Lorentz invariance with $a_\kappa\bar{\psi}\gamma^\kappa\psi$ and $b_\kappa\bar{\psi}\gamma_5\gamma^\kappa\psi$ CPT odd; all the other terms are CPT even. In this model the conventional figure of merit $r_g^\mu \equiv |g_{\mu^+} - g_{\mu^-}|/g_{\text{average}}$ is zero at leading order; however, effects on the anomalous spin precession frequency ω_a do exist in lowest order [3]. The frequency ω_a is proportional to the magnetic field and therefore to ω_p, so the sidereal variation of $\mathcal{R} = \omega_a/\omega_p$ is analyzed, rather than ω_a directly.

To compare results from different experiments, it is convenient to work in the nonrotating standard celestial equatorial frame $\{\hat{X}, \hat{Y}, \hat{Z}\}$ [6]. The $\hat{Z}$ axis is along the earth's rotational north pole, with the $\hat{X}$ and $\hat{Y}$ axes lying in the plane of Earth's equator. The precession of the Earth's rotational axis can be ignored because its precession period is 26 000 years. In this frame the correction to the (standard-model) muon anomalous precession frequency $\omega_a^{\mu^\pm}$ in Eq. (1) is calculated to be

$$\delta\omega_a^{\mu^\pm} \approx 2\check{b}_Z^{\mu^\pm}\cos\chi + 2(\check{b}_X^{\mu^\pm}\cos\Omega t + \check{b}_Y^{\mu^\pm}\sin\Omega t)\sin\chi; \quad (4)$$

$$\check{b}_J^{\mu^\pm} \equiv \pm\frac{b_J}{\gamma} + m_\mu d_{J0} + \frac{1}{2}\varepsilon_{JKL}H_{KL} \qquad (J = X, Y, Z), \quad (5)$$

where χ is the geographic colatitude ($= 90° -$ latitude) of the experiment location. For E821, $\chi_{\text{BNL}} = 49.1°$. The sidereal angular frequency is $\Omega = 2\pi/T_s$, where $T_s \approx$ 23 h 56 min. Equation (4) predicts two signatures of Lorentz and CPT violation: a difference between the time averages of $\omega_a^{\mu^+}$ and $\omega_a^{\mu^-}$, and oscillations in the values of $\omega_a^{\mu^+}$ and $\omega_a^{\mu^-}$ at the sidereal angular frequency.

The E821 muon $(g-2)$ data have been analyzed for these two signatures. Data from the 1999 run (μ^+) [7], 2000 (μ^+) [8], and 2001 (μ^-) [9] runs were used. Since bounds on clock comparisons of ^{199}Hg and ^{133}Cs [6,10] place limits on the Lorentz-violating energy shifts in the proton precession frequency (ω_p) of $\sim 10^{-27}$ GeV, any shifts in the NMR measurements are at the mHz level, which is negligible compared to the uncertainty in the amplitude of any sidereal variation in ω_a. A feedback system based on the reading from NMR probes at fixed locations around the ring keeps the field constant to within 3 ppm. Both the proton frequency ω_p and the muon frequency ω_a are measured relative to a clock stabilized to a LORAN C 10 MHz frequency standard [11], via a radio signal. The LORAN C frequency standard is based on cesium hyperfine transitions with $m_F = 0$, which are insensitive to orientation of the clock. The Michelson-Morley–type experiment of Brillet and Hall [12] establishes that the fractional frequency shift, $\Delta f/f$, of the LORAN radio signals due to Earth's rotation is less than 10^{-14}, so at the level of the precision of this experiment, the measured value of ω_p is independent of sidereal time. Moreover, since LORAN C is the frequency standard for ω_a as well, any sidereal variation in the LORAN C standard would cancel in $\mathcal{R}$.

Comparison of the time averages of ω_a over many sidereal days [Eq. (4)] gives $\Delta\omega_a \equiv \langle\omega_a^{\mu^+}\rangle - \langle\omega_a^{\mu^-}\rangle = (4b_Z/\gamma)\cos\chi$. For measurements made at different χ and/or ω_p,

$$\Delta\mathcal{R} = \frac{2b_Z}{\gamma}\left(\frac{\cos\chi_1}{\omega_{p1}} + \frac{\cos\chi_2}{\omega_{p2}}\right) + 2(m_\mu d_{Z0} + H_{XY})\left(\frac{\cos\chi_1}{\omega_{p1}} - \frac{\cos\chi_2}{\omega_{p2}}\right). \quad (6)$$

The colatitudes for the E821 μ^+ and μ^- measurements were identical, and the slightly different values of ω_p [4] can be neglected. The E821 result [4] is $\Delta\mathcal{R} = -(3.6 \pm 3.7) \times 10^{-9}$, which corresponds to

$$b_Z = -(1.0 \pm 1.1) \times 10^{-23}\ \text{GeV} \quad \text{or} \qquad r_{\Delta\omega_a} \equiv \frac{\Delta\omega_a}{m_\mu} = -(8.7 \pm 8.9) \times 10^{-24}, \quad (7)$$

a factor of 22 improvement over the limit that can be obtained from the CERN muon $(g-2)$ experiment [13].

The second signature of Lorentz and *CPT* violation would be a variation of $\mathcal{R}(t)$ with a period of one sidereal day. The data are collected in "runs" of approximately 60-minute duration, each with a time stamp at the beginning and end of the run. Data from each of these different time intervals are fitted to the 5-parameter function [Eq. (2)] to obtain ω_a, with the center of the time interval assigned as the time of that interval. The average magnetic field, ω_p, for that time interval is used to determine $\mathcal{R}(t)$. Figure 1 shows $\omega_a(t)$, $\omega_p(t)$, and $\mathcal{R}(t)$ as a function of time for the 2001 data collection period.

A sidereal variation of ω_a can be written as $\omega_a(\omega_p(t_i), t_i) = K\omega_p(t_i) + A_\Omega \cos(\Omega t_i + \phi)$. Dividing by ω_p gives

$$\mathcal{R}(t_i) = K + \frac{A_\Omega}{\omega_p(t_i)}\cos(\Omega t_i + \phi), \quad (8)$$

where $K = \lambda a_\mu/(1 + a_\mu)$ is a constant, and A_Ω is the amplitude of the sidereal variation with the sidereal period $2\pi/\Omega$. Two analysis techniques were used to search for an oscillation at the sidereal frequency: a multiparameter fit to Eq. (8), and the Lomb-Scargle test [14], a spectral analysis technique developed for unevenly sampled data such as those displayed in Fig. 1. With evenly sampled data it reduces to the usual Fourier analysis. For the time series $\{h_i\}$ with $i = 1, \ldots, N$, the normalized Lomb power at frequency ω is defined as

$$P_N(\omega) \equiv \frac{1}{2\sigma^2}\left\{\frac{[\sum_{i=1}^N (h_i - \bar{h})\cos[\omega(t_i - \tau)]]^2}{\sum_{i=1}^N \cos^2[\omega(t_i - \tau)]} + \frac{[\sum_{i=1}^N (h_i - \bar{h})\sin[\omega(t_i - \tau)]]^2}{\sum_{i=1}^N \sin^2[\omega(t_i - \tau)]}\right\}, \quad (9)$$

where $\bar{h}$, σ, and τ are defined as

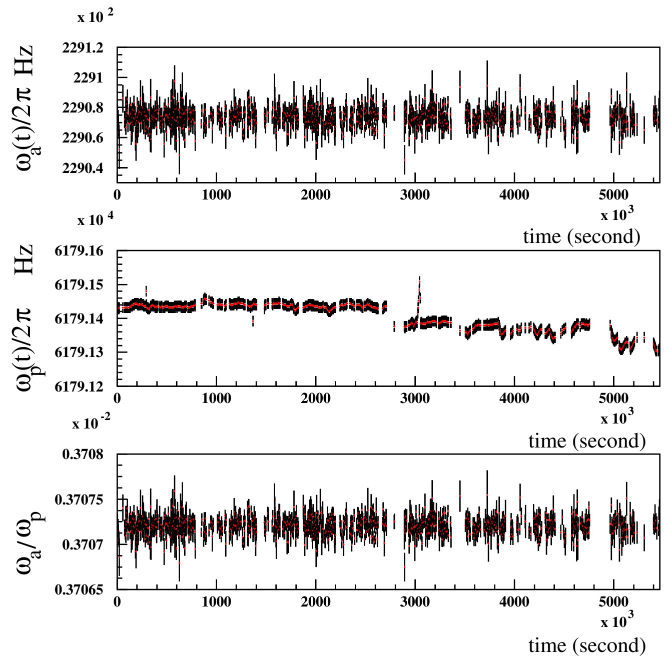

FIG. 1 (color online). Values of $\omega_a(t)$, $\omega_p(t)$, and $\mathcal{R}(t) \equiv \frac{\omega_a}{\omega_p}(t)$ from the 2001 μ^- run. The uncertainty on each ω_a point is about 20 ppm.

$$\bar{h} \equiv \frac{\sum_{i=1}^N h_i}{N}, \qquad \sigma^2 \equiv \frac{1}{N-1}\sum_{i=1}^N (h_i - \bar{h})^2, \qquad \tan(2\omega\tau) \equiv \left(\sum_{i=1}^N \sin 2\omega t_i \Big/ \sum_{i=1}^N \cos 2\omega t_i\right). \quad (10)$$

In searching for a periodic signal, the Lomb power is calculated over a set of frequencies. For a single frequency, with no corresponding periodic signal, the Lomb power is distributed exponentially with unit mean. If *M independent* frequencies are scanned, the probability that none of them are characterized by a Lomb power greater than z is $(1 - e^{-z})^M$, assuming there is no signal present. The significance (confidence) level of any peak in $P_N(\omega)$ is $1 - (1 - e^{-z})^M$, which is the probability of the frequencies being scanned giving a Lomb power greater than z due to a statistical fluctuation. A small value of this probability therefore indicates the presence of a significant periodic signal. In the case of equally separated points, the number of independent frequencies is almost equal to the number of time values. The lowest independent frequency, f_0, is the inverse of the data's time span and the highest is roughly $(N/2)f_0$, but, because of the uneven time sampling, may be somewhat greater. More generally, the number of independent frequencies, which depends on the number and spacing of the points, as well as the number of frequencies scanned, can be determined by Monte Carlo simulation, using Eq. (9) to fit for M.

The frequency spectrum for the 2001 data is shown in Fig. 2. The Lomb power at the sidereal frequency is 3.4.

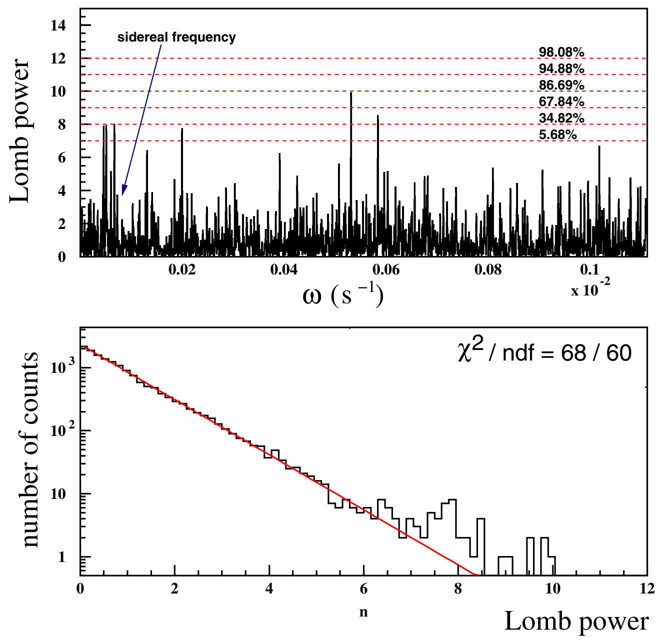

FIG. 2 (color online). The Lomb-Scargle test on the 2001 data (top plot). The horizontal lines show the confidence level associated with each Lomb power. At the sidereal frequency the Lomb power is 3.4, corresponding confidence level less than 10^{-3}%. The distribution of the Lomb power of the scanned frequencies (bottom plot) is consistent with an exponential distribution, indicating there is no time-varying signal in the real data.

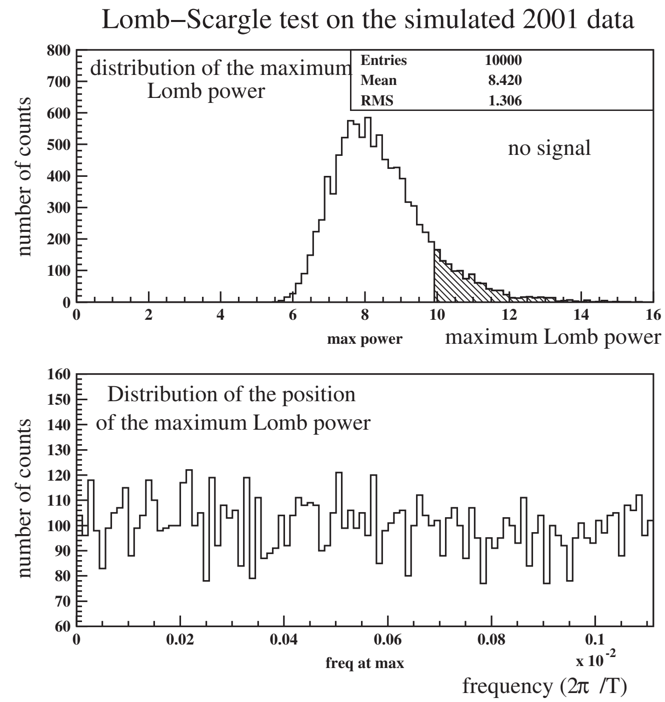

FIG. 3. The Lomb-Scargle test on the simulated 2001 data with no signal. The shaded area shows the maximum Lomb powers greater than that of the real data. The locations of the maximum Lomb powers are randomly distributed (bottom plot).

The probability that this is inconsistent with the null hypothesis is negligible. The Lomb power distribution of scanned frequencies (for $M = 3144$) shown in the lower half of Fig. 2 is consistent with an exponential distribution, indicating there is no significant time-varying signal in the data. The Lomb power spectrum reaches only a modest maximum of 9.93 at $f_{\max} = 8.4 \times 10^{-5}$ Hz, and is not present in the 1999 or 2000 data sets. If there were no oscillation signal at $f_{\max}$, then 14% of the time the measured power is expected to be that large. Moreover, only a few of the Lomb power spectra taken from data subsets reveal a peak at that frequency.

The significance of a possible sidereal signal in the power spectrum was carefully studied with a Monte Carlo simulation. First, a large number of artificial time spectra were generated. The distribution of times of the data points is chosen to be the same as for the real data. The value of each data point was distributed randomly with a central value equal to an average over all the actual data values, while having a standard deviation equal to that of each individual data point. The Lomb-Scargle test was then applied to 10 000 simulated data groups. The distribution of maximum Lomb power and the corresponding frequencies are shown in Fig. 3.

Next, sidereal oscillation signals of different amplitudes A_Ω were introduced into the simulated data, and the resulting spectra were analyzed with both the multiparameter fit and Lomb-Scargle test. Table I lists the signal amplitude required so that 95% of the simulated data sets yield a larger A_Ω or $P(\omega)$ than the real data. The two analysis methods give consistent results.

Several potential systematic effects were studied. Since the sidereal period is very close to 1 solar day, the Lomb-Scargle test was applied to the ω_p data to check for false sidereal variations that might be produced by diurnal temperature changes. The upper limits on the amplitude of a sidereal variation in ω_p were 0.04, 0.03, and 0.08 ppm for the 1999, 2000, and 2001 data sets, respectively, significantly smaller than the limit on A_Ω presented above. Additional studies were carried out on the 2001 data set. The data were folded back over a four sidereal day time period (i.e., modulo four sidereal days), and then analyzed. To search for systematic effects, other subwindow time

TABLE I. The signal amplitude in ppm needed for 95% of the simulated data to have larger A_Ω, or $P(\Omega)$, than that of the real data. MPF means multiparameter fit, L-S stands for Lomb-Scargle.

Data set	MPF amplitude (ppm)	L-S amplitude (ppm)
1999 μ^+	5.5	5.2
2000 μ^+	2.2	2.0
1999/2000 μ^+	2.2	2.0
2001 μ^-	4.2	4.2

periods, where no variation was expected, were also used, e.g., 24 hours (solar day), or an arbitrary number of minutes.

No significant sidereal variation in $\mathcal{R}$, and hence in ω_a, is found in the E821 muon $(g-2)$ data. The limits on A_Ω from the multiparameter fit in Table I give at the 95% confidence level:

$$\check{b}_\perp^{\mu^+} = \sqrt{(\check{b}_X^{\mu^+})^2 + (\check{b}_Y^{\mu^+})^2} < 1.4 \times 10^{-24}\ \text{GeV},$$
$$\check{b}_\perp^{\mu^-} = \sqrt{(\check{b}_X^{\mu^-})^2 + (\check{b}_Y^{\mu^-})^2} < 2.6 \times 10^{-24}\ \text{GeV}. \quad (11)$$

For the dimensionless figure of merit obtained by dividing by m_μ [3], we obtain $r_{A_\Omega}^{\mu^+} \equiv 2\sin\chi b_\perp^{\mu^+}/m_\mu < 2.0 \times 10^{-23}$ and $r_{A_\Omega}^{\mu^-} < 3.8 \times 10^{-23}$, which can be compared with the ratio of the muon to Planck mass, $m_\mu/M_P = 8.7 \times 10^{-21}$. Using the E821 and CERN [13] values of $\mathcal{R}$ for μ^+ and μ^- along with Eq. (6), we find

$$(m_\mu d_{Z0} + H_{XY}) = (1.8 \pm 6.0 \times 10^{-23}\ \text{GeV}. \quad (12)$$

An experiment that searched for sidereal variation in transitions between muonium hyperfine energy levels [15] obtained $r^\mu \le 5.0 \times 10^{-22}$. Penning trap experiments with a single trapped electron obtained $r^e \le 1.6 \times 10^{-21}$ [16].

We find no significant Lorentz and *CPT* violation signatures in the E821 muon $(g-2)$ data, which we interpret in the framework of the standard-model extension [3]. Limits on the parameters are of the order 10^{-23} to 10^{-24} GeV, with the dimensionless figures of merit $\sim 10^{-23}$. These results represent the best test of this model for leptons. Both $r_{\Delta\omega_a}^\mu$ and $r_{A_\Omega}^\mu$ are much less than m_μ/M_P, so E821 probes Lorentz and *CPT* violation signatures beyond the Planck scale.

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