# Kerr Black Holes Are Not Unique to General Relativity

Dimitrios Psaltis,[1,2] Delphine Perrodin,[1] Keith R. Dienes,[1] and Irina Mocioiu[3]

[1]*Department of Physics, University of Arizona, Tucson, Arizona 85721, USA*

[2]*Department of Astronomy, University of Arizona, Tucson, Arizona 85721, USA*

[3]*Department of Physics, Pennsylvania State University, University Park, Pennsylvania 16802, USA*

(Received 30 April 2007; published 3 March 2008; publisher error corrected 17 March 2008)

Considerable attention has recently focused on gravity theories obtained by extending general relativity with additional scalar, vector, or tensor degrees of freedom. In this Letter, we show that the black-hole solutions of these theories are essentially indistinguishable from those of general relativity. Thus, we conclude that a potential observational verification of the Kerr metric around an astrophysical black hole cannot, in and of itself, be used to distinguish between these theories. On the other hand, it remains true that detection of deviations from the Kerr metric will signify the need for a major change in our understanding of gravitational physics.

DOI: 10.1103/PhysRevLett.100.091101 PACS numbers: 04.70.Bw, 04.50.−h, 97.60.Lf

Black holes are among the most extreme astrophysical objects predicted by general relativity. They are vacuum solutions of the Einstein field equations realized astrophysically at the end stages of the collapse of massive stars. According to a variety of no-hair theorems, a general relativistic black hole is characterized only by three parameters identified with its gravitational mass, spin, and charge. Any additional ''hair'' on the black hole, associated with the properties of the progenitor star or the collapse itself, are radiated away in the form of gravitational waves over a finite amount of time.

Black holes might look different if general relativity is only an effective theory of gravity, valid at the curvature scales probed by current terrestrial and astrophysical experiments. If the more fundamental gravity theory has additional degrees of freedom, they might appear as additional hair to the black hole. This would be important for a number of reasons. First, additional degrees of freedom appear naturally in all attempts to quantize gravity, either in a perturbative approach [1] or within the context of string theory [2]. Detecting observational signatures of these additional degrees of freedom in black-hole spacetimes would serve as a confirmation of quantum gravity effects. Second, black-hole solutions not described by the Kerr-Newman metric may follow a set of thermodynamic relations different from those calculated by Bekenstein [3] and Hawking [4] with important implications for string theory [5]. Finally, the external spacetimes of astrophysical black hole will soon be mapped with gravitational-wave [6] and high-energy observations [7] and the means for searching for black holes with additional degrees of freedom will become readily available.

Introducing additional degrees of freedom to the Einstein-Hilbert action of the gravitational field does not necessarily alter the resulting field equations and hence the black-hole solutions. For example, the addition of a Gauss-Bonnet term to the action leaves the field equation completely unchanged [1]. Moreover, a large class of gravity theories in the Palatini formalism for which the action is a general function $f(R)$ of the Ricci scalar curvature $R$, lead to field equations that are indistinguishable from the general relativistic ones [8]. In all these situations, no astrophysical observation of a classical phenomenon, such as test particle orbits or gravitational lensing, can distinguish between these theories. Nevertheless, this leaves a large number of Lagrangian gravity theories that incorporate general relativity as a limiting case but are described by more general field equations.

The most widely studied such extension of general relativity is the Brans-Dicke gravity, which incorporates a dynamical scalar field in addition to the metric tensor. Black-hole solutions in this theory were studied by Thorne and Dykla [9]. Following a conjecture by Penrose, these authors showed that the Kerr solution of general relativity is also an exact solution of the field equations in Brans-Dicke gravity and offered a number of arguments to support the claim that the collapse of a star in this gravity theory will produce uniquely a Kerr black hole. Additional analytic [4,10,11] and numerical [12] arguments were offered by other authors providing further evidence for the uniqueness of the Kerr solution in Brans-Dicke gravity.

In this Letter, we show that black-hole solutions of the general relativistic field equations are indistinguishable from solutions of a wide variety of gravity theories that arise by adding dynamical vector and tensor degrees of freedom to the Einstein-Hilbert action. Although we do not prove that the general relativistic vacuum solution is the *unique* solution of the extended Lagrangian theories, we use our results to argue that an observational verification of the Kerr solution for an astrophysical object cannot be used in distinguishing between general relativity and other Lagrangian theories such as those considered here. Note that we are only considering four-dimensional theories that obey the equivalence principle, and hence we are not studying theories with prior geometry [13], that are Lorentz violating [15], or braneworld gravity theories

 

[16]. Although several of these extensions lead to predictions of an unstable quantum vacuum and of ghosts, we are focusing here on their classical black-hole solutions.

In general relativity, the external spacetimes of black holes that are astrophysically relevant, i.e., with zero charge, are completely specified by the relation

$$R_{\mu\nu} = \frac{R}{4} g_{\mu\nu}, \tag{1}$$

with $R_{,\mu} = 0$. Here $R_{\mu\nu}$ is the Ricci tensor and $R$ is the Ricci scalar curvature. When the cosmological constant $\Lambda$ is considered to be nonzero, then $R = 4\Lambda$.

It is our aim to show that the external spacetimes of general relativistic black holes, which satisfy Eq. (1), are practically indistinguishable from solutions in a number of gravity theories that arise by adding vector or tensor degrees of freedom to the Einstein-Hilbert action

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g}(-2\Lambda + R). \tag{2}$$

It is important to make here a distinction between the external spacetimes of black holes and those of stellar objects, which also satisfy Eq. (1) in general relativity. The field equation of a gravity theory is a high-order partial differential equation and its solutions depend on the boundary conditions imposed. In particular, when solving for the external spacetime of a stellar object, a number of regularity conditions need to be satisfied at the stellar surface, inside which the field equations are altered by the presence of matter. As a result, proving that the external spacetime of a general relativistic star satisfies the vacuum field equation of a different gravity theory is not a guarantee that it will be a valid solution for that theory, as well. It also needs to meet the altered regularity conditions at the stellar surface. This issue was recently explored for $1/R$ gravity in the metric [17] and in the Palatini formalism [18] with important implications for the viability of this theory. This concern, however, is not relevant for black-hole solutions, in which there is no matter anywhere outside the horizon and hence no regularity conditions need to be met. Indeed the vacuum field equation is valid throughout the entire spacetime accessible to a distant observer and only the boundary conditions at radial infinity need to be checked.

*$f(R)$ gravity in the metric formalism.*—A self-consistent theory of gravity can be constructed for any Lagrangian action that obeys a small number (four) of simple requirements [19]. Of all the possibilities, the field equations that are derived from the Einstein-Hilbert action (2) are the only ones that are also linear in the Riemann tensor and result in field equations that are of second order. However, any other action $f(R)$ that depends only on the Ricci curvature scalar will also satisfy the above four requirements [20], while being free of the Ostrogradski instability [21].

The field equation that results from extremizing an action that is a general function of the Ricci scalar $f(R)$ is

$$(-R_{;k}R_{;l} + g_{kl}R_{;m}R^{;m})f'''(R) + (-R_{;kl} + g_{kl}\Box R)f''(R) + R_{kl}f'(R) - \tfrac{1}{2}g_{kl}f(R) = 0, \tag{3}$$

where primes denote differentiation with respect to $R$ and we have used the sign convention of Ref. [19].

A general relativistic black-hole solution, i.e., one that satisfies Eq. (1) with $R_{,\mu} = 0$, will also be a solution of the field equation (3) if

$$\tfrac{1}{2}Rf'(R) - f(R) = 0. \tag{4}$$

We will now consider nonpathological functional forms of $f(R)$ that can be expanded in a Taylor series of the form

$$f(R) = a_0 + R + a_2R^2 + a_3R^3 + \cdots + a_nR^n + \cdots, \tag{5}$$

where we have normalized all coefficients with respect to the coefficient of the linear term. The Einstein-Hilbert action is the specific case of Eq. (5) for $a_0 = -2\Lambda$, and $a_{n\geq 2} = 0$. We can then write the condition (4) for the existence of a constant-curvature solution as

$$-a_0 - \frac{1}{2}R + \frac{1}{2}a_3R^3 + \cdots + \frac{n-2}{2}a_nR^n + \cdots = 0. \tag{6}$$

There are three cases to consider: (i) If $a_0 = 0$, then the Kerr solution, which corresponds to $R = 0$, will always be a solution of the field equations of a general $f(R)$ theory. Thus, in the absence of a cosmological constant, we conclude that the Kerr solution of general relativity remains an exact solution to all $f(R)$ theories as long as $f(R)$ has a Taylor expansion of the form in Eq. (5). (ii) Moreover, independent of the value of $a_0$, all of the constant-curvature solutions of general relativity in vacuum—including the Kerr solution—remain exact solutions of the $f(R)$ theory, if the Taylor series for $f(R)$ terminates after the quadratic term (i.e., if $a_{n\geq 3} = 0$). Indeed, this statement remains true independently of the value of $a_0$, and thus holds for both vanishing and nonvanishing cosmological constants. (iii) Finally, if $a_0 \neq 0$ and the Taylor expansion extends beyond the quadratic term, then Kerr-like black-hole solution will always be possible. The only change is that the value of its constant curvature will be shifted relative to the value predicted in general relativity. Since terrestrial and solar-system tests require any extra nonlinear terms in the gravity action to be perturbative, this shift in the curvature will also be correspondingly small. However, even in this case, it is straightforward to show that the corrections to the curvature are actually suppressed by additional powers of the cosmological constant relative to what might naively have been expected on the basis of dimensional analysis. For example, given the expansion for $f(R)$ in Eq. (5), we would have expected the curvature term to have a leading correction term which scales as $R = -2a_0[1 + O(a_0a_2) + \ldots]$. However, explicitly solving Eq. (6), we find that the true leading correction is actually given by

$$R = -2a_0(1 + 4a_0^2a_3 + \ldots). \tag{7}$$

Thus the deviations of the vacuum curvature solutions of $f(R)$ gravity from those of general relativity are particularly suppressed.

*$f(R)$ gravity in the Palatini formalism.*—In deriving the field equation (3), we extremized the action of the gravitational field with respect only to variations in the metric. In the so-called Palatini formalism, field equations of lower order can be derived from the same action of the gravitational field, by extremizing it over both the metric and the connection [22]. A large class of $f(R)$ theories in the Palatini formalism are known to result in the same field equations as general relativity [8].

Applying this procedure for a gravitational action that is a general function $f(R)$ of the Ricci scalar curvature, we obtain the well-known set of equations [22]

$$R_{kl}f'(R) - \tfrac{1}{2}g_{kl}f(R) = 0, \tag{8}$$

$$\nabla_\sigma[\sqrt{-g}f'(R)g^{\mu\nu}] = 0. \tag{9}$$

In order to look for constant-curvature solutions in vacuum for this theory, we first take the trace of Eq. (8). The result is simply the algebraic equation (4), which we can solve for the value of the constant curvature (7) as before. For a solution with constant curvature, the factor $f'(R)$ in Eq. (9) is a constant, and the solutions to this equation are simply the Christoffel symbols of general relativity. As a result, any general relativistic solution of constant curvature, such as the black-hole solutions with cosmological constant, will also be solutions (with the same or slightly different value of the cosmological constant) to the field equations of an $f(R)$ gravity in the Palatini formalism.

*General quadratic gravity.*—We shall now consider a gravitational action that incorporates all combinations of the Ricci curvature, Ricci tensor, and Riemann tensor, up to second order, i.e.,

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(-2\Lambda + R + \alpha R^2 + \beta R_{\sigma\tau}R^{\sigma\tau} + \gamma R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}), \tag{10}$$

with $\alpha$, $\beta$, and $\gamma$ the parameters of the theory. Such terms appear naturally as radiative corrections to the Einstein-Hilbert action in perturbative approaches to quantum gravity [1] or in string theory [2]. Note, however, that in general such theories are not free of the Ostrogradski instability [21].

Because of the Gauss-Bonnet identity, the predictions of the theory described by the action (10) in calculating classical properties of astrophysical black holes are identical to those of the action [23]

$$S = \frac{1}{16\pi G}\int \sqrt{-g}(-2\Lambda + R + \alpha' R^2 + \beta' R_{\sigma\tau}R^{\sigma\tau}), \tag{11}$$

where $\alpha' = \alpha - \gamma$ and $\beta' = \beta + 4\gamma$.

The field equation for this action in the metric formalism is

$$R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} + \alpha' K_{\mu\nu} + \beta' L_{\mu\nu} + \Lambda g_{\mu\nu} = 0, \tag{12}$$

where

$$K_{\mu\nu} \equiv -2R_{;\mu\nu} + 2g_{\mu\nu}\Box R - \tfrac{1}{2}R^2g_{\mu\nu} + 2RR_{\mu\nu}, \tag{13}$$

$$L_{\mu\nu} \equiv -2R^\sigma_{\mu;\sigma\nu} + \Box R_{\mu\nu} + \tfrac{1}{2}g_{\mu\nu}\Box R - \tfrac{1}{2}g_{\mu\nu}R_{\sigma\tau}R^{\sigma\tau} + 2R^\alpha_\mu R_{\alpha\nu}. \tag{14}$$

It is trivial to show that, for any black-hole solution satisfying Eq. (1), $K_{\mu\nu} = L_{\mu\nu} = 0$ and the field equation of quadratic gravity reduces to that of general relativity. As a result, the Kerr solution is also a solution of the general quadratic theory considered here.

*Vector-tensor gravity.*—We finally consider a gravitational theory that incorporates a dynamical vector field in addition to the metric tensor. *A priori*, such an addition to the Einstein-Hilbert action appears to have the highest probability of requiring black-hole solutions that are not described by the Kerr metric. This is because the vector field has the same spin as photons, the geodesics of which are used to define the event horizon of a black hole. We restrict our attention to Lagrangian theories that are linear and at most of second order in the vector field. The most general action for such a theory is [14]

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(-2\Lambda + R + \omega RK_\mu K^\mu + \eta K^\mu K^\nu R_{\mu\nu} - \epsilon F_{\mu\nu}F^{\mu\nu} + \tau K_{\nu;\mu}K^{\mu;\nu}), \tag{15}$$

with

$$F_{\mu\nu} = K_{\nu;\mu} - K_{\mu;\nu}. \tag{16}$$

The vector field $K_\mu$ at large distances from an object is meant to asymptote smoothly to a background value determined by a cosmological solution. Note that the values of the model parameters $\omega$, $\eta$, $\epsilon$, and $\tau$ are not independent [14].

As in the case of previous investigations of scalar-tensor gravity [9], we will be seeking vacuum solutions that are characterized by constant curvature, as well as by a constant vector $K_\mu$. In this case, the field equations that are derived from the action (15) are [14]

$$R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} + \omega\Theta^{(\omega)}_{\mu\nu} + \eta\Theta^{(\eta)}_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \tag{17}$$

$$\omega K_\mu R + \eta K^\alpha R_{\mu\alpha} = 0, \tag{18}$$

where $K^2 \equiv K_\mu K^\mu$,

$$\Theta^{(\omega)}_{\mu\nu} = K_\mu K_\nu R + K^2 R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} K^2 R, \tag{19}$$

$$\Theta^{(\eta)}_{\mu\nu} = 2K^\alpha K_\mu R_{\nu\alpha} - 2K^\alpha K_\nu R_{\mu\alpha} - \tfrac{1}{2} g_{\mu\nu} K^\alpha K^\beta R_{\alpha\beta}. \tag{20}$$

We now multiply Eq. (18) by $K_\nu$, combine it with Eq. (17), and look for the constant-curvature solution (1) to obtain

$$[\Lambda - \tfrac{R}{4}(1 + \omega K^2)] g_{\mu\nu} - \eta \tfrac{R}{4}(K_\mu K_\nu + \tfrac{1}{2} K^2 g_{\mu\nu}) = 0. \tag{21}$$

Contracting Eq. (21) with $g^{\mu\nu}$, we obtain for the constant curvature

$$R = \frac{16\Lambda}{4 + (4\omega + 3\eta)K^2} \simeq 4\Lambda\left[1 - \left(\omega + \frac{3\eta}{4}\right)K^2\right]. \tag{22}$$

As in the previous cases, a black-hole solution that differs only in the value of the constant curvature from the general relativistic one is possible for the vector-tensor gravity theory that we have considered.

*Discussion.*—Our results have important implications for current attempts to test general relativity in the strong-field regime using astrophysical black holes. On the one hand, we appear to be lacking a parametric theoretical framework with which to interpret observational data and quantify possible deviations from the general relativistic predictions for astrophysical black holes. On the other hand, the detection of deviations from the Kerr metric in the spacetime of an astrophysical black hole will be a very strong indication for the need of a major change in our understanding of gravitation.

D. P. is supported in part by the Astrophysics Theory Program of NASA under Grant No. NAG 513374. K. R. D. is supported in part by the U.S. National Science Foundation under Grant No. PHY/0301998, by the U.S. Department of Energy under Grant No. DE-FG02-04ER-41298, and the Research Corporation. I. M. is supported by NSF Grant No. PHY-0555368. We are happy to thank Z. Chacko, C.-K. Chan, K. Sigurdson, and U. van Kolck for discussions.

[1] C. P. Burgess, Living Rev. Relativity **7**, 5 (2004).
[2] M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory* (Cambridge University Press, Cambridge, U.K., 1987), Vols. I and II; J. Polchinski, *String Theory* (Cambridge University Press, Cambridge, U.K., 1988), Vols. I and II.
[3] J. Bekenstein, Phys. Rev. D **7**, 2333 (1973); **9**, 3292 (1974).
[4] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975); Phys. Rev. D **13**, 191 (1976).
[5] J. Maldacena, A. Strominger, and E. Witten, J. High Energy Phys. 12 (1997) 002.
[6] F. D. Ryan, Phys. Rev. D **52**, 5707 (1995); N. A. Collins and S. A. Hughes, *ibid.* **69**, 124022 (2004); K. Glampedakis and S. Babak, Classical Quantum Gravity **23**, 4167 ( 2006).
[7] D. Psaltis, Living Rev. Relativity (to be published).
[8] M. Ferraris, M. Francaviglia, and I. Volovich, Classical Quantum Gravity **11**, 1505 (1994); G. Allemandi, M. Francaviglia, M. L. Ruggiero, and A. Tartaglia, Gen. Relativ. Gravit. **37**, 1891 (2005).
[9] K. S. Thorne and J. J. Dykla, Astrophys. J. **166**, L35 (1971).
[10] J. D. Bekenstein, Phys. Rev. D **5**, 2403 (1972).
[11] J. D. Bekenstein and A. Meisels, Phys. Rev. D **18**, 4378 (1978).
[12] M. A. Scheel, S. L. Shapiro, and S. A. Teukolsky, Phys. Rev. D **51**, 4208 (1995); **51**, 4236 (1995).
[13] For example, Rosen's bimetric theory allows for naked singularities as discussed in Ref. [14].
[14] C. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, U.K., 1993).
[15] C. Eling and T. Jacobson, Classical Quantum Gravity **23**, 5643 (2006); J. W. Moffat, arXiv:gr-qc/0702070.
[16] R. Gregory, R. Whisker, K. Beckwith, and C. Done, J. Cosmol. Astropart. Phys. 10 (2004) 013.
[17] A. L. Erickcek, T. L. Smith, and M. Kamionkowski, Phys. Rev. D (to be published).
[18] E. Barausse, T. P. Sotiriou, and J. C. Miller, Classical Quantum Gravity **25**, 062001 (2008).
[19] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W. H. Freeman and Co., San Francisco, 1973).
[20] R. Wald, *General Relativity* (University of Chicago Press, Chicago, 1984).
[21] R. P. Woodard, arXiv:astro-ph/0601672.
[22] T. P. Sotiriou and S. Liberati, Ann. Phys. (N.Y.) **322**, 935 (2007).
[23] J. D. Barrow and A. C. Ottewill, J. Phys. A **16**, 2757 (1983).