No-Local-Broadcasting Theorem for Multipartite Quantum Correlations

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We prove that the correlations present in a multipartite quantum state have an operational quantum character even if the state is unentangled, as long as it does not simply encode a multipartite classical probability distribution. Said *quantumness* is revealed by the new task of local broadcasting, i.e., of locally sharing preestablished correlations, which is feasible if and only if correlations are strictly classical. Our operational approach leads to natural definitions of measures for quantumness of correlations. It also reproduces the standard no-broadcasting theorem as a special case.

The characterization of correlations present in a quantum state has recently drawn much attention [1–6]. In particular, efforts have been made to analyze whether and how correlations can be understood, quantified, and classified as either classical or quantum. Even if such distinction may not be possible in clear-cut terms, understanding to some extent the quantumness of correlations is not only relevant from a fundamental point of view, but also in order to make more clear the origin of the quantum advantage [6,7], in fields like quantum computing and quantum information [8]. Therefore, while entanglement [9] may be considered the essential feature of quantum mechanics, it is relevant to study how and in what sense even correlations present in unentangled states may exhibit a certain quantum character.

In this Letter we provide an operational characterization of those multipartite states whose correlations may be considered as completely classical, hence, by contrast, also of quantumness. We do this in two ways. First, we consider the process of extracting classical correlations from a quantum state, and we prove that said correlations amount to the total correlations if and only if the quantum state can be interpreted from the very beginning as a classical joint probability distribution. Second, we consider local broadcasting, i.e., the procedure of locally distributing preestablished correlations in order to have more copies of the original state [10]. Again, we prove that local broadcasting is feasible if and only if the state is just a classical probability distribution. We further generalize the latter approach, showing that what really counts is the amount of correlations, as measured by mutual information. All the results presented here are valid for the multipartite case, when bipartite mutual information is substituted by one of its most natural multipartite versions. For the sake of clarity, we derive them in the bipartite case.

We start by recalling [4,6,13] several definitions that make clear what we mean when we discuss classicality and quantumness. A bipartite state ρ is: (i) separable if it can be written as $\sum_i p_i \sigma_i^A \otimes \sigma_i^B$, where p_i is a probability

distribution and each σ_i^X is a quantum state, and entangled if nonseparable; (ii) classical-quantum (CQ) if it can be written as $\sum_i p_i |i\rangle\langle i| \otimes \sigma_i^B$, where $\{|i\rangle\}$ is an orthonormal set, $\{p_i\}$ is a probability distribution and σ_i^B are quantum states; (iii) classical-classical (CC), or (strictly) classically correlated [4,6], if there are two orthonormal sets $\{|i\rangle\}$ and $\{|j\rangle\}$ such that $\rho = \sum_{ij} p_{ij} |i\rangle\langle i| \otimes |j\rangle\langle j|$, with p_{ij} a joint probability distribution for the indexes (i, j). One could consider a CC state to correspond simply to the embedding into the quantum formalism of a classical joint probability distribution.

Every operation on states, corresponding to a measurement or not, will be described by a trace-preserving and completely positive Λ; i.e., Λ admits an expression $\Lambda[\rho] = \sum_i K_i \rho K_i^\dagger$, $\sum_i K_i^\dagger K_i = \mathbb{1}$. A (quantum-to-classical) measurement map $\mathcal{M}$ acts as $\mathcal{M}[X] = \sum_i \mathrm{Tr}(M_i X)|i\rangle\langle i|$, where $\{M_i\}$ is a POVM, i.e., $0 \leq M_i \leq \mathbb{1}$ and $\sum_i M_i = \mathbb{1}$, and $\{|i\rangle\}$ is a set of orthonormal states. A measurement map describes a POVM measurement, with the results written in a classical register (i.e., that can be perfectly read and copied), thus any POVM corresponds to a measurement map. Hence, to any bipartite state ρ and any POVM $\{M_i\}$ (on A, in this case) we can associate a CQ state $\rho^{CQ}(\{M_i\}) = (\mathcal{M}_A \otimes \mathrm{id}_B)[\rho] = \sum_i p_i |i\rangle\langle i| \otimes \sigma_i^B$, where $\mathcal{M}_A$ is the measurement map associated to the POVM, so that $p_i = \mathrm{Tr}(M_i^A \rho)$ and $\sigma_i^B = \mathrm{Tr}_A(M_i^A \rho)/p_i$. Similarly, given POVMs $\{M_i\}$ and $\{N_j\}$ on A and on B, respectively, we can associate to ρ the CC state $\rho^{CC}(\{M_i\}, \{N_j\}) = (\mathcal{M}_A \otimes \mathcal{N}_B)[\rho] = \sum_{ij} p_{ij} |i\rangle\langle i| \otimes |j\rangle \times \langle j|$, with $\mathcal{M}_A$, $\mathcal{N}_B$ the two local measurement maps associated to the two POVMs, and $p_{ij} = \mathrm{Tr}(M_i^A \otimes N_j^B \rho)$. Notice that in this case one may always think that the passage from the initial state ρ to the CC state $\rho^{CC}(\{M_i\}, \{N_j\})$ happens in two separate (and commuting) steps corresponding to the two local POVMs.

(Quantum) mutual information [(Q)MI] $I(\rho_{AB})$ of a bipartite quantum state ρ_{AB} is a measure of total correlations, both from an axiomatic and an operative point of view

[3,5]. It is defined as $I(\rho_{AB}) = S(A) + S(B) - S(AB)$, where $S(X) = S(\rho_X) = -\text{Tr}(\rho_X \log \rho_X)$ is the von Neumann entropy of ρ_X. QMI is the generalization to the quantum scenario of the classical MI for a joint probability distribution $\{p_{ij}^{AB}\}$: $I(\{p_{ij}^{AB}\}) = H(\{p_i^A\}) + H(\{p_i^B\}) - H(\{p_{ij}^{AB}\})$, with $p_i^A = \sum_j p_{ij}^{AB}$ the marginal distribution for A (similarly for B), and $H(\{q_k\}) = -\sum_k q_k \log q_k$ is the Shannon entropy of the classical distribution $\{q_k\}$. QMI can be written as the relative entropy between the total bipartite state and the tensor product of its reductions, i.e. $I(\rho_{AB}) = S(\rho_{AB} || \rho_A \otimes \rho_B)$, with $\rho_X = \text{Tr}_Y(\rho_{XY})$ and $S(\sigma || \tau) = \text{Tr}(\sigma(\log \sigma - \log \tau))$. Thus, QMI is positive, and vanishes only for factorized states. Most importantly, it cannot increase under local operations (LO), i.e., maps of the form $\Lambda_A \otimes \Gamma_B$: $I(\rho_{AB}) \geq I[(\Lambda_A \otimes \Gamma_B)(\rho_{AB})]$ [8]. Operationally, QMI provides the classical capacity of a noisy quantum channel when entanglement is a free unlimited resource [14]. Moreover, in the asymptotic setting $I(\rho_{AB})$ gives the smallest rate of classical randomness necessary and sufficient to erase all correlations present in ρ_{AB} [5].

We will consider two other measures of correlations for a bipartite state, the CQ mutual information and the CC mutual information, defined as $I_{CQ}(\rho_{AB}) = \max_{\{M_i\}} I[\rho^{CQ}(\{M_i\})]$, and $I_{CC}(\rho_{AB}) = \max_{\{M_i\},\{N_j\}} I[\rho^{CC}(\{M_i\}, \{N_j\})]$, respectively. The maxima are taken with respect to (local) measurements. Notice that both CQ mutual information and CC mutual information correspond to the standard QMI of the state after the application of local measurement maps. I_{CC} corresponds exactly to the classical MI of the joint classical distribution $p_{ij} = \text{Tr}(M_i \otimes N_j \rho)$. I_{CQ} was considered—though not in terms of MI—in [3] as a measure of classical correlations, but one may argue that there is still some quantumness in the CQ state entering in its definition. I_{CC} was first defined in [15] and provides the maximum amount of the correlations that are present in the state and that can be considered classical, in the sense that they can be revealed by means of local measurements, and in this way transferred from the quantum to the classical domain (i.e., recorded in classical registers). We have already seen that MI does not increase under LO. In [3] this was proved also for I_{CQ}, and the same holds for I_{CC}, as LO on both sides can be absorbed in the measurements, making the latter nonoptimal. Indeed, given a (local) map $\Lambda[X] = \sum_k A_k X A_k^\dagger$ and a POVM $\{M_i\}$, then $\{\Lambda^\dagger[M_i]\}$, with $\Lambda^\dagger[M_i] = \sum_k A_k^\dagger M_i A_k$, is also a POVM. Moreover, I, I_{CQ}, and I_{CC} are related by LO themselves and each of them vanishes only for uncorrelated states [3,16]. We collect these results in the following:

Observation 1.—Mutual information functions I, I_{CQ}, I_{CC}: (i) are nonincreasing under LO; (ii) satisfy $I \geq I_{CQ} \geq I_{CC} \geq 0$; (iii) vanish if and only if the state is factorized.

We will prove, with the help of simple lemmas, that all quantum states, that are not CC from the beginning, contain correlations that are not classical, in the sense made precise by Theorem 1.

Lemma 1.—Given a CQ state $\rho = \sum_i p_i |i\rangle\langle i| \otimes \sigma_i^B$, we have $I(\rho) = I_{CQ}(\rho) = \chi(\{p_i, \sigma_i\})$, with the Holevo quantity $\chi(\{p_i, \sigma_i\}) = S(\sum_i p_i \sigma_i) - \sum_i p_i S(\sigma_i)$. Moreover, we have $I(\rho) = I_{CC}(\rho)$ if and only if the states σ_i^B commute, and thus ρ is CC.

Proof.—To see that $I(\rho) = I_{CQ}(\rho)$, consider on A a complete projective measurement in the basis comprising the orthogonal states $\{|i\rangle\}$. For the given ρ, one checks that $I(\rho) = \chi(\{p_i, \sigma_i\})$. Moreover, $I_{CC}(\rho)$ is the classical MI between two parties, where party A sends a state σ_i labeled by i with probability p_i, and B performs a measurement that gives outputs j with conditional probabilities $p(j|i)$ [8]. It is known [17] that χ is an upper bound to the classical MI of $\{p_{ij} = p_i p(j|i)\}$, that is saturated if and only if the states σ_i commute, i.e., can be diagonalized in the same basis. □

Lemma 2.—Given a bipartite state $\rho = \rho_{AB}$ and (local) operations Λ_A, Γ_B, if $I[(\Lambda_A \otimes \Gamma_B)(\rho)] = I(\rho)$, then there exist maps Λ_A^* and Γ_B^* such that $(\Lambda_A^* \otimes \Gamma_B^*)[(\Lambda_A \otimes \Gamma_B) \times [\rho]] = \rho$.

Proof.—A theorem [18,19] by Petz states that, given two states ρ, σ and a map $\Lambda[Y] = \sum_i K_i Y K_i^\dagger$, then $S(\rho || \sigma) = S(\Lambda[\rho] || \Lambda[\sigma])$ if and only if there exists a map Λ^* such that $\Lambda^*[\Lambda[\rho]] = \rho$ and $\Lambda^*[\Lambda[\sigma]] = \sigma$. Moreover, the action of Λ^* on the support of $\Lambda[\sigma]$ can be given the explicit expression $\Lambda^*[X] = \sigma^{1/2} \Lambda^\dagger[(\Lambda[\sigma])^{-1/2} X (\Lambda[\sigma])^{-1/2}] \sigma^{1/2}$, where $\Lambda^\dagger[Y] = \sum_i K_i^\dagger Y K_i$. With this result, if furthermore $\sigma = \sigma_A \otimes \sigma_B$ and $\Lambda = \Lambda_A \otimes \Gamma_B$, one easily checks that $\Lambda^* = \Lambda_A^* \otimes \Gamma_B^*$. □

We are now ready to state our first main result.

Theorem 1.—$I_{CC}(\rho) = I(\rho)$ if and only if ρ is CC.

Proof.—If the state is CC, it is immediate to check that $I_{CC} = I$. On the other hand, let us assume $I(\rho) = I_{CC}(\rho) = I(\rho^{CC}(\{M_i\}, \{N_j\}))$, with $\rho^{CC}(\{M_i\}, \{N_j\}) = \sum_{ij} p_{ij} |i\rangle \times \langle i| \otimes |j\rangle\langle j|$ for some optimal $\{M_i\}$, $\{N_j\}$. Thanks to Lemma 2 we have that there exist maps $\mathcal{M}^*$ and $\mathcal{N}^*$ which invert the measurement maps, i.e., such that $\rho = (\mathcal{M}^* \otimes \mathcal{N}^*)[\rho^{CC}] = \sum_{ij} p_{ij} \mathcal{M}^*[|i\rangle\langle i|] \otimes \mathcal{N}^*[|j\rangle\langle j|]$. Let us consider $\tilde{\rho}^{QC} = (\mathcal{M}^* \otimes \text{id})[\rho^{CC}] = \sum_j p_j^B \sigma_j^A \otimes |j\rangle\langle j|$, where $p_j^B = \sum_i p_{ij}$ and $\sigma_j^A = \sum_i p_{ij}/p_j^B \mathcal{M}^*[|i\rangle\langle i|]$. This is a QC state such that $I(\tilde{\rho}^{QC}) = I_{CC}(\tilde{\rho}^{QC}) = I_{CC}(\rho) = I(\rho)$, because of Observation 1.(i). Therefore, all $\sigma_j^A = \sum_k q_k^{(j)} |\phi_k\rangle\langle\phi_k|$ are diagonal in the same basis $\{|\phi_k\rangle\}$ by Lemma 1. The original state can now be written as $\rho = \sum_j p_j^B \sigma_j^A \otimes \mathcal{N}^*[|j\rangle\langle j|] = \sum_k r_k |\phi_k\rangle\langle\phi_k| \otimes \tau_k$, where $r_k = \sum_j p_j^B q_k^{(j)}$ and $\tau_k = \sum_j \frac{p_j^B q_k^{(j)}}{r_k} \mathcal{N}^*[|j\rangle\langle j|]$. Thus we have found that ρ is a CQ state with $I = I_{CC}$, therefore it is CC, again by Lemma 1. □

We depict here another operational way to characterize CC states which regards local broadcastability. Given a state ρ we say that $\tilde{\rho}_{XY}$ is a broadcast state for ρ if $\tilde{\rho}_X = \tilde{\rho}_Y = \rho$ [11]. In the bipartite scenario $\rho \equiv \rho_{AB}$, one can consider two cuts: one between the copies, and one between the parties. The latter defines locality. Thus, the

copies are labeled by $X = AB$ and $Y = A'B'$, while the two parties are (A, A') and (B, B').

Definition 1.—We say that the state $\rho = \rho_{AB}$ is locally broadcastable (LB) if there exist LO $\Theta_A: A \to AA'$, $\Theta_B: B \to BB'$ such that $\sigma_{AA',BB'} \equiv (\Theta_A \otimes \Theta_B)[\rho_{AB}]$ is a broadcast state for ρ.

No entangled state is LB, as it cannot be broadcast, in particular, by LO and classical communication, i.e., by operations more powerful than LO (see Proposition 1 in [20]). On the contrary, every CC state is LB by cloning locally its eigenbasis $\{|i\rangle \otimes |j\rangle\}$. We provide now a condition for local broadcastability in terms of QMI.

Theorem 2.—A state $\rho \equiv \rho_{AB}$ is LB if and only if there exists a broadcast state $\sigma_{AA',BB'}$ for ρ such that $I(\rho_{A:B}) = I(\sigma_{AA':BB'})$. Any broadcast state satisfying the latter condition can be obtained from ρ by LO.

Proof.—If ρ is LB then there exists a broadcast state $\sigma \equiv \sigma_{AA':BB'} \equiv (\Theta_A \otimes \Theta_B)[\rho_{AB}]$. Since σ is obtained from ρ by LO, we have that $I(\sigma) \leq I(\rho)$, because of Observation 1.(i). Moreover, since σ is a broadcast state, ρ can be obtained from it by local tracing, $\rho = (\text{Tr}_{A'} \otimes \text{Tr}_{B'})[\sigma]$. Therefore, $I(\sigma) \geq I(\rho)$ and we get $I(\rho_{A:B}) = I(\sigma_{AA':BB'})$. On the other hand, let us now suppose there exists a broadcast state σ for ρ such that $I(\rho_{A:B}) = I(\sigma_{AA':BB'})$. We want to show that it can be obtained by local broadcasting. Indeed, by taking $\Lambda_{AA'} = \text{Tr}_{A'}$ and $\Lambda_{BB'} = \text{Tr}_{B'}$, we have $I(\sigma) = I(\rho) = I((\Lambda_{AA'} \otimes \Lambda_{BB'}) \times [\sigma])$. By applying Lemma 2, we see there are LO $\Theta_A = \Lambda^*_{AA'}$ and $\Theta_B = \Lambda^*_{BB'}$ that locally broadcast ρ into σ. $\square$

Given that local broadcastability is related to the existence of broadcast states with the same MI as the initial state, we can derive our second main result.

Theorem 3.—A state $\rho \equiv \rho_{AB}$ is LB if and only if it is CC. More strongly, there exists a state $\sigma_{AA'BB'}$ with $I(\sigma_{AB}) = I(\sigma_{A'B'}) = I(\rho)$, that can be obtained from ρ by means of LO, if and only if ρ is CC.

Proof.—Given a LB state $\rho \equiv \rho_{AB}$, consider any broadcast state $\sigma \equiv \sigma_{AA'BB'}$ satisfying $I(\rho) = I(\sigma)$, and let measuring maps $\mathcal{M}$ and $\mathcal{N}$ be optimal for the sake of $I_{\text{CC}}(\rho)$. Applying $\mathcal{M}$ and $\mathcal{N}$ on subsystems A and B of σ, we obtain: $\tilde{\sigma} = (\mathcal{M}_A \otimes \mathcal{N}_B)[\sigma] = \sum_{ij} p_{ij} |i_A j_B\rangle\langle i_A j_B| \otimes \rho^{ij}_{A'B'}$. Here, $p_{ij} = \text{Tr}(M^A_i \otimes N^B_j \otimes \mathbb{1}_{A'B'} \sigma)$ coincides with the optimal classical probability distribution for ρ, $\text{Tr}(M^A_i \otimes N^B_j \rho)$, because of the broadcasting condition, and $\rho^{ij}_{A'B'} = \text{Tr}_{AB}(M^A_i \otimes N^B_j \sigma)/p_{ij}$. For the same reason, $\text{Tr}_{AB}(\tilde{\sigma}) = \text{Tr}_{AB}(\sigma) = \sigma_{A'B'} = \rho_{AB}$. Thus, $I(\tilde{\sigma}) = I(\rho)$, and at the same time

$$I(\tilde{\sigma}) = I(\{p_{ij}\}) + \sum_i p^A_i S(\tau^i_{A'}) + \sum_j p^B_j S(\tau^j_{B'})$$
$$- \sum_{ij} p_{ij} S(\rho^{ij}_{A'B'})$$
$$\geq I_{\text{CC}}(\rho) + \sum_{ij} p_{ij} I(\rho^{ij}_{A'B'}), \quad (1)$$

where $p^A_i = \sum_j p_{ij}$, $\tau^i_{A'} = \sum_j p_{ij}/p^A_i \rho^{ij}_{A'}$ (similarly for p^B_i

and $\tau^j_{B'}$). The inequality comes from the concavity of entropy: $\sum_i p^A_i S(\tau^i_{A'}) \geq \sum_{ij} p_{ij} S(\rho^{ij}_{A'})$ (similarly for B), and we have used the fact that $I(\{p_{ij}\}) = I_{\text{CC}}(\rho)$. Consider now any other measurement maps $\tilde{\mathcal{M}}$ and $\tilde{\mathcal{N}}$, and let them act on the (still quantum) systems A' and B' of $\tilde{\sigma}$, getting a state σ^{CC}. This corresponds simply to transforming each $\rho^{ij}_{A'B'}$ into some CC state $(\rho^{ij})^{\text{CC}}_{A'B'}(\{\tilde{M}\}, \{\tilde{N}\})$. Thus, we have $I_{\text{CC}}(\sigma) \geq I(\sigma^{\text{CC}}) \geq I_{\text{CC}}(\rho) + \sum_{ij} p_{ij} I[(\rho^{ij})^{\text{CC}}_{A'B'}(\{\tilde{M}\}, \{\tilde{N}\})]$, for arbitrary $\{\tilde{M}\}$, $\{\tilde{N}\}$, because the measurement maps $\mathcal{M}_A \otimes \tilde{\mathcal{M}}_{A'}$ and $\mathcal{N}_B \otimes \tilde{\mathcal{N}}_{B'}$ may not be the optimal ones to get $I_{\text{CC}}(\sigma)$. By the assumptions and by Theorem 2, σ can be obtained from ρ via local broadcasting, and by Observation 1.(i) it must be $I_{\text{CC}}(\sigma) \leq I_{\text{CC}}(\rho)$. Therefore, we have $I_{\text{CC}}(\sigma) = I_{\text{CC}}(\rho)$. This means that $I[(\rho^{ij})^{\text{CC}}_{A'B'}(\{\tilde{M}\}, \{\tilde{N}\})]$ must be zero for any nonvanishing p_{ij}. Choosing $\{\tilde{M}\}, \{\tilde{N}\}$ repeatedly to be optimal for every $\rho^{ij}_{A'B'}$, one concludes that it must be $I_{\text{CC}}(\rho^{ij}_{A'B'}) = 0$ for every i, j such that $p_{ij} > 0$, so that, according to Observation 1. (iii) it must be $\rho^{ij}_{A'B'} = \rho^{ij}_{A'} \otimes \rho^{ij}_{B'}$. Moreover to have equality in (1), it must be that $\rho^{ij}_{A'} = \rho^i_{A'}$ and $\rho^{ij}_{B'} = \rho^j_{B'}$, because of the strong concavity of entropy. Thus, we have found that actually $\tilde{\sigma}$ is CC: $\tilde{\sigma} = \sum_{ij} p_{ij}(|i_A\rangle\langle i_A| \otimes \rho^i_{A'}) \otimes (|j_B\rangle\langle j_B| \otimes \rho^j_{B'})$. Therefore, the following sequence of relations holds: $I_{\text{CC}}(\sigma) \geq I_{\text{CC}}(\tilde{\sigma}) = I(\tilde{\sigma}) \geq I(\rho) = I(\sigma) \geq I_{\text{CC}}(\sigma)$, because of Observation 1. (i) of the classicality of $\tilde{\sigma}$, of the broadcasting condition $\tilde{\sigma}_{A'B'} = \rho$, of the assumption $I(\rho) = I(\sigma)$, and of Observation 1. (ii), respectively. Hence, all the appearing quantities coincide, and, as $I_{\text{CC}}(\sigma) = I_{\text{CC}}(\rho)$, we have $I_{\text{CC}}(\rho) = I(\rho)$. Therefore, according to Theorem 1, ρ is also CC.

The essential assumptions used to prove that ρ_{AB} is CC are: (i) $\sigma_{AA'BB'}$ is obtained from ρ by LO; (ii) $I(\sigma_{AB}) = I(\sigma_{A'B'}) = I(\rho_{AB})$. Indeed, thanks to Lemma 2, we get that then ρ_{AB}, $\sigma_{AA'BB'}$, σ_{AB}, $\sigma_{A'B'}$ are all connected by LO. Thus, with slight changes in the proof above, one can obtain the second part of the theorem. $\square$

This result represent a no-local-broadcasting theorem for quantum correlations as measured by a scalar number, mutual information. It points out a fundamental difference between classical and quantum mutual information: correlations measured by the latter cannot be shared, in the local broadcasting sense, as soon as the involved state does not simply describe a classical joint probability distribution. We remark that our result regards single states ρ_{AB} of a bipartite system. The standard no-broadcasting theorem [11] refers to a set of two or more states $\{\rho^B_i\}$ of a single system B, and says that there is a map $\Gamma: B \to AB$ such that $\text{Tr}_A(\Gamma[\rho_i]) = \text{Tr}_B(\Gamma[\rho_i]) = \rho^B_i$, for all i, if and only if the states ρ^B_i commute. This latter condition may also be interpreted in terms of classicality of the ensemble of states: when all ρ^B_i are diagonal in the same basis, they may be considered as distribution probabilities over different states of the same classical register. We notice that Theorem 3

implies the standard no-broadcasting theorem. In order to see this, let us consider a CQ state $\sigma = \sum_i p_i |i\rangle\langle i| \otimes \rho_i^B$, with $p_i > 0$ for each i. If states $\{\rho_i^B\}$ can be broadcast, then also σ can be locally broadcast; conversely, our results say that σ is LB if and only if it is CC, i.e., if and only if states ρ_i^B commute.

All the results can be extended to the multipartite setting, considering the multipartite version of mutual information given by $I(A_1{:}A_2{:}\ldots{:}A_n) = S(\rho_{A_1 A_2 \ldots A_n} \| \rho_{A_1} \otimes \rho_{A_2} \otimes \cdots \otimes \rho_{A_n})$, which vanishes if and only if the state of the n subsystems is completely factorized, and does not increase under LO. All the other definitions are trivially extended to the multipartite case: (i) a strictly classical correlated state is a classical probability multidistribution embedded in the quantum formalism; (ii) given a state $\rho_{A_1 A_2 \ldots A_n}$, we say that $\tilde{\rho}_{A_1 A_1' A_2 A_2' \ldots A_n A_n'}$ is a broadcast state for ρ if $\tilde{\rho}$ satisfies $\tilde{\rho}_{A_1 A_2 \ldots A_n} = \tilde{\rho}_{A_1' A_2' \ldots A_n'} = \rho_{A_1 A_2 \ldots A_n}$; (iii) a state can be made classical on chosen parties by local measuring maps; (iv) optimizing mutual information for the states obtained acting by measuring maps over an increasing number of parties, gives rise to a whole family of mutual information quantities. All Theorems remain valid, as Observation 1 and Lemma 2 are immediately extended, while Lemma 1 can be generalized to the case of a state that is classical with respect to all the parties but one.

In conclusion, we characterized operationally the set of CC states, i.e., states that correspond essentially to the description of correlated classical registers. We showed that they are the only states for which correlations, as measured by mutual information, can be totally transferred from the quantum to the classical world. Furthermore, they are the only states that can be locally broadcast. An even stronger result was derived in terms of mutual information alone: correlations, as quantified by such a scalar quantity, can be locally broadcast only for CC states. Thus, our results show that also correlations of separable non-CC states have quantum features, and suggest some natural ways to quantify such nonclassicality. E.g., one may consider the gap $\Delta_{CC}(\rho) = I(\rho) - I_{CC}(\rho)$, or the minimal difference [21] $\Delta_b(\rho_{AB}) = \min_{\sigma_{AA'BB'}} I(\sigma_{AA'{:}BB'}) - I(\rho_{AB})$, between the mutual information of a two-copy broadcast state $\sigma_{AA'BB'}$ and of the state ρ_{AB} itself. Theorems 1, and 2 and 3, respectively, make sure that such quantities are strictly positive for all non-CC states, and, in particular, for entangled states. The gap Δ_{CC} resembles the discord introduced in [2]: the latter corresponds to the gap $I - I_{\tilde{C}Q}$, where $\tilde{C}$ means that the measuring map which gives rise to $I_{\tilde{C}Q}$ is chosen among complete projective measurements rather than POVMs, as in the case of I_{CQ}. The value of the different mutual information measures here considered, e.g., of I_{CQ} [3], and of the respective gaps, as well as of

Δ_b, is in general hard to compute, but analytical solutions exist in special cases, e.g., for some CQ states. It is worth mentioning that I_{CQ} is related to the amount of common randomness that two parties can distill from a shared quantum state by means of one-way classical communication [22]. In this Letter, we have presented fundamental no-go results for multipartite quantum correlations; we hope that they will help in searching for new tasks based on quantum correlations. The actual quantification of quantumness, as well as the specific role of entanglement, will be treated in a forthcoming publication.

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