Collective Cooper-Pair Transport in the Insulating State of Josephson-Junction Arrays

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We investigate collective Cooper-pair transport of one- and two-dimensional Josephson-junction arrays. We derive an analytical expression for the current-voltage characteristic revealing thermally activated conductivity at small voltages and threshold voltage depinning. The activation energy and the related depinning voltage represent a dynamic Coulomb barrier for collective charge transfer over the whole system and scale with the system size. We show that both quantities are nonmonotonic functions of the magnetic field. We propose that formation of the dynamic Coulomb barrier and its size scaling are consequences of the mutual Josephson phase synchronization across the system. We apply the results for interpretation of experimental data in disordered films near the superconductor-insulator transition.

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Recent experimental studies of the superconductor-insulator transition (SIT) (see [1] for a review) in thin disordered superconducting films proved formation of a collective insulating state exhibiting thermally activated Arrhenius-like conductivity at low biases [2–4] and the threshold voltage depinning [3,5] behavior. Below the depinning voltage, V_T, a film falls into a zero-conductivity phase and abruptly switches to a finite conductance regime when the bias achieves V_T. The discovery of the novel phase existing in a narrow window of disorder strength near SIT calls for a comprehensive theory.

Josephson-junction arrays (JJAs) serve as a perfect testing ground for SIT studies (see, e.g., [6–10]). A salient similarity of the voltage threshold behavior in superconducting films [3,5] and the voltage depinning in one-dimensional JJAs [10] suggests an intimate relation between these systems. Further parallel appears from the striking observations of voltage threshold dependence on the array length in [10], the sample size dependent activation energy, $k_B T_0$, observed in [4], and the connection between V_T and T_0 revealed in [3]. An advantage of a JJA as a model system is that it offers a straightforward theoretical description of the current-voltage characteristics, which is what is measured in all the major experimental studies of SIT. In this Letter we develop a theory of the collective transport of large Josephson-junction arrays in the insulating state and apply our results for interpretation of experimental data on SIT.

The current-voltage characteristics of Josephson systems in an insulating state were discussed in a single junction [11–13] and two-junction [14,15] systems. Each junction is characterized by the Josephson coupling energy, $E_J = \hbar I_c/2e$, where I_c is the Josephson critical current, and by charging energies E_c related to interisland capacitance and E_{c0} associated with capacitance to ground, C_0. We consider an insulating state, with charging energies E_c, $E_{c0} \gg E_J$, in the vicinity of SIT, where the superconducting gap $\Delta > E_c$ [16]. This implies that the transport is mediated by the thermally activated motion of the Cooper pairs [18]. We show that in the regular and disordered arrays a collective current state develops. This state is characterized by the energy gap, Δ_c, stemming from the collective Coulomb blockade effect involving all junctions and extending over the whole system. We derive low-bias I-V dependence in the temperature interval $E_c < k_B T < \Delta_c$:

$$I \propto \exp\left[-\frac{(\Delta_c - eV)^2}{2\Delta_c k_B T}\right]. \tag{1}$$

Equation (1) reveals that there are two dynamic regimes: first, thermally activated charge transfer with the resistance

$$R \propto \exp[\Delta_c/(2k_B T)] \tag{2}$$

at $eV \ll \Delta_c$ and, second, threshold behavior at $V \approx V_T \simeq \Delta_c/e$, where the activated conductivity turns to a finite nonactivated transport. We find that in a regular 1D array $\Delta_c \simeq E_c L/d$, while in a 2D array $\Delta_c \simeq E_c \ln(L/d)$, where L is the array length and d is the size of the elemental cell of the array. We demonstrate, finally, that the magnetic field perpendicular to the film gives rise to a nonmonotonic $\Delta_c(B)$ dependence in an excellent agreement with the experimental data on both 1D artificial Josephson arrays [10] and superconducting films near SIT [3,5].

Let us consider $N \times M$ superconducting islands comprising a two-dimensional array closed by a small (as compared to the quantum resistance for Cooper pairs $R_{CP} = h/4e^2 \simeq 6.45\ \mathrm{k\Omega}$) external resistance, R_{ext}; see Fig. 1. We assign the fluctuating order parameter phase $\chi_{ij}(t)$ to the $\{i, j\}$th superconducting island (see Fig. 1). The phases of the left- and right leads, $\chi_L(t)$ and $\chi_R(t)$, respectively, are fixed by the dc voltage V across the array:

$$\chi_R - \chi_L = 2eVt/\hbar + \psi(t), \tag{3}$$

where $\psi(t)$ describes fluctuations in the leads. We single out the leftmost, $i = 1$, and rightmost, $i = N$, columns of

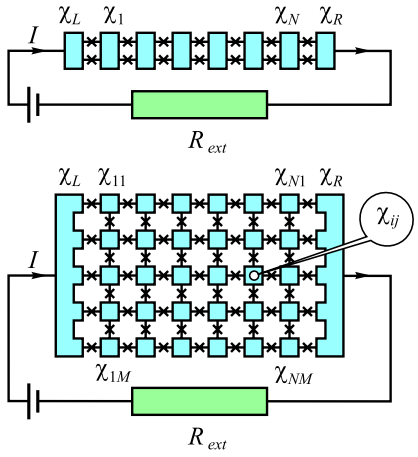

FIG. 1 (color online). Sketch of the considered array geometries. An external current I is injected from the left through the electrode having the superconducting phase χ_L and extracted through the right electrode with the phase χ_R. Upper panel: One-dimensional array of N superconducting islands (squares) connected by two Josephson junctions (crosses) to neighbors corresponding to experimental system of [10]. Lower panel: Two-dimensional $M \times N$ Josephson-junction array.

islands directly coupled to leads and represent the array Hamiltonian in a form:

$$H = H_0 + H_{\rm int} + \frac{\hbar^2}{8E_c}\sum_{j=1}^{M}[\dot\chi_{1j}(t) + \dot\chi_{Nj}(t)]^2 - 2E_J\sum_{j=1}^{M}\cos\left[\frac{\chi_{1j}(t) + \chi_{Nj}(t)}{2}\right] \times \cos\left[\frac{2eVt/\hbar + \psi(t) + \chi_{1j}(t) - \chi_{Nj}(t)}{2}\right].$$

Here

$$H_0 = \sum_{\langle ij,kl\rangle}\left[\frac{\hbar^2}{4E_c}(\dot\chi_{ij} - \dot\chi_{kl})^2 - E_J\cos(\chi_{ij} - \chi_{kl})\right] + \sum_{ij}\frac{\hbar^2}{4E_{c0}}\dot\chi_{ij}^2, \quad (4)$$

the brackets $\langle ij, kl\rangle$ denote summation over the pairs of adjacent junctions, and the last term in (4) represents the self-charge energies of superconducting islands. The $H_{\rm int}$ term in (4) describes coupling of phases on the leads to the thermal heat bath [11].

The dc Josephson current through the array is

$$I_s(V) = I_c\lim_{\tau\to\infty}\frac{1}{\tau}\int_0^\tau dt\sum_{j=1}^{M}\left\langle\cos\left[\frac{\chi_{1j}(t) + \chi_{Nj}(t)}{2}\right] \times \sin\left[\frac{2eVt/\hbar + \psi(t) + \chi_{1j}(t) - \chi_{Nj}(t)}{2}\right]\right\rangle, \quad (5)$$

where the brackets $\langle\ldots\rangle$ stand for an averaging over thermal fluctuations in the leads and quantum mechanical averaging over phases of internal junctions $[\chi_{ij}(t)]$ and over the variable $\phi_j = (\chi_{1j} + \chi_{Nj})/2$. We construct the time-dependent perturbation theory with respect to small parameter E_J/E_c, similar to the case of a Cooper-pair two-junction transistor [14,15], omitting the last term in (4) since in most experiments $C \gg C_0$, and thus $E_c \ll E_{c0}$. In the first order one finds

$$\langle\cos\phi_j\rangle = \frac{E_J}{2E_c}\cos\left[\frac{2eVt/\hbar + \psi(t) + \chi_1(t) - \chi_N(t)}{2}\right]. \quad (6)$$

In the second order, using the approach developed in [11], one arrives at

$$I_s(V) = MI_c\frac{E_J}{\hbar}\left(\frac{E_J}{2E_c}\right)^2\Im m\int_0^\infty dt e^{-t\delta/\hbar}K(t)e^{i[(2eVt)/\hbar]}, \quad (7)$$

where $\delta = 4e^2R_{\rm ext}k_BT$ reflects the Gaussian character of the current noise in the leads due to thermal fluctuations [11,19]. The correlation function of internal phases is defined as

$$K(t) = \langle\exp\{i[\chi_{1j}(t) - \chi_{1j}(0) - \chi_{Nj}(t) + \chi_{Nj}(0)]\}\rangle_{H_0}. \quad (8)$$

In the two-junction system (single Cooper-pair transistor), $\chi_{1j} \equiv \chi_{Nj}$, $K(t) \equiv 1$, and we recover the results of [15]. In the zero approximation one neglects the Josephson coupling inside the array, and $K(t)$ can be found in a closed form as an analytical continuation of $K(\tau)$, where τ is the imaginary time:

$$K(\tau) = \int D[\chi_{ij}]\exp\{i[\chi_{1j}(\tau) - \chi_{1j}(0) - \chi_{Nj}(\tau) + \chi_{Nj}(0)]\} \times\exp\left(-\frac{\hbar}{4}\int_0^{\hbar/k_BT}d\tilde\tau\left[\sum_{\langle ij,kl\rangle}\frac{[\dot\chi_{ij}(\tilde\tau) - \dot\chi_{kl}(\tilde\tau)]^2}{E_c} + \sum_{ij}\frac{[\dot\chi_{ij}(\tilde\tau)]^2}{E_{c0}}\right]\right). \quad (9)$$

Expanding phases $\chi_{ij}(\tilde\tau)$ over the Matsubara frequencies $\omega_m = 2\pi k_BTm/\hbar$ as $\chi_{ij}(\tilde\tau) = \sum\exp(i\omega_n\tilde\tau)\chi_{ij}(\omega_m)$, and going over to charge representation, $\chi_{ij}(\omega_m) = n_{ij}(2E_ck_BT/(\hbar\omega_m)^2)[\exp(-i\omega_m\tau) - 1]$, with n_{ij} being the number of Cooper pairs localized on the $\{ij\}$th island, one eventually obtains the correlation function $K(t)$:

$$K(t) = \exp(-2\Delta_ck_BTt^2/\hbar^2 - 2i\Delta_ct/\hbar), \quad (10)$$

where Δ_c is the barrier for the Cooper-pair propagation through the whole system defined by the relation

$$\exp\left(-\frac{\Delta_c}{k_BT}\right) = \int D[n_{ij}]\exp\frac{E_c}{k_BT}\left[i(n_{1j} - n_{Nj}) - \sum_{\langle ij,kl\rangle}\frac{1}{2}(n_{ij} - n_{kl})^2 - \sum_{ij}\frac{E_cn_{ij}^2}{2E_{c0}}\right]. \quad (11)$$

When deriving Eqs. (9)–(11), the nonzero winding numbers $W_{ij} = [\chi_{ij}(\hbar/k_BT) - \chi_{ij}(0)]/(2\pi)$ were neglected,

which is justified only at temperatures $T > E_c/k_B$. At $T = E_c/k_B$ a Berezinskii-Kosterlitz-Thouless–like transition into a low temperature superinsulating state occurs. Accounting for nonzero windings numbers at $T < E_c/k_B$, one finds the double-exponential resistivity $R \propto \exp\{(\Delta_c/E_c)\exp[E_c/(2k_BT)]\}$ in the superinsulating state [20]. In what follows we restrict ourselves to moderate temperatures, $E_c < k_BT < \Delta_c$.

Plugging (10) into (7), one derives the current-voltage characteristics of the insulating state as given by Eq. (1). At low biases, $eV \ll \Delta_c$, it gives an Arrhenius activation temperature dependence (2) for the resistance, with the activation energy, Δ_c, defined by Eq. (11). In 1D case, one writes $n_1 - n_N \equiv n_1 - n_2 + n_2 - n_3 + \ldots + n_{N-1} - n_N$. Provided that $E_{c0} \ll E_c$, the last term in the exponent in a functional integral can be neglected, and the right-hand side of Eq. (11) splits into a product of independent Gaussian integrals. This immediately gives $\Delta_c \simeq NE_c$. In the 2D case, the functional integral is determined by a saddle point configuration of n_{ij}. Since the first sum in the exponent in Eq. (11) is taken over nearest neighbors, the latter obeys the 2D discrete Laplace equation $\nabla^2_{ij} n_{ij} = 0$. Finally, one finds a general expression:

$$\Delta_c = \begin{cases} E_c \min\{\lambda_c, L\}/d, & \text{for 1D arrays,} \\ (E_c/2)\ln(\min\{\lambda_c, L\}/d), & \text{for 2D arrays,} \end{cases} \quad (12)$$

where $\lambda_c \simeq d\sqrt{E_{c0}/E_c}$ is the screening length related to capacitance to ground [21]. In experiments on 2D films E_{c0} is so high that λ_c can well exceed the sample size L.

The study done on thin films of amorphous indium oxide InO_x near the SIT on the insulating side [4] revealed the activation behavior of the resistance. A single batch of the film with a common width 500 μm showed the activation energy T_0 between 3.4 and 13.5 K depending on the length L between the electrodes. It is noteworthy that T_0 well exceeds the temperature of the superconducting transition $T_c \approx 1$ K for films on the superconducting side of the SIT. Plotted in Fig. 2(a) are activation energies, $T_0 \equiv \Delta_c/k_B$, extracted from Fig. 4a of [4], vs log$L$. The logarithmic scaling of T_0 is precisely as Eq. (12) predicts.

The nature of the Coulomb barrier Δ_c and its size scaling can be understood in terms of the mutual phase locking or phase synchronization in the Josephson-junction array. In the Coulomb blockade regime the charge at each junction is fixed, and, therefore, conjugated phases fluctuate freely. Yet, the exponentially small dc Josephson current couples phases of the adjacent junctions to provide a minimal power dissipation in the array. This establishes a global phase-synchronized state, and transport occurs as a simultaneous thermal activation of Cooper pairs through the whole array. The probability of such a process in a 1D array is proportional to $[\exp(-E_c/k_BT)]^N$, giving the total Coulomb barrier as $\Delta_c \simeq E_cN$. Another way of thinking is to say that synchronization builds on the large screening length λ_c, which allows for the small charge fluctuations at each junction to interact over the whole system. In this sense the linear (logarithmic) scaling of Δ_c reflects linear (logarithmic) growth of the Coulomb energy in 1D (2D) systems. As a result, synchronization is rigid with respect to disorder: even large (of the order of the quantity itself, but Gaussian) fluctuations in E_c, E_{c0}, and E_J, as well as the offset charges, are negligible as compared to the huge magnitude of Δ_c. That is why this scaling of Δ_c holds even in the amorphous superconducting films [4], where the granularity is of a self-induced nature [3,22] and the variations in Josephson coupling strength are small.

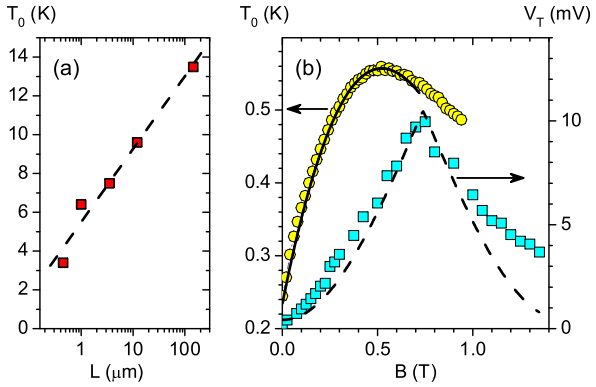

FIG. 2 (color online). (a) Activation energy $T_0 \equiv \Delta_c/k_B$ plotted as a function of logarithm of the sample length (squares are the data from [4]). (b) The experimental data from [3]: activation energy (circles, left axis) and voltage threshold (squares, right axis) as functions of the magnetic field B. The lines are according to Eq. (14): $E_J^{2D}(B)$, with values $\alpha E_J/E_c = 0.8$ and $A_{\text{loop}} = 1.4 \times 10^{-3}$ μm^2 fits T_0 (solid line); $V_T(B)$ is the 1D quantity and is fitted by $E_J^{1D}(B)$ with the same A_{loop} and $\tilde{\alpha}E_J/E_c = 0.96$ (dashed line) reflecting slightly different geometric factors.

The current-voltage characteristic of Eq. (1) is valid as long as $(\Delta_c - eV)^2 \gg 2\Delta_c k_BT$. At temperatures of interest, $T < \Delta_c/k_B$, this gives an accurate estimate for the threshold voltage of the regular Josephson-junction array as

$$eV_T \simeq \Delta_c, \quad (13)$$

with Δ_c from (12). This result holds in disordered systems as well. However, in both 1D and 2D systems the threshold voltage scales linearly with the size of the sample, i.e., the 1D scenario works. In the 1D chain the current state forms at the threshold voltage as a result of the dielectric breakdown where the collective charge transfer over the whole array occurs. The associated Coulomb energy $\Delta_c \sim (L/d)\langle E_c\rangle$ scales linearly with the system size as long as the distribution in E_c is not exponentially broad and the average $\langle E_c\rangle$ is well defined. The size dependence of V_T on the sample size was observed on the chain of SQUIDs [10], schematically shown in the upper panel of Fig. 1. One sees from Fig. 2d of [10] that for the two largest samples indeed $V_T \propto L$. In 2D arrays with disorder the dielectric breakdown occurs along the "lowest resistance" path connecting the leads. This retains a 1D scaling of V_T. Consequently, in 2D films the energy eV_T is much larger than

the activation energy determined from the low-bias resistance behavior (2). Indeed, at the magnetic field 0.7 T, the ratio $eV_T/k_BT_0 \approx 220$ [3]. The dielectric breakdown in 2D JJA is identical to that in 2D arrays of metallic dots investigated numerically in [23].

Next, we discuss the effect of the magnetic field on the activation energy and voltage depinning threshold. The field modulates the effective Josephson coupling: in the 1D SQUID chain one has $E_J^{1D}(B) = E_J|\cos(\pi f)|$, while in the 2D array $E_J^{2D}(B) = E_J\{1 - 4f\sin^2[\pi(1-f)/4]\}$ [24], where $f = eBA_{\rm loop}/\pi h$, $A_{\rm loop}$ is the area of either the elemental SQUID or the plaquette in the 2D array. The correction to the Coulomb barrier in the first order perturbation theory with respect to E_J/E_c follows from (10) and (11):

$$\Delta_c(B) = \Delta_c[1 - \alpha E_J(B)/E_c], \tag{14}$$

where the parameter α is of the order of unity, and depends on the geometry of the lattice. The field modulation of $E_J(B)$ yields nonmonotonic field behaviors of T_0 and $V_T(B)$. Shown in Fig. 2(b) are fits to activation energy T_0 and V_T vs B dependencies to the experimental data from [3]. The quantity $\alpha E_J/E_c = 0.8$ is chosen to match $T_0(0)$ to $B = 0$ experimental value and reflects that experiments were carried out in the vicinity of SIT (still allowing "borderline" estimates within the perturbation theory). The loop area is defined unambiguously by the position of the maximum in T_0 (only the branch corresponding to $0 \le f \le 1/2$ should be taken in E_J^{2D} [24]). With the same $A_{\rm loop}$, the theoretical $V_T(B)$ matches the data of [3] [Fig. 2(b)]. The above fit in the actual absence of fitting parameters confirms the 2D nature of activation energy and the 1D scenario of threshold depinning.

In conclusion, we have developed a theory of collective Cooper-pair transport in the insulating state of one- and two-dimensional Josephson-junction arrays. We have obtained the Arrhenius low-bias resistance and derived the corresponding activation energy in the temperature interval $E_c < k_BT < \Delta_c$. We have shown that both the activation energy and the voltage depinning threshold represent the dynamic Coulomb barrier Δ_c, controlling collective charge transfer in the insulating state. In Josephson-junction chains the activation energy and voltage threshold coincide and both scale linearly with the chain length. In two-dimensional arrays the activation energy scales logarithmically with the sample length, while threshold voltage, V_T, exhibits the 1D linear scaling, since disorder sets the dielectric breakdown mechanism of charge depinning. We have proposed that the physical origin of the energy gap and its scaling is the mutual phase locking in junction arrays, which maintains even in disordered systems. We expect that at temperatures above the energy gap Δ_c/k_B the synchronized state breaks down and collective activation transport transforms into a variable range hopping as observed in [3,22]. We have demonstrated that modulating Josephson coupling by the magnetic field leads to a peaked $V_T(B)$ dependence in agreement with the experimental findings for 1D Josephson arrays [10] and for superconducting films near the SIT [3,5].

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