Stability of Precessing Superfluid Neutron Stars

K. Glampedakis,[1] N. Andersson,[2] and D. I. Jones[2]

[1]*SISSA/International School for Advanced Studies and INFN, via Beirut 2-4, 34014 Trieste, Italy*
[2]*School of Mathematics, University of Southampton, Southampton SO17 1BJ, United Kingdom*

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We discuss a new superfluid instability occurring in the interior of mature neutron stars with implications for free precession. This instability is similar to the instability which is responsible for the formation of turbulence in superfluid helium. We demonstrate that the instability is unlikely to affect slowly precessing systems with weak superfluid coupling. In contrast, fast precession in systems with strong coupling appears to be generically unstable. This raises serious questions about our understanding of neutron star precession and complicates attempts to constrain neutron star interiors using such observations.

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Introduction.—Neutron stars tend to be extremely stable rotators, with stability that sometimes rivals that of the best atomic clocks. Yet a growing sample of pulsars exhibits spin irregularities, like glitches and timing noise. They may also be undergoing free precession. From the theory point of view, one might expect precession to be generic. Nevertheless, for reasons still to be understood, compelling evidence for long-period precession has been found only in the timing data of a few pulsars. The best candidate is PSR B1828-11 [1] which exhibits a ~500 d high-quality periodicity, with an amplitude of a few degrees. The paucity of precessing neutron stars is one of the mysteries of pulsar physics. To explain why precession is so rare is difficult. A modestly realistic neutron star model requires the fusion of much of modern theoretical physics, since it should account for strong gravity, supranuclear density matter, superfluidity or superconductivity and potentially very strong magnetic fields.

In the standard picture of a mature neutron star the bulk of the neutrons are superfluid and rotate by forming a dense array of vortices. Meanwhile the outer core protons, which are expected to form a type II superconductor, are electromagnetically coupled to the normal electrons or muons. This leads to a model with two distinct fluid components (loosely referred to as neutrons and "protons" in the following). Their coupling is usually assumed to have the same form as in the case of superfluid helium, see [2–5]. However, this model is based on the assumption that the neutron vortex array is (locally) straight. This may not be the case. In a body that undergoes a more complex motion one might expect to find that the vortices get tangled up to form a state of superfluid turbulence. In helium, the formation of a vortex tangle is assumed to follow the onset of an instability in the vortex array [6]. It has recently been suggested that an analogue of this so-called Donnelly-Glaberson instability may be relevant for neutron stars [7–9]. If this is the case, one would expect it to have interesting repercussions for neutron star precession. In this Letter we confirm this expectation by demonstrating that short-wavelength instabilities are generic in fast precessing superfluid neutron stars.

Plane-wave analysis.—Our main objective is to investigate whether analogues of the Donnelly-Glaberson instability are likely to occur in a neutron star interior. The smooth-averaged hydrodynamics of the system is governed by two coupled Euler-type equations (one for the neutrons and one for the protons, variables associated with each fluid will be labeled by $\mathrm{x} = \{\mathrm{n}, \mathrm{p}\}$), see [5] for more details. In a frame rotating with angular velocity Ω^i we have

$$D_t^{\mathrm{n}} v_i^{\mathrm{n}} + \nabla_i \psi_{\mathrm{n}} = 2\epsilon_{ijk} v_{\mathrm{n}}^j \Omega^k + f_i^{\mathrm{mf}}, \tag{1}$$

$$D_t^{\mathrm{p}} v_i^{\mathrm{p}} + \nabla_i \psi_{\mathrm{p}} = 2\epsilon_{ijk} v_{\mathrm{p}}^j \Omega^k - f_i^{\mathrm{mf}}/x_{\mathrm{p}} + \nu_{\mathrm{ee}} \nabla^2 v_i^{\mathrm{p}}. \tag{2}$$

Here the fluid velocities are denoted by v_{x}^i, we have introduced the convective derivatives $D_t^{\mathrm{x}} = \partial_t + v_{\mathrm{x}}^j \nabla_j$ and $x_{\mathrm{p}} = \rho_{\mathrm{p}}/\rho_{\mathrm{n}}$ is the density fraction. The scalars ψ_{x} are the sums of specific chemical potentials and the gravitational potential [5]. For simplicity, we assume that both fluids are incompressible; i.e., we have $\nabla_i v_{\mathrm{x}}^i = 0$. In the interest of clarity, we ignore the entrainment effect and vortex tension in this study. (In fact, it is clear from the results in [9] that the latter is only important for extremely short-wavelength oscillations.) A key property of the system is that neutrons and protons are coupled via mutual friction, a force f_i^{mf} mediating the interaction between the quantized neutron vortices and the proton fluid or magnetic flux tubes. The standard expression for this force is [2–5]

$$f_i^{\mathrm{mf}} = \mathcal{B}\epsilon_{ijk}\epsilon^{kml}\hat{\omega}_{\mathrm{n}}^j \omega_m^{\mathrm{n}} w_l^{\mathrm{np}} + \mathcal{B}'\epsilon_{ijk}\omega_{\mathrm{n}}^j w_{\mathrm{np}}^k, \tag{3}$$

where $w_{\mathrm{np}}^i = v_{\mathrm{n}}^i - v_{\mathrm{p}}^i$ and the neutron vorticity is given by $\omega_{\mathrm{n}}^i = 2\Omega_{\mathrm{n}}^i + \epsilon^{ijk}\nabla_j v_k^{\mathrm{n}}$. A "hat" denotes a unit vector. This form for the mutual friction force results from balancing the Magnus force that acts on the neutron vortices and a resistive "drag" force which represents the interaction between the vortices and the charged fluid [5].

Representing the drag force by a dimensionless coefficient $\mathcal{R}$, one finds that

$$\mathcal{B} = \frac{\mathcal{R}}{1+\mathcal{R}^2} \quad \text{and} \quad \mathcal{B}' = \frac{\mathcal{R}^2}{1+\mathcal{R}^2}. \tag{4}$$

In the most commonly considered case, the mutual friction arises from scattering of electrons off the vortex's intrinsic magnetic field. This leads to a relatively weak coupling [5,10], with $\mathcal{R} \approx 4\times 10^{-4}$. This means that we have $\mathcal{B} \approx \mathcal{R} \ll 1$ and $\mathcal{B}' \approx \mathcal{B}^2$. It is, however, not established that it is this limit that applies. Hence, one must also consider the case of strong coupling that follows from taking $\mathcal{R} \gg 1$. This translates into $\mathcal{B} \ll 1$ and $\mathcal{B}' \approx 1 - \mathcal{B}^2$. The strong coupling limit is relevant if the interaction between neutron vortices and fluxtubes is efficient [11–13], if there is a fluxtube cluster associated with each neutron vortex [14], or if there is significant vortex pinning [15] (in the limit $\mathcal{R} \to \infty$ the vortices can be considered as perfectly "pinned").

Returning to the Euler equations (1) and (2), only the proton equation contains a shear viscosity term. This is because the dominant process is expected to be electron-electron scattering. The upshot of this is that the neutron fluid is not directly affected by shear viscosity. For a uniform density star with $M = 1.4M_\odot$, $R = 10$ km and $x_p = 0.1$ (our canonical values) we have $\nu_{ee} \approx 10^7(T/10^8\ \text{K})^{-2}\ \text{cm}^2/\text{s}$, see [16,17].

We consider perturbations of Eqs. (1) and (2) for a background configuration where both fluids rotate rigidly with $v^i_{x0} = \epsilon^{ijk}(\Omega^x_j - \Omega_j)x_k$. By allowing for an arbitrary orientation of the angular velocity vectors, this configuration can represent the standard free precession modes of a two-fluid star [18,19]. We then linearize the Euler equations, focusing on short-wavelength motion by making the standard plane-wave decomposition

$$\delta v^i_x = A^i_x e^{i(\sigma t + k_j x^j)}, \qquad A^i_x = \text{const}, \tag{5}$$

and similarly for all other variables. Since we expect the flow along the background vortex array to play a central role [8,9], we carry out the perturbation calculation in the neutron frame. That is, we take $\Omega^i = \Omega^i_n = \Omega_n \hat{n}^i$. To simplify the problem, without any real loss of generality [9], we consider only waves propagating along the vortices, i.e., $k^i = k_\parallel \hat{n}^i$. Then the fact that we have assumed the fluids to be incompressible means that the waves are transverse, $\hat{n}_i A^i_x = 0$. After some algebra (cf. [9] for a similar analysis), the perturbed versions of (1) and (2) lead to a 4×4 system, the determinant of which provides the dispersion relation for short-wavelength waves. A detailed analysis will be provided elsewhere. Here we focus on the modes that may become unstable.

Let us first consider the weak drag limit. Then we find a mode with frequency (with viscous corrections of order $1/k_\parallel$)

$$\sigma \approx 2\Omega_n + (i\mathcal{B} - \mathcal{B}')(2\Omega_n - k_\parallel w_\parallel). \tag{6}$$

Here $w_\parallel$ represents the relative linear flow along the (background) neutron vortex array. In our case we have $w_\parallel = -\hat{n}^i\epsilon_{ijk}\Omega^j_p x^k$, and we have taken $w_\parallel$ to be constant. Hence, our analysis is only consistent for short-wavelength motion. From (6) we see that the system is unstable ($\text{Im}\sigma < 0$) if

$$w_\parallel > 2\Omega_n/k_\parallel. \tag{7}$$

As discussed in [9], the solution (6) represents inertial waves in the neutron fluid. This instability belongs to the general class of two-stream instabilities, and is the exact analogue of the Donnelly-Glaberson instability in helium [6]. Hence, its existence should come as no real surprise. As in helium, one would expect the onset of the instability to lead to the formation of tangled vortices, reconnection, and superfluid turbulence. Since turbulence alters the form of the macroscopic mutual friction force [7,8], it is not yet clear how the system will evolve once the unstable waves grow to large amplitude.

As far as we are aware, the strong drag problem has not been considered previously. Interestingly, there are unstable modes also in this case. The nature of the instability is, however, more complex. Taking $\mathcal{B} = 0$ and $\mathcal{B}' = 1$, we find a mode with frequency

$$\sigma \approx \Omega_n\left(1 - \frac{1}{x_p}\right) + k_\parallel w_\parallel + i\frac{\nu_{ee}k_\parallel^2}{2} - \left\{\frac{\Omega_n^2(1+x_p)^2}{x_p^2} - \frac{2\Omega_n k_\parallel w_\parallel}{x_p} - \frac{\nu_{ee}^2 k_\parallel^4}{4} - i\frac{(1-x_p)}{x_p}\nu_{ee}k_\parallel^2\Omega_n\right\}^{1/2}. \tag{8}$$

This result clearly shows that there will be unstable waves (representing coupled inertial waves in the neutron and proton fluids). In the inviscid ($\nu_{ee} = 0$) limit the instability is active provided that

$$w_\parallel > \frac{\Omega_n(1+x_p)^2}{2k_\parallel x_p}. \tag{9}$$

As in the weak drag case, one would expect the onset of this instability to lead to tangled vortices.

Implications for precessing neutron stars.—To discuss the implications of the above results we need to make contact between our background configuration and the global precession motion. Fortunately, this is straightforward. The precession of a two-component neutron star model, including mutual friction coupling, has already been discussed in [18]. The simplest model consists of two components that rotate rigidly. The neutron component is assumed spherical with moment of inertia I_n. At the same time, the protons (including the crust) are assumed to be slightly deformed in such a way that $I_p = I_1 = I_2 = I_3/(1+\epsilon)$ with $\epsilon \ll 1$ (in a principal coordinate system where the deformation axis is along $\hat{x}_3$). When perturbed away from alignment of the two rotation axes Ω^i_x, the crust precesses with a certain frequency and observable wobble angle θ_w (the angle between the deformation axis $\hat{x}_3$ and the total angular momentum axis) [19].

The plane-wave analysis is consistent for the precessing system provided that the two rotation vectors Ω_x^i can be considered fixed. This is true as long as the precession period P_{pr} is significantly longer than the time scale associated with the local waves. As already mentioned, the wavelength of the waves we consider must also be short enough that $w_\|$ can be treated as a constant. If these conditions hold then we are simply considering local perturbations of a given precession model. To check whether this system is locally stable we only need to work out $w_\|$ from the precession solution. If an instability is present, then the precession solution must be considered questionable. It certainly cannot be the case that the two components rotate rigidly, a key assumption in the standard analysis [18].

Weak drag slow precession.—In the weak drag limit, there exists a long-period precession solution that is slowly damped by mutual friction. We have [18]

$$P_{pr} \approx P/\epsilon \quad \text{and} \quad t_d \approx \frac{P}{2\pi\mathcal{B}\epsilon}\frac{I_p}{I_n}, \tag{10}$$

where P_{pr} is the precession period, t_d is the damping time, and P is the rotation period of the star. We then find that

$$w_\| \approx 2\pi\epsilon\theta_w x_2/P, \tag{11}$$

where $x_2(<R)$ is one of the coordinates associated with the crust system. This estimate can be used in (7) to show that all waves with wavelength ($\lambda = 2\pi/k_\|$) shorter than

$$\lambda_{max} = 5\times 10^{-4}\left(\frac{\theta_w}{1^\circ}\right)\left(\frac{\epsilon}{10^{-8}}\right)\left(\frac{R}{10^6\ \text{cm}}\right)\ \text{cm} \tag{12}$$

are unstable. However, there is a short-wavelength cutoff for the instability. Our analysis obviously becomes invalid once the wavelength is so short that the fluid description is no longer relevant. It is natural to assume that this cutoff corresponds to a wavelength

$$\lambda_{min} \approx 100 d_n \approx \left(\frac{P}{1\text{s}}\right)^{1/2}\ \text{cm} \tag{13}$$

where d_n is the intervortex spacing. Since we need to have $\lambda_{max} > \lambda_{min}$ in order to argue that the instability is relevant, we see that we must have

$$\left(\frac{\theta_w}{1^\circ}\right) > 1900\left(\frac{P}{1\text{s}}\right)^{1/2}\left(\frac{\epsilon}{10^{-8}}\right)^{-1}\left(\frac{R}{10^6\ \text{cm}}\right)^{-1}. \tag{14}$$

What does this result tell us? It suggests that, if the drag is weak, the superfluid instability is unlikely to play a role in slowly spinning systems. For the archetypal precessor PSR B1828-11 [1] the spin period is 0.4 s and in order to have precession with the observed period one would need $\epsilon \approx 10^{-8}$. It is then clear from (14) that precession with a wobble angle of a few degrees is safely in the stable regime. Nevertheless, the weak drag result is not without interest. Consider, for example, a millisecond pulsar with a maximally strained crust. From (14) we see that if the spin period is 1 ms, then precession with θ_w larger than a degree would be unstable provided that $\epsilon > 6\times 10^{-7}$. Since the theoretically predicted range for crustal deformations has $\epsilon < 10^{-5}$ [20], we see that our result puts a constraint on slow precession in very fast spinning neutron stars.

Strong drag fast precession.—In the strong drag limit, the relevant precession solution is such that [18]

$$P_{pr} \approx \frac{I_p}{I_n}P \quad \text{and} \quad t_d \approx \frac{P}{2\pi\mathcal{B}}\frac{I_p}{I_n}, \tag{15}$$

and we find that

$$w_\| \approx \frac{2\pi\theta_w x_2}{P}\frac{I_n}{I_p}. \tag{16}$$

The (inviscid) instability criterion (9) then implies that waves with wavelength shorter than

$$\lambda_{max} = 2\times 10^5\left(\frac{\theta_w}{1^\circ}\right)\left(\frac{R}{10^6\ \text{cm}}\right)\ \text{cm} \tag{17}$$

(we have assumed $I_p/I_n \approx x_p$) will be unstable.

However, as is clear from (8), the unstable strong drag modes are affected by viscosity. To unveil the detailed behavior we have solved the dispersion relation numerically for a range of parameter values. Typical results are shown in Fig. 1. This figure shows the growth time scale for the instability, τ_{grow}, as a function of the wavelength λ and illustrates how the importance of shear viscosity varies with temperature. The results for core temperatures $T = 10^9$ K and 10^7 K show a clear transition from a regime where the inviscid approximation to (8) is valid (above $\lambda \approx 20$ cm and 10^4 cm, respectively) to a short-wavelength regime where viscosity alters the result. However, a surprising feature appears as one proceeds towards shorter wavelengths for a fixed temperature. As $k_\|$ becomes large, it turns out that there is a cancellation of the leading order viscosity terms, cf. (8). For short wavelengths, the mode frequencies are instead accurately (with errors of order $1/k_\|$) described by (6) with $\mathcal{B}' = 1$. Hence, for $\lambda \ll \lambda_{max}$ the short-wavelength modes grow on a time scale given by

$$\tau_{grow} \approx \frac{\lambda}{2\pi\mathcal{B}|w_\||}. \tag{18}$$

For typical parameters, we have

$$\tau_{grow} > 140\left(\frac{x_p}{0.1}\right)\left(\frac{\theta_w}{1^\circ}\right)^{-1}\left(\frac{P}{1\ \text{s}}\right)\left(\frac{\mathcal{R}}{10^3}\right)\left(\frac{\lambda}{R}\right)\ \text{s}. \tag{19}$$

For consistency the unstable waves need to grow on a time scale that is short compared to the precession period. If we require (say) $\tau_{grow} < 0.1 P_{pr}$, then we have

$$\lambda < 70\left(\frac{\mathcal{R}}{10^3}\right)^{-1}\left(\frac{\theta_w}{1^\circ}\right)\ \text{cm}. \tag{20}$$

The corresponding instability region is indicated by a I in

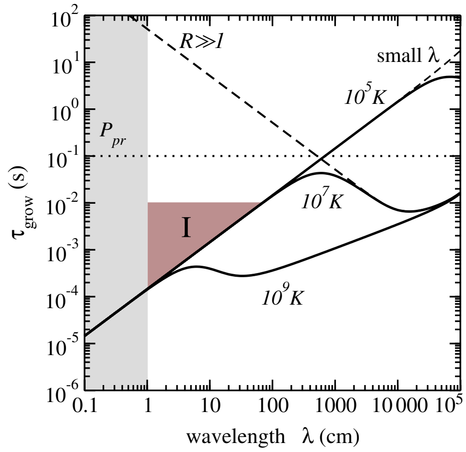

FIG. 1 (color online). Growth time scales $\tau_{\mathrm{grow}}(\lambda)$ for unstable superfluid waves in a precessing neutron star. This particular model has $\mathcal{R} = 10^3$, representing the strong drag regime, $P = 1$ s and $\theta_w = 1°$. The dotted horizontal line shows the fast precession period P_{pr} that follows if we take $x_{\mathrm{p}} = I_{\mathrm{p}}/I_{\mathrm{n}} = 0.1$. We compare our numerical results for three different core temperatures (solid lines) to two approximations. The short-wavelength approximation and the $\mathcal{R} \gg 1$ approximation (8) are both indicated by dashed lines. The region where a short-wavelength instability operates (I) for $T = 10^7$ K is highlighted. Finally, the $\lambda < \lambda_{\mathrm{min}}$ region where we assume that the hydrodynamical description fails is shaded.

Fig. 1. Moreover, in order to have $\lambda > \lambda_{\mathrm{min}}$ (noting that the short-wavelength cutoff remains as in the weak drag case) we must have

$$\mathcal{R} < 7 \times 10^4 \left(\frac{\theta_w}{1°}\right)\left(\frac{P}{1\ \mathrm{s}}\right)^{-1/2}. \tag{21}$$

This shows that the short-wavelength instability constrains a wide range of fast precession models. From the results in Fig. 1 it is also clear that there may exist a medium wavelength instability [well approximated by (8)]. As we will discuss in detail elsewhere, this instability regime is relevant for temperatures above 10^7 K or so, and will dominate for young neutron stars with very large $\mathcal{R}$.

Brief discussion.—In this Letter we have demonstrated that short-wavelength superfluid instabilities may operate in freely precessing neutron stars. In the weak drag regime, the instability affects only rapidly spinning stars that have significantly deformed crusts. PSR B1828-11, the best candidate precessor, lies well within the stable regime. In contrast, our results have serious implications for systems in the strong drag regime. We predict that these systems will suffer local instabilities, possibly leading to the formation of superfluid turbulence, for a wide range of the relevant parameter space. This calls into question the standard precession model, which is based on two coexisting fluids rotating as solid bodies [18], and any conclusions drawn from it. In particular, one would note Link's argument [12,13] that the coupling between vortices and fluxtubes ought to lead to fast precession according to (15). Since this is contradicted by the observed slow precession of PSR B1828-11, Link suggests that our understanding of the neutron star core physics is wrong and that the protons would actually form a type I superconductor (without fluxtubes). Our results add an element of doubt. We have essentially shown that the strong drag fast precession solution may be inconsistent for a neutron star spinning at the rate of PSR B1828-11. If the precessing motion triggers a range of unstable short-wavelength waves then the original solid-body assumption that led to (15), cf. [18], cannot hold. The precession problem may thus be more complex than usually assumed, and a consistent description of fast precession must properly include superfluid wave dynamics and potential turbulence.

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