Semiclassical Theory of the Anderson Transition

Antonio M. García-García

Physics Department, Princeton University, Princeton, New Jersey 08544, USA

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We study analytically the metal-insulator transition in a disordered conductor by combining the self-consistent theory of localization with the one parameter scaling theory. We provide explicit expressions of the critical exponents and the critical disorder as a function of the spatial dimensionality d. The critical exponent ν controlling the divergence of the localization length at the transition is found to be $\nu = \frac{1}{2} + \frac{1}{d-2}$ thus confirming that the upper critical dimension is infinity. Level statistics are investigated in detail. We show that the two level correlation function decays exponentially and the number variance is linear with a slope which is an increasing function of the spatial dimensionality. Our analytical findings are in agreement with previous numerical results.

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The recent experimental realization of disorder in ultracold atoms [1] together with the rapid progress in numerical calculations [2–7] has revived the interest in the metal-insulator transition (MIT) [8]. The statistical analysis of the spectrum and eigenvectors plays a central role in the identification and characterization of a MIT. Typical signatures of a MIT include (a) multifractal [2,5] eigenstates (for a review, see [9,10]), namely, the scaling of $\mathcal{P}_q = \int d^d r |\psi(\mathbf{r})|^{2q} \propto L^{-D_q(q-1)}$ with respect to the sample size L is anomalous with $D_q < d$ a set of exponents describing the transition, (b) scale invariance [3] of the spectral correlations, (c) level repulsion of neighboring eigenvalues as in a disordered metal, (d) linear number variance like in a disordered insulator, $\Sigma^2(\ell) = \langle (N_\ell - \langle N_\ell \rangle)^2 \rangle \sim \chi \ell$ (N_ℓ is the number of eigenvalues in an interval of length $\ell \gg 1$ and $\langle \ldots \rangle$ stands for ensemble average) but with a slope $\chi < 1$. In $3d$, $\chi \approx 0.27$ for higher dimensions, see Refs. [4,7]. Level statistics with these features are usually referred to as critical statistics [3,11,12].

Theoretical progress has been much slower in recent times. With the exception of the case of a Cayley tree [13] geometry we are indeed far from a quantitative analytical theory of the MIT. Below we briefly review the main analytical approaches to the MIT problem. In the original Anderson's paper the critical disorder at which the MIT occurs was estimated by looking at the limits of applicability of a locator expansion [8,14]. In this approach [8] the metal-insulator transition is induced by increasing the hopping amplitude of an initially localized particle. A more refined self-consistent [15] condition, still within the locator formalism, provided a similar answer. This method is only exact in the case of a Cayley tree but it is believed to be accurate in the $d \to \infty$ limit. In both cases a MIT in $3d$ is predicted correctly. However, the estimated critical disorder is considerably smaller than the one found in numerical simulations [2]. This disagreement persists in higher dimensions [7].

The one parameter scaling theory [16] (OPT) provides a completely different approach to the MIT. A key concept in this theory is the dimensionless conductance $g = E_c/\Delta$ [17] where E_c, the Thouless energy, is an energy scale related to the diffusion time to cross the sample and Δ is the mean level spacing. In a metal, $E_c = \hbar D_{\text{clas}}/L^2$ ($D_{\text{clas}} = v_F l/d$ is the classical diffusion constant with l the mean free path and v_F the velocity of the particle) and therefore $g \propto L^{d-2}$. In an insulator the particle is exponentially localized and $g \propto e^{-L/\xi}$ where ξ is the localization length. The OPT is based on the following two simple assumptions: (a) $\beta(g) = \frac{\partial \log g(L)}{\partial \log L}$ is continuous and monotonic, (b) The change in the conductance with the system size only depends on the conductance itself. With this input the OPT predicts correctly a MIT for $d > 2$ characterized by a size independent dimensionless conductance $g = g_c$ such that $\beta(g_c) = 0$ and $\beta(g) > (<) 0$ for $g > (<) g_c$. In order to make more quantitative statements about the MIT it is necessary to understand in detail how the system approaches the transition.

In the case of $d = 2 + \epsilon$ ($\epsilon \ll 1$) the MIT occurs at weak disorder. A rigorous analytical treatment is possible by combining diagrammatic perturbation theory and field theory techniques [18–20]. The results thus obtained for critical exponents and critical disorder are in agreement with numerical calculations [10]. Later on Vollhardt and Wolfle [21] proposed an extension of this theory valid for any d. The idea was to go beyond perturbation theory around the metallic limit by solving a self-consistent equation for a renormalized diffusion coefficient. The self-consistent condition was obtained by relating ladder diagrams associated with diffusions with crossed diagrams associated with Cooperons. Time reversal invariance is a required condition for this relation to hold. As in the $d = 2 + \epsilon$ case, the MIT is induced by the growing effect of constructive interference on an otherwise metallic state. However, a similar self-consistent condition was also obtained starting from a locator expansion [22]. Unfortunately some of the results of the Vollhardt-Wolfle theory do not agree with recent numerical results: for instance it is predicted that the upper critical dimension for localization

is $d = 4$ and that the critical exponent ν controlling the divergence of the localization length is $\nu = 1/(d-2)$. However numerical results indicate the upper critical dimension for localization is $d > 6$ and $\nu \approx 1.5, 1$ in $3d$, $4d$, respectively [4,7].

From the above discussion it seems clear that except in the $d = 2 + \epsilon$ limit the MIT cannot be described by any perturbation theory around the metallic or the insulator side. The ultimate reason for this is that, according to the OPT, the MIT is a fixed point, $\beta(g_c) = 0$, of the motion. It is a well known fact that in these situations the Hamiltonian may have universal properties completely different from the ones observed in the proximity of the transition. The above theoretical approaches neither can be extrapolated to the physically relevant case of $d = 3$ nor predict the dependence with the spatial dimensionality of different parameters describing the transition.

In order to have a better understanding of this problem, a deeper knowledge of the dynamics precisely at the transition is needed. This Letter is a first step in that direction. We solve the self-consistent condition of Vollhardt-Wolfle [20,21] but including the spatial dependence of the diffusion constant predicted by the OPT. With this simple input we obtain explicit expressions of different parameters characterizing the MIT such as critical exponents and the slope of the number variance as a function of the spatial dimensionality d. The analytical expressions for critical exponents and level statistics are in agreement with previous numerical calculations [4,5,7]. Throughout the Letter we assume time reversal invariance, and periodic boundary conditions. The results of this Letter are thus not applicable to the MIT of the integer quantum Hall effect [23] or to the one occurring in a $2d$ disordered system with spin-orbit interactions [24]. The extension of this semiclassical formalism to these cases will be published elsewhere [25].

One parameter scaling theory and anomalous diffusion at the MIT.—In the metallic limit, $g \to \infty$, the dynamics of a single particle in a random potential is well described by a normal diffusion process. The density of probability $P(\vec{r}, t)$ of finding a particle, initially at the origin, around the position $\vec{r}$ at time t is described by the solution of the diffusion equation, $P(\vec{r}, t) = \frac{e^{-|\vec{r}|^2/D_{\text{clas}}t}}{(2D_{\text{clas}}t)^{d/2}}$ in real space and $P(\omega, q) = \frac{1}{-i\omega + D_{\text{clas}}q^2}$ in Fourier space where $|\vec{q}|^2 \equiv q^2$. Since $\langle r^2 \rangle = D_{\text{clas}}t$, $g \propto L^{d-2} \gg 1$ for $d > 2$. These expressions are the starting point to study transport properties and level statistics in the metallic limit or when the localization corrections are small [10]. The MIT for $d \geq 3$ occurs in the strong disorder region which is beyond the range of applicability of perturbation theory. However, the OPT can predict the type of motion at the critical point for any dimension: at the MIT $g = E_c/\Delta$ does not depend on the system size. Since $\Delta \propto 1/L^d$, the Thouless energy must scale as $E_c \propto 1/L^d$. This can happen only if the diffusion at the MIT is anomalous with, $\langle r^{2m} \rangle \sim t^{2m/d}$ [26] where m is a positive integer. This result can likewise be interpreted as that the diffusion constant becomes scale dependent $D(L) \propto 1/L^{d-2}$ or, in momentum space, $D(q) \propto q^{d-2}$. The OPT is only capable to predict $\langle r^2 \rangle$ but not the distribution function $P(\omega, q)$. However, it is precisely this function the one needed for the evaluation of critical exponents or level statistics. On the other hand it is evident that any perturbation theory around the metallic or insulator limit will fail if it cannot take into account the anomalous diffusion predicted by the OPT. In fact, anomalous diffusion in low dimensional systems has been related to a power-law decay of the eigenstates [5,27]. This strongly suggests [25] that a new basis for the localization problem given by eigenstates with a power-law decay is the starting point for a meaningful perturbation theory at the MIT.

In order to proceed we have to come up with an expression for $P(\omega, q)$ consistent with the OPT prediction, $\langle r^2 \rangle \sim t^{2/d}$ and which at the same time can describe the dynamics in the proximity of the MIT. The simplest alternative is to assume that the classical diffusion coefficient D_{clas} gets renormalized to $D = \tilde{D}(\omega)\tilde{D}(q)$ with $\tilde{D}(q) = D_0 q^{d-2}$. The function $\tilde{D}(\omega)$ is given by the solution of the self-consistent condition of Vollhardt-Wolfle, $\frac{\tilde{D}(\omega)}{D_{\text{clas}}} = 1 - \frac{\Delta}{\pi\hbar V D_{\text{clas}}} \sum_q [1/(-\frac{i\omega}{\tilde{D}(\omega)\tilde{D}(q)} + q^2)]$.

As a first step to solve this self-consistent equation we replace the sum by an integral and write down Δ and D_{clas} as a function of $k_F = m v_F/\hbar$ and l, $\frac{\tilde{D}(\omega)}{D_{\text{clas}}} = 1 - \frac{dk_F^{2-d}}{\pi k_F l} \int_0^{1/l} dq [|q|^{d-1}/(\frac{-i\omega}{\tilde{D}(\omega)D(q)} + q^2)]$, using $\frac{x^{2d-3}}{a^d + x^d} = a^d x^{d-3}[\frac{1}{a^d} - \frac{1}{a^d + x^d}]$ and noting that the effective disorder strength is controlled by the parameter $\lambda \equiv 1/k_F l$ (the metallic limit corresponds thus with $\lambda \to 0$),

$$\frac{\tilde{D}(\omega)}{D_{\text{clas}}} = 1 - \frac{d}{(k_F l)^{d-1}(d-2)\pi} + \frac{1}{D_0 \xi^2} \frac{dk_F^{2-d}}{\pi k_F l} \times \int_0^{1/l} dq \frac{|q|^{d-3}}{\frac{1}{D_0\xi^2} + q^d}, \quad (1)$$

where we have used that the localization length $\xi = \lim_{\omega \to 0} \sqrt{-\tilde{D}(\omega)/i\omega}$. This expression is obtained by matching the predictions for the conductivity and density response function on the metallic and insulating side of the transition [20,21].

The third term in Eq. (1) vanishes ($\xi \to \infty$) as we approach the MIT. We use this fact to compute the critical disorder $\lambda = \lambda_c$ and the critical exponent s related to the vanishing of the conductivity. The critical disorder $\lambda_c = (\frac{d-2}{\pi^{d-2} d})^{1/d-1}$ is obtained by solving λ in Eq. (1) with $\lim_{\omega \to 0} \tilde{D}(\omega) = 0$. On the metallic side of the transition $\lim_{\omega \to 0} \tilde{D}(\omega) = 0$ since $\tilde{D}(\omega) \propto \sigma(\omega)$ and the conductivity $\sigma(0) \propto g_c/L = 0$ for $L \to \infty$. Likewise it is straightforward to show $\sigma(0) \propto |\lambda - \lambda_c|^s$ with $s = 1$. Both results agree with the prediction of Vollhardt and Wolfle [21]. Therefore anomalous diffusion does not affect the behavior of the conductivity and λ_c close to the transition. We note

that $\lambda_c = 1/\pi > 0$ in the limit$d \to \infty$. This is consistent with the fact that the MIT in the Cayley tree occurs at finite disorder [13].

The situation is different in the case of the critical exponent ν related to the divergence of the localization length. As we approach the transition from the insulator side ($\lambda > \lambda_c$) the localization length diverges as $\xi \propto |\lambda - \lambda_c|^{-\nu}$. The conductivity also vanishes on the insulator side of the transition $\lim_{\omega\to 0}\sigma(\omega) \propto i\omega$ [20,21]. Combining the results for the metallic and insulator sides of the transition it turns out $\lim_{\omega\to 0}\tilde{D}(\omega) \propto i\omega = 0$. Using this fact the critical exponent ν is obtained by simply solving Eq. (1) for ξ with $\omega \to 0$,

$$\nu = \frac{1}{d-2} + \frac{1}{2}. \tag{2}$$

A few comments are in order: this expression (a) only agrees with the Vollhardt-Wolfle prediction for $d \sim 2$, (b) agrees with numerical calculations for any $d > 2$ dimensions [7], (c) shows that the upper critical dimension for localization is infinity since $\nu > 1/2$ for any finite d. This is the most important result of the Letter.

Finally, we study the critical dimensional conductance $g_c = \frac{\hbar\tilde{D}(L)}{L^2\Delta}$. In order to proceed we have to compute $\tilde{D}(L)$. In practical terms this can be carried out by including a lower-cutoff $\sim 1/L$ in the integral over momentum Eq. (1). For $\lambda > \lambda_c$, and $\omega \to 0$, $\tilde{D}(\omega) \to 0$ and, $\tilde{D}(L) = \frac{\Delta}{\pi}\frac{S_d}{(2\pi)^d} \times \int_0^{1/L}[|q|^{2d-3}/(\frac{1}{D_0\xi^2} + q^d)]$. After performing the integration, and taking the limit $\xi \to \infty$ for a fixed L we obtain

$$g_c = \frac{S_d}{\pi(d-2)(2\pi)^d}, \tag{3}$$

where S_d is the surface of the d sphere. We note that $g_c \ll 1$ for $d \gg 1$. This result agrees with previous predictions based on a self-consistent condition [21] or simple one loop perturbation theory [28]. Thus anomalous diffusion does not affect the value of g_c. However corrections to the above g_c due to a finite L or ξ will be in general different from the predictions of Refs. [21,28].

Level statistics at the MIT.—We study analytically the number variance and the two level correlation function (TLCF) at the MIT. Our main result is that the number variance is linear $\Sigma^2(\ell) \sim \chi\ell$ with $\chi \approx 1 - 2/d$.

We are now interested in the properties of the system precisely at the MIT. Key in our argument is again the fact that diffusion is anomalous at the MIT. Our starting point is the connected TLCF, $R_2(\epsilon_1, \epsilon_2) = \Delta\langle\rho(\epsilon_1)\rho(\epsilon_2)\rangle$, ($\langle\rangle$ denotes averaging over disorder realizations and ρ stands for the spectral density). In the metallic limit, $g \gg 1$ and for $s \equiv \frac{\epsilon_1-\epsilon_2}{\Delta} \gg g$, the TLCF is related to $P(\omega, q_{n_i}) = \frac{1}{-i\omega + D_{\rm clas}q_{n_i}^2}$ by, $R_2(s) = -\frac{\Delta^2}{\pi^2}\Re\sum_{n_i}P^2(s\Delta, q_{n_i})$, where the sum runs over all momentum eigenstates q_{n_i}. This result is semiclassical in the sense that interference corrections represented by maximally crossed diagrams are not taken into account. As disorder increases diffusion becomes slower as a consequence of the growing interferences effects. Corrections to the metallic results above are thus expected.

The anomalous diffusion $\langle r^2\rangle \sim t^{2/d}$ at the MIT is reproduced by simply replacing the standard diffusion pole $\sim q^2$ in $P(\omega, q)$ by $\sim q^d$. We thus propose that for $s \gg g_c$ the TLCF at the transition is given by

$$R_2(s) = -\frac{1}{\pi^2}\Re\sum_{n_i}\frac{1}{(is + g_c|q_{n_i}|^d)^2}, \tag{4}$$

where $|q_{n_i}| = \sqrt{\sum_{i=1}^d n_i^2}$. In this approach we assume all interference effects are included in the renormalization of the diffusion coefficient $D_{\rm clas} \to D_0 q^{d-2}$. Corrections to this result are expected due to the multifractality of the eigenstates. However, such corrections cannot modify the scale invariance of g_c at the MIT.

We are now ready to compute the number variance, $\Sigma^2(\ell) = \langle(N_\ell - \langle N_\ell\rangle)^2\rangle = 2\int_0^\ell(s-\ell)R_2(s)$ with N_ℓ the number of eigenvalues in an interval of length ℓ in units of the mean level spacing. Carrying out this integral and replacing the sum over momenta by an integral,

$$\Sigma^2(\ell) = \frac{1}{\pi^2}\frac{S_d}{(2\pi)^d}\int_0^\infty dt|t|^{d-1}\ln\left[\frac{\ell^2}{g_c^2t^{2d}} + 1\right]. \tag{5}$$

Performing the integral and using Eq. (3),

$$\Sigma^2(\ell) \approx \chi\ell, \qquad \chi = 1 - 2/d, \qquad \ell \gg g_c. \tag{6}$$

A linear number variance with $\chi < 1$ is considered a signature of a MIT. The origin of this linear behavior was predicted heuristically [12] by using OPT and making the plausible approximation that eigenvalues interact only if their separation is smaller than g_c. The value of the slope was later estimated to be $\chi = \frac{d-D_2}{2d}$ [29]. We do not fully understand the relation between this result and ours $\chi = 1$–$2/d$. However, we note our expression for χ reproduces correctly the limits $d \sim 2$ and $d \to \infty$. For $d \geq 3$ our prediction is around 10%–15% off the numerical value [7], the predictions of Ref. [29] fail for $d > 3$.

We now turn to the discussion of the TLCF in the region $s \sim g_c$. We aim to examine the heuristic arguments of Ref. [12] where it was suggested that for $s > g_c$ there must be a sharp suppression of the spectral correlations. If localization corrections are negligible the use of the supersymmetry method permits an explicit evaluation of the nonperturbative part of the TLCF [30]. $R_2^{\rm NP}(s) \propto D^2(s, g)$ where $D(s, g) = \prod_{n_i\neq 0}(1 + \frac{s^2}{q_{n_i}^{2d}g^2})^{-1}$ is the spectral determinant associated to the classical diffusion operator $P(\omega, q)$. It is plausible to expect that such an expression for $R_2^{\rm NP}$ can still be used at the MIT provided that the spectral determinant is modified to take into account the anomalous diffusion predicted by the OPT, namely, $|q_{n_i}| = \sqrt{\sum_{i=1}^d n_i^2}$

and $g \to g_c$. This is again a semiclassical approximation, we suggest that all the quantum interference effects at the MIT are included by an appropriate redefinition of the classical spectral determinant. The spectral determinant $D(s, g_c)$ can then be estimated analytically by exponentiation of the product, replacing sums by integrals and using Eq. (3). The final result is simply,

$$R_2^{\mathrm{NP}}(s) \propto e^{-2\pi^2 s(d-2)/d}. \tag{7}$$

We note (a) an exponential decay with a similar prefactor has been observed in numerical calculation [7], (b) the sharp decay of the TLCF does not occur for $s \sim g_c \ll 1$ but rather for $s \sim 2d/(d-2)\pi \gg g_c$, (c) the conformal symmetry predicted by the OPT at the MIT is only consistent with an exponential or a power-law decay of the TLCF, (b) the power-law decay of $R_2(s)$ observed in Ref. [31] is related to how the system approach the transition rather than to the transition itself.

Limits of applicability.—According to numerical and heuristic arguments [32] it is expected that $P(q, \omega)$ will depend on the multifractal dimension D_2 rather than d for times and distances much shorter than the Heisenberg time and the system size, respectively. The proposed renormalization of the diffusion coefficient $[D(q) \propto q^{d-2}]$ is in principle restricted to the region $q, \omega \to 0$. However we note this region is the only one relevant in the calculation of the critical exponents. In the evaluation of the number variance $\Sigma^2(\ell)$ other momentum regions may also be relevant. Therefore our expression of the slope as a function of d may get corrections depending on the multifractal dimension D_2 [29]. These corrections point to the limit of applicability of our approach. The OPT is based on the scaling of the moments not on the distribution function itself. However, multifractality is a property of the distribution function and consequently beyond the reach of the OPT formalism.

In conclusion, we have put forward an analytical approach to the metal-insulator transition based on the anomalous diffusion predicted by the OPT at the critical point. With this simple input we have shown that the upper critical dimension is infinity and found explicit expressions for the critical exponents and critical disorder as a function of the spatial dimensionality. Moreover, we have shown that the number variance is asymptotically linear with a slope which is a simple increasing function of the spatial dimensionality. All our analytical predictions are in fair agreement with numerical simulations.

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