Radiative and Collisional Jet Energy Loss in the Quark-Gluon Plasma at the BNL Relativistic Heavy Ion Collider

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We calculate and compare bremsstrahlung and collisional energy loss of hard partons traversing a quark-gluon plasma. Our treatment of both processes is complete at leading order in the coupling and accounts for the probabilistic nature of the jet energy loss. We find that the nuclear modification factor R_{AA} for neutral π^0 production in heavy ion collisions is sensitive to the inclusion of collisional and radiative energy loss contributions while the averaged energy loss only slightly increases if collisional energy loss is included for parent parton energies $E \gg T$. These results are important for the understanding of jet quenching in Au + Au collisions at 200A GeV at the Relativistic Heavy Ion Collider (RHIC). Comparison with data is performed applying the energy loss calculation to a relativistic ideal (3 + 1)-dimensional hydrodynamic description of the thermalized medium formed at RHIC.

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Introduction.—Relativistic heavy ion collisions are designed to produce and study strongly interacting matter at high temperatures and densities. Experiments at the Relativistic Heavy Ion Collider (RHIC) have demonstrated that high p_T hadrons in central $A + A$ collisions are significantly suppressed in comparison with those in binary $p + p$ collisions, scaled to nucleus-nucleus collisions [1–3]. This result has been referred to as jet quenching and has been attributed to the energy loss of hard p_T partons due to induced gluon bremsstrahlung in a hot quark-gluon plasma. Bremsstrahlung energy loss has been calculated in several theoretical formalisms before [4–9]. Recently such bremsstrahlung calculations were implemented in models employing relativistic ideal (3 + 1)-dimensional hydrodynamics in order to calculate the nuclear modification factor R_{AA} of neutral pions at RHIC [10–12]. Early estimates of the collisional energy loss which used asymptotic arguments indicated that the radiative energy loss is much larger than elastic energy loss [13]. Zakarhov compared radiative energy loss in the light-cone path integral approach and collisional energy loss employing the Bjorken method and concluded collisional energy loss is relatively small in comparison to the radiative one [7]. Renk derives phenomenological limits on radiative vs collisional energy loss by considering quadratic vs linear path-length dependence and concludes that any elastic energy loss component has to be small [14]. In contrast, Mustafa and Thoma find that collisional energy loss has a significant influence on jet quenching [15,16]. Recent studies by Gyulassy and collaborators also point in this direction; see, e.g., [17–19].

The purpose of this study is to consistently incorporate collisional and radiative energy loss in the same formalism and to employ this formalism in a realistic description of energy loss of hard p_T leading partons in the soft nuclear medium as described by (3 + 1)-dimensional hydrodynamics in 200A GeV Au + Au collisions at RHIC.

We will emphasize three points (the first two of which have been elucidated earlier [20] for bremsstrahlung energy loss). First, in many previous approaches the *average* energy loss is computed and applied to the primary partons. Bremsstrahlung and collisional energy loss are not well described by a (path length dependent) average energy loss alone. Bremsstrahlung energy loss is dominated by hard emissions. Therefore, if a sample of partons initially has the same energy, then after traversing some path length of the medium, the distribution of final energies will be in general broad and not sharply centered at an average energy. This will be illustrated in Fig. 1. While the average energy loss has some value in judging the importance for observable consequences in jet quenching, the evolution of the probability density distributions of partons until fragmentation is the decisive quantity for such studies. To account for this we directly evolve the spectrum of partons as they undergo bremsstrahlung and collisional energy loss. Second, radiative energy loss depends on a coherence effect: the Landau-Pomeranchuk-Migdal (LPM) suppression. While some approaches take the LPM effect as a parametrically large suppression, this is true only when the parent parton and the emitted gluon are highly energetic, E_{parton}, $E_{\mathrm{gluon}} \gg T$. For small radiated gluon energies $E_{\mathrm{gluon}} \ll E_{\mathrm{parton}}$ the LPM suppression can be significantly less, and those bremsstrahlung events are of significant importance in jet quenching [20] due to the steeply falling initial parent parton spectrum. We therefore employ the Arnold, Moore, and Yaffe (AMY) formalism [9] for radiative energy loss to treat the LPM effect at all energies $E_{\mathrm{gluon}} > g_s T$ correctly up to $O(g_s)$. Third, while there has been considerable theoretical effort to improve our understanding of jet modification in the quark-gluon medium,

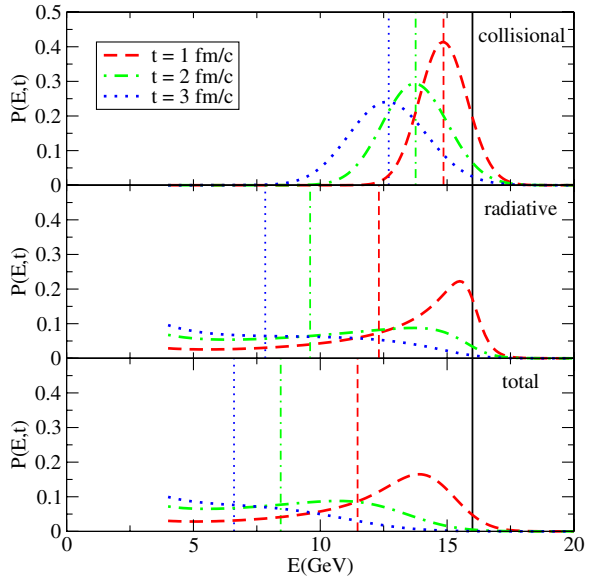

FIG. 1 (color online). The evolution of a quark jet with initial energy $E_i = 16$ GeV propagating through a static medium of temperature $T = 400$ MeV, where the vertical lines represent the values of mean energy related to the corresponding distributions.

early jet quenching calculations often relied on an elementary description of the soft medium. Until recently most jet quenching calculations used simple medium models only loosely constrained by the value of bulk observables. Previously we presented a calculation of R_{AA} in central and noncentral collisions using the AMY formalism and a (3 + 1) dimensional hydrodynamical model constrained by soft observables at RHIC [12]. Here we incorporate collisional energy loss into this framework.

Brief review of the formalism.—For details of the calculation of the initial distributions of hard partons in the early stage of the collision and the subsequent propagation through the hot and dense quark-gluon plasma as well as the fragmentation we refer the reader to [12] and references therein. We concentrate here on the incorporation of collisional energy loss in the formalism.

The jet momentum distribution $P(E, t) = \frac{dN(E,t)}{dE}$ evolves in the medium according to a set of coupled Fokker-Planck type rate equations, which have the following generic form [12]:

$$\frac{dP(E)}{dt} = \int_{-\infty}^{\infty} d\omega \left[P(E+\omega)\frac{d\Gamma(E+\omega,\omega)}{d\omega} - P(E)\frac{d\Gamma(E,\omega)}{d\omega}\right], \tag{1}$$

where $d\Gamma(E, \omega)/d\omega$ is the transition rate for processes where partons of energy E lose energy ω. The $\omega < 0$ part of the integration incorporates processes which increase a particle's energy. The radiative part of the transition rate is discussed in [12,20,21].

Now we must add the contribution from collisional energy loss. Compared with radiative loss, collisional losses are more dominated by small energy transfers; the contribution to the mean energy loss rate dE/dt from elastic collisions is logarithmically sensitive to large energy transfers, while the radiative contribution is power-law dominated by large radiations. Therefore it should be an adequate procedure to approximate elastic collisions by a mean energy loss, *provided* we include a momentum diffusion term as dictated by detailed balance.

The leading order mean collisional energy loss rate is

$$\frac{dE}{dt} = \frac{g_k}{2E}\int\frac{d^3k}{(2\pi)^3 2k}\int\frac{d^3p'}{(2\pi)^3 2E'}\int\frac{d^3k'}{(2\pi)^3 2k'} \times (2\pi)^4\delta^4(P+K-P'-K') \times (E-E')|\bar{M}|^2 f(k)[1 \pm f(k')], \tag{2}$$

where f is the thermal distribution of the medium partons. $|\bar{M}|^2$ is the t-channel scattering matrix element squared calculated in leading order, and g_k is the degeneracy factor for the initial thermal partons. Equation (2) is infrared logarithmic divergent, screened by plasma effects which are incorporated by including hard thermal loop corrections for soft momenta $\sim gT$. The resulting differential energy loss $dE/dt|_{ab}$ for the scattering of a light hard parton a off a soft parton b are [22]

$$\begin{aligned}
\left.\frac{dE}{dt}\right|_{qq} &= \frac{2}{9}n_f\pi\alpha_s^2T^2\left[\ln\frac{ET}{m_g^2} + c_f + \frac{23}{12} + c_s\right],\\
\left.\frac{dE}{dt}\right|_{qg} &= \frac{4}{3}\pi\alpha_s^2T^2\left[\ln\frac{ET}{m_g^2} + c_b + \frac{13}{6} + c_s\right],\\
\left.\frac{dE}{dt}\right|_{gq} &= \frac{1}{2}n_f\pi\alpha_s^2T^2\left[\ln\frac{ET}{m_g^2} + c_f + \frac{13}{6} + c_s\right],\\
\left.\frac{dE}{dt}\right|_{gg} &= 3\pi\alpha_s^2T^2\left[\ln\frac{ET}{m_g^2} + c_b + \frac{131}{48} + c_s\right],
\end{aligned} \tag{3}$$

where $c_b = -\gamma_E + \zeta'(2)/\zeta(2)$, $c_f = c_b + \ln(2)$, and $c_s \approx -1.66\,246$ are constants and $m_g^2 = 2\pi\alpha_s T^2(1 + n_f/6)$ is the thermal gluon mass [24].

We can incorporate these dE/dt results in Eq. (1) by introducing the drag term, $(dE/dt)dP(E)/dE$, and the diffusion term, $T(dE/dt)d^2P(E)/dE^2$. We discretize Eq. (1), such that $\int d\omega \to \Delta\omega\sum_{\omega=n\Delta\omega}$, and

$$\begin{aligned}
\Gamma(E+\Delta\omega,\Delta\omega) &= (1+f_B(\Delta\omega))(\Delta\omega)^{-1}dE/dt,\\
\Gamma(E,-\Delta\omega) &= f_B(\Delta\omega)(\Delta\omega)^{-1}dE/dt,
\end{aligned} \tag{4}$$

which yields the right energy loss rate and preserves detailed balance. $f_B(\Delta\omega)$ is the Bose-Einstein distribution function.

Results.—In order to illustrate how collisional and radiative energy loss influence the time evolution of the leading parton distributions, we first consider a static infinite medium with $T = 400$ MeV and $\alpha_s = 1/3$ and an initial single light quark jet of energy $E_i = 16$ GeV propagating through it.

In Fig. 1 we compare the evolution of the jet parton distribution $P(E, t)$ under three different approximations: (1) with only collisional energy loss, (2) with only radiative energy loss (already calculated in [20]), and (3) with both energy loss mechanisms. The first moment in energy of these distributions defines the mean energy (indicated by vertical lines) and indicates the average energy loss. The figure indicates as expected that radiation lead to a larger mean energy loss than with elastic collisions only. As pointed out earlier, small differences in the average energy loss do not necessarily imply small differences in the parton distributions. While the time evolution of $P(E, t)$ in case (3) resembles qualitatively the case (2) in which only radiative energy loss has been considered, quantitative differences especially at energies closer to E_i can be large.

We will model the behavior of the quark-gluon medium using relativistic fluid dynamics, which has been shown to give a good description of bulk properties at RHIC. We use a fully (3 + 1) dimensional hydrodynamical model for the description of 200A GeV Au + Au collisions at RHIC [25]. The initial momentum distribution of jets $dN^j/d^2p_T^j dy|_{\rm ini}$ is computed from perturbative quantum chromodynamics in the factorization formalism, for details see [12,26,27]. The final hadron spectrum $dN^h/d^2p_T dy$ at high p_T is obtained by the fragmentation of jets in the vacuum after their passing through the (3 + 1) dimensional expanding medium

$$\frac{dN^h}{d^2p_T dy} = \sum_j \int d^2\vec{r}_\perp \mathcal{P}(b, \vec{r}_\perp) \int \frac{dz_j}{z_j^2} D_{h/j}(z_j, Q_F) \times \frac{dN^j(b, \vec{r}_\perp)}{d^2p_T^j dy}\Big|_{\rm fin}, \quad (5)$$

where $dN^j(b, \vec{r}_\perp)/d^2p_T^j dy|_{\rm fin}$ is the final momentum distribution of the jet initially created at transverse position $\vec{r}_\perp$ after passing through the medium. This distribution is calculated for every specific path through the medium by solving Eq. (1) incorporating collisional and radiative energy loss. The fragmentation function $D_{h/j}(z_j, Q_F)$ [28] gives the average multiplicity of the hadron h with momentum fraction $z_j = p_T/p_T^j$ produced from a jet j at fragmentation scale Q_F. $\mathcal{P}(b, \vec{r}_\perp)$ is the initial jet density distribution at the transverse position $\vec{r}_\perp$ in collisions with impact parameter $\vec{b}$. For further details, see [12] where radiative energy loss has been studied in an analogous manner.

The final hadron spectrum directly enters the calculation of the nuclear modification factor R_{AA} which is defined as the ratio of the hadron yield in $A + A$ collisions to that in $p + p$ interactions scaled by the number of binary collisions $N_{\rm coll}$:

$$R_{AA}^h(b, \vec{p}_T, y) = \frac{1}{N_{\rm coll}(b)} \frac{dN^h(b)/d^2p_T dy}{dN_{pp}^h/d^2p_T dy}. \quad (6)$$

Once temperature evolution is fixed by the initial conditions and subsequent evolution of the (3 + 1) dimensional expansion, the strong coupling constant α_s is the only quantity which is not uniquely determined by the model. In this work we take its value to be constant at $\alpha_s = 0.27$, which reproduces the most central data [29]. We confirmed that changing the value of α_s leaves the shape of R_{AA} as a function of p_T almost unchanged. The overall magnitude of the suppression is, however, affected.

In Fig. 2 we show the mean energy loss of quark jets passing through the nuclear medium created in central collisions ($b = 2.4$ fm) at RHIC, as a function of their initial energy E_i. In this figure, the jets are assumed to be created at the center and propagating along the in-plane direction. In agreement with [7] we find that the average energy loss is not strongly changed by accounting for elastic collisions. In Fig. 3 we present the calculation of R_{AA} for neutral pions measured at midrapidity for two different impact parameters, 2.4 and 7.5 fm, compared with PHENIX data for the most central (0%–5%) and midcentral (20%–30%) collisions [2]. We present R_{AA} for purely collisional (1) and purely radiative (2) energy loss, as well as the combined case (3). One finds that while the shape of R_{AA} for case (3) is not strongly different from case (2) that has only radiative energy loss, the overall magnitude of R_{AA} changes significantly. We checked (comparison not shown) that the stronger influence on R_{AA} stems from the differences in the evolution of the parton distributions in cases (2) and (3). This has already been discussed in the static case (compare Fig. 1). The magnitude of R_{AA} is therefore sensitive to the actual form of the parton distribution functions at fragmentation and not only to the average energy loss of single partons (compare Fig. 2). In [12], the observational implications on R_{AA} measurements due to only radiative energy loss were studied. Recalculating R_{AA} versus reaction plane including elastic collisions we found only small differences (after adjusting the coupling strength from $\alpha_s = 0.33$ to $\alpha_s = 0.27$) in the shape of R_{AA} as a function of p_T and the azimuth.

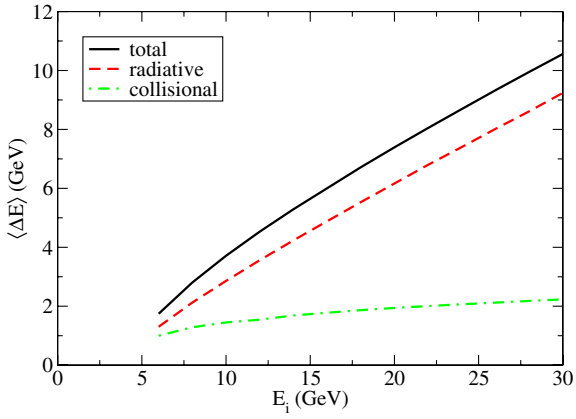

FIG. 2 (color online). The mean energy loss of a quark jet with initial energy E_i passing through the nuclear medium created in central collisions ($b = 2.4$ fm) at RHIC. The jet starts from the center of the medium and propagates in-plane.

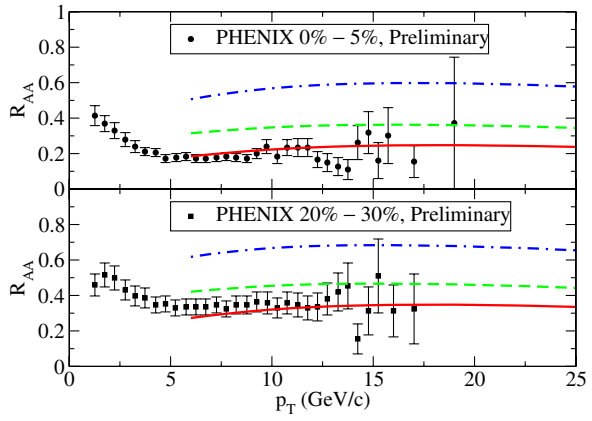

FIG. 3 (color online). Nuclear suppression factor R_{AA} for neutral pions in central and midcentral collisions. Here the dashed curves includes only radiative energy loss, the dash-dotted curve includes only collisional energy loss, and the solid curve includes both radiative and collisional energy loss.

Conclusions.—We calculated collisional and radiative jet energy loss of hard partons in the hot and dense medium created at RHIC. We treated the LPM effect using the AMY formalism [9] and treated collisions using a drag plus diffusion term reproducing dE/dt and detailed balance. While we find in accordance with [7] that the additionally induced average energy loss due to elastic collisions is small in comparison to the radiative one, the time evolution of the parton distributions $P(E, t)$ in both cases differ significantly. This is especially true for energies close to the initial parton energy. Since the initial parton spectrum is steeply falling, stronger differences in R_{AA} can result. We find that the inclusion of collisional energy loss significantly influences the quenching power quantified as the overall magnitude of neutral pion R_{AA} at RHIC, but that the shape as a function of p_T does not show a strong sensitivity. We emphasize that the description of R_{AA} is not enough to prove the consistency of a specific energy loss mechanism with data, if assumptions about the medium evolution can be rather freely adjusted. Therefore folding the jet energy loss mechanism with a dynamical evolution model which has been well tested to describe soft observables is necessary.

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[1] K. Adcox *et al.* (PHENIX Collaboration), Phys. Rev. Lett. **88**, 022301 (2001).
[2] C. Adler *et al.* (STAR Collaboration), Phys. Rev. Lett. **89**, 202301 (2002).
[3] M. Gyulassy and X. Wang, Nucl. Phys. **420**, 583 (1994).
[4] R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne, and D. Schiff, Nucl. Phys. **B483**, 291 (1997).
[5] M. Gyulassy, P. Levai, and I. Vitev, Nucl. Phys. **B594**, 371 (2001).
[6] A. Kovner and U. A. Wiedemann, *Quark Gluon Plasma 3*, edited by R. C. Hwa and X. N. Wang (World Scientific, Singapore, 2003), p. 192.
[7] B. G. Zakharov, arXiv:0708.0816.
[8] X. N. Wang and X. F. Guo, Nucl. Phys. **A696**, 788 (2001); A. Majumder, E. Wang, and X. N. Wang, arXiv:nucl-th/0412061.
[9] P. Arnold, G. D. Moore, and L. G. Yaffe, J. High Energy Phys. 11 (2001) 057; 12 (2001) 009; 06 (2002) 030.
[10] T. Renk, J. Ruppert, C. Nonaka, and S. A. Bass, Phys. Rev. C **75**, 031902(R) (2007).
[11] A. Majumder, C. Nonaka, and S. A. Bass, arXiv:nucl-th/0703019.
[12] G. Y. Qin, J. Ruppert, S. Turbide, C. Gale, C. Nonaka, and S. A. Bass, arXiv:0705.2575.
[13] J. D. Bjorken, Fermilab Report No. FERMILAB-PUB-82-059-THY, No. FERMILAB-PUB-82-059-T, 1982.
[14] T. Renk, arXiv:0708.4319.
[15] M. G. Mustafa and M. H. Thoma, Acta. Phys. Hung. A **22**, 93 (2005).
[16] M. G. Mustafa, Phys. Rev. C **72**, 014905 (2005).
[17] S. Wicks, W. Horowitz, M. Djordjevic, and M. Gyulassy, Nucl. Phys. **A784**, 426 (2007).
[18] A. Adil, M. Gyulassy, W. A. Horowitz, and S. Wicks, Phys. Rev. C **75**, 044906 (2007).
[19] S. Wicks and M. Gyulassy, arXiv:nucl-th/0701088.
[20] S. Jeon and G. D. Moore, Phys. Rev. C **71**, 034901 (2005).
[21] S. Turbide, C. Gale, S. Jeon, and G. D. Moore, Phys. Rev. C **72**, 014906 (2005).
[22] The collisional energy loss as calculated by [23] differs by constant terms in the brackets in Eqs. (3) since $u \approx -s$ was employed in the numerator of the matrix elements squared for the hard scattering there. The differences are not of phenomenological importance.
[23] M. H. Thoma, Phys. Lett. B **273**, 128 (1991).
[24] Joseph I. Kapusta and Charles Gale, *Finite-Temperature Field Theory: Principles and Applications* (Cambridge University Press, Cambridge, U.K., 2006).
[25] C. Nonaka and S. A. Bass, Phys. Rev. C **75**, 014902 (2007).
[26] H. L. Lai *et al.* (CTEQ Collaboration), Eur. Phys. J. C **12**, 375 (2000).
[27] K. J. Eskola, V. J. Kolhinen, and C. A. Salgado, Eur. Phys. J. C **9**, 61 (1999).
[28] B. A. Kniehl, G. Kramer, and B. Potter, Nucl. Phys. **B582**, 514 (2000).
[29] Recently there has been some discussion about effects of running coupling on collisional energy loss [30], especially at high energies $E \to \infty$. However, in such calculations the results are found to be sensitive to the choice of input parameters [19].
[30] A. Peshier, Phys. Rev. Lett. **97**, 212301 (2006); Phys. Rev. C **75**, 034906 (2007).