

Superconductivity-Induced Macroscopic Resonant Tunneling

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We show analytically and by numerical simulations that the conductance through π -biased chaotic Josephson junctions is enhanced by several orders of magnitude in the short-wavelength regime. We identify the mechanism behind this effect as macroscopic resonant tunneling through a macroscopic number of low-energy quasidegenerate Andreev levels.

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Resonant tunneling is the process by which quantum tunneling is enhanced by resonant transfer through intermediate quasibound states [1]. The paradigmatic example is a one-dimensional double-barrier structure, where the transmission probability is given by

$$T_{\text{res}}(E) = \frac{T_1 T_2}{1 + R_1 R_2 - 2\sqrt{R_1 R_2} \cos[\Theta(E)]}, \quad (1)$$

in terms of the transmission and reflection probabilities $T_{1,2} = 1 - R_{1,2}$ of the individual barriers. In the tunneling regime, $T_{1,2} \ll 1$, narrow quasibound states exist between the two barriers, with well resolved quantized energies, ϵ_m . When the energy of the tunneling particle coincides with one of these energies, $\Theta(E = \epsilon_m) = 0$, and in the case of symmetric barriers, $T_1 = T_2$, the transmission is perfect, $T_{\text{res}}(\epsilon_m) = 1$. This is to be contrasted with the transmission probability $T(E) = T_1 T_2 / 4$ away from resonance, and the incoherent transmission probability $T(E) = T_1 T_2 / (T_1 + T_2)$ one obtains when inelastic scattering occurs between the two barriers [2].

Resonant tunneling also occurs in higher dimensions. In chaotic systems with no spatial symmetry, there is no degeneracy of the intermediate states. Therefore, considering linear transport at low temperature, resonance occurs with at most one state at a time, leading at best to an increase of the conductance by an amount $G_0 = 2e^2/h$ —it is a microscopic effect of order one. In this Letter, we show that the proximity of the intermediate system to two superconductors can lead to a totally different phenomenology, where resonant tunneling through a macroscopic number $\propto N_n$ of intermediate levels occurs at the Fermi energy. At resonance, the conductance is insensitive to the transparency Γ_n of the tunnel barriers, $G \propto G_0 N_n$, and therefore much larger than the resonant conductance $\propto G_0 \Gamma_n N_n$ in absence of superconductivity. The resonance condition is met when the phase difference between the two superconductors is $\phi = \pi$.

The system we investigate is sketched in Fig. 1. A ballistic metallic quantum dot is connected to two metallic electrodes (L and R , each carrying $N_n \gg 1$ channels) and two superconducting electrodes (S_1 and S_2 , each carrying $N_s \gg 1$ channels). The electrodes are coupled to the dot

via tunnel contacts of transparency $0 < \Gamma_{n,s} \leq 1$, such that $1 \lesssim \Gamma_n N_n \ll N_n$, and $\Gamma_s N_s \gg 1$. We are interested in transport between the two normal leads as a function of the phase difference ϕ between the two superconductor's pair potentials, $\Delta_{S_1} = \Delta_{S_2} \exp[-i\phi]$, $|\Delta_{S_{1,2}}| = \Delta$.

The physics in our system is to a large extent governed by Andreev reflection [3]. At low-energy, this is the dominant reflection process at an interface between a metal and a superconductor, where an electron is retroreflected into a hole, and vice-versa. The process is sketched in Fig. 1. When the excitation energy ϵ is negligible against Δ and the Fermi energy E_F , the retroreflection is perfect, and the hole (electron) exactly retraces the path previously followed by the electron (hole), with an additional Andreev reflection phase of $-\pi/2 \pm \phi/2$. In absence of normal lead (one then has an *Andreev billiard* [4]), Andreev reflection renders all classical paths periodic in a cavity that would be chaotic otherwise.

When the cavity is weakly connected to external leads, transport can be resonantly mediated through those periodic orbits that touch the contacts to the leads and both

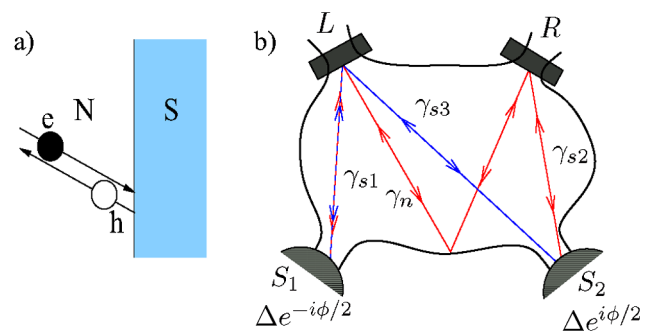


FIG. 1 (color online). (a) Andreev reflection by a superconductor (S). An incoming electron (e) in a normal metal (N) near the Fermi energy E_F is reflected as a hole (h) with opposite velocity. (b) Schematic of our system. A ballistic metallic cavity is connected to two normal (L and R) metallic leads and two superconducting (S_1 and S_2) leads. The superconducting pair potentials have a phase difference of ϕ . Two periodic Andreev orbits contributing to macroscopic resonant tunneling at $\phi = \pi$ are drawn.

superconductors. Two such orbits are depicted in Fig. 1. Each such orbit represents a family of scattering trajectories constructed from a *primitive trajectory*, and an *Andreev loop* that can be traveled p times, $p = 0, 1, 2, \dots$. We indicate segments of trajectories as $\gamma_a^{(\alpha)}$ with a superscript $\alpha = e, h$ denoting whether an electron or a hole travels on the trajectory, and a subscript identifying whether the segment touches a superconducting lead ($a = s_i$) or only normal leads ($a = n$). With this convention, the shorter trajectory in Fig. 1 represents trajectories coded in Eq. (5). At $\epsilon = 0$, the action phase accumulated along such trajectories is given by $S_\gamma(p) = p(-\pi \pm \phi) + \varphi$, where φ is a constant phase that is irrelevant for transport. The crucial element is that the phase difference between the two superconductors cancels out the accumulated Andreev scattering phase when $\pi = \phi$. Then all members of a family interfere constructively with each other because $S_\gamma(p) - S_\gamma(p') = 0$. This holds simultaneously for *all* families of trajectories that touch both superconductors, with the topology of the trajectories sketched in Fig. 1. As there are infinitely many such trajectories, the result is macroscopic resonant tunneling with a conductance $G(\phi = \pi) \propto G_0 N_n$, independent of $\Gamma_n \ll 1$. Similar resonant behaviors were reported for nongeneric cavities without mode mixing [5]; however, neither macroscopic resonant tunneling nor the associated massive quasidegeneracy of energy levels around $\epsilon = 0$ for $\phi = \pi$ have been noticed in earlier investigations of the density of states of chaotic Andreev billiards [6–10] and of transport through chaotic Josephson junctions as the one shown in Fig. 1 [11–13].

We sketch our analytical calculation. In the symmetric configuration we consider, where each normal lead carries the same number of channels, connected to the cavity with the same transparency, the average conductance from L to R reads, to leading order in $\Gamma_n N_n$ [14]

$$\langle G \rangle / G_0 = \langle T_{RL}^{ee} \rangle + \langle T_{LL}^{he} \rangle. \quad (2)$$

Here, $T_{ji}^{\beta\alpha}$ gives the transmission probability for a quasiparticle of type α from the normal lead i to a quasiparticle of type β into the normal lead j . To evaluate the resonant contributions to $\langle T_{RL}^{ee} \rangle$ and $\langle T_{LL}^{he} \rangle$, we follow the semiclassical approach of Ref. [15] (see also Refs. [16]). We first write the transmission probabilities as

$$T_{ji}^{\beta\alpha} = \frac{1}{2\pi\hbar} \int_i dy_0 \int_j dy \sum_{\gamma_1, \gamma_2} A_{\gamma_1} A_{\gamma_2}^* \exp[i\delta S / \hbar]. \quad (3)$$

This expression sums over all classical trajectories γ_1 and γ_2 entering the cavity at y_0 on a cross section of lead i and exiting at y on a cross section of lead j , while converting an α quasiparticle into a β quasiparticle. The phase $\delta S = S_{\gamma_1} - S_{\gamma_2}$ gives the difference in action phase accumulated along γ_1 and γ_2 . In presence of tunnel barriers, the stability amplitude A_γ is given by [17,18]

$$A_\gamma = B_\gamma t_j \prod_k [r_k]^{l_\gamma(k)}, \quad (4)$$

where $l_\gamma(k)$ gives the number of times that γ is reflected back into the system from the tunnel barrier $k = L, R$, the transmission and reflection amplitudes at the normal leads satisfy $|t_i|^2 = (1 - |r_i|^2) = \Gamma_{n,s}$ (for $i = L, R, S_1$, or S_2), and $B_\gamma^2 = (dp_{y_0}/dy)_\gamma$ measures the rate of change of the initial momentum p_{y_0} as the exit position y of γ is changed, for a fixed sequence of transmissions and reflections at the tunnel barriers.

We use Eqs. (3) and (4) to evaluate the contributions to the conductance arising from trajectories touching both superconductors, topologically identical to those sketched in Fig. 1. These trajectories are ϕ -dependent, and we subdivide them into class I trajectories, contributing to $\langle T_{LL}^{he} \rangle$ (shorter trajectory), and class II trajectories, contributing to $\langle T_{RL}^{ee} \rangle$ (longer trajectory on Fig. 1). Here, we focus our discussion on class I trajectories, and will present the calculation of class II contributions elsewhere. Class I trajectories are made of two legs (denoted γ_{s1} and γ_{s3} in Fig. 1) which connect the left, normal lead to both superconducting terminals S_1 and S_2 . This still allows γ_{s1} and γ_{s3} to include intermediate bounces at any tunnel barrier, either to a normal or a superconducting contact. All possible number of such reflections are effectively resummed in our approach, in the spirit of Ref. [17]. The key point, however, is that only trajectories with the same legs (with the same sequence of reflections in $\gamma_{s1, s3}$) resonate with each other. Class I trajectories are made of the following sequence

$$\gamma_I^{(p)} = \gamma_{s1}^{(e)} + \gamma_{s1}^{(h)} + p[\gamma_{s3}^{(h)} + \gamma_{s3}^{(e)} + \gamma_{s1}^{(e)} + \gamma_{s1}^{(h)}], \quad (5)$$

where $s1$ and $s3$ can be interchanged, and $p = 0, 1, 2, \dots$. They undergo $2p + 1$ Andreev reflections, $2p$ reflections at tunnel barriers, and accumulate an action phase

$$S_{\gamma, I} = p(-\pi + \phi + \epsilon t_{\ell, I}) + 2\epsilon t_{\gamma_{s1}} - (\pi/2 - \phi/2). \quad (6)$$

One should substitute $\phi \rightarrow -\phi$ when interchanging segments $s1$ and $s3$, but the relative sign between π and ϕ does not affect the final result. Here, $t_{\ell, I}$ gives the duration of the Andreev loop [the sequence between bracket in Eq. (5)] and $t_{\gamma_{s1}}$ the duration of the segment γ_{s1} . We see that at $\epsilon = 0$ and $\phi = \pi$, the phase difference accumulated by any two members (with different p and p') of a given family vanishes, so that all pairs of trajectories within a given family resonate. There is, however, no resonance between members of different families.

In normal chaotic billiards, the stability B_γ^2 of periodic orbits decreases exponentially with the number of times the orbit is traveled [19]. The situation is fundamentally different in the presence of superconductivity, where Andreev reflections refocus the dynamics. The stability of a trajectory is then given by the product of the stabilities along the primitive segments (γ_{s1} and γ_{s3} for class I, γ_{s1} , γ_n , and γ_{s2} for class II) that the trajectories are made of,

independent of p [20]. This is true as long as half the duration of the Andreev loop is shorter than the Ehrenfest time τ_E , i.e., the time beyond which an initially narrow wave packet can no longer fit inside a superconducting lead [8,9]. For a quantum dot of linear size L_c and Lyapunov exponent λ (in absence of superconductivity), one has $\tau_E = \lambda^{-1} \ln[N_s^2/k_F L_c]$, which determines the

relative measure of trajectories contributing to macroscopic resonant tunneling, together with the average time τ_D between two consecutive Andreev reflections.

We are now ready to evaluate the dominant contributions to conductance close to resonance at $\epsilon = 0$ arising from class I trajectories. We start from Eq. (3), and, following the above considerations, we substitute

$$\sum_{\gamma_1, \gamma_2} A_{\gamma_1} A_{\gamma_2}^* [\dots]_{\gamma_1, \gamma_2} \rightarrow \Gamma_n^2 \sum_{\gamma=\text{prim}} B_\gamma^2 \sum_{p, p'=0}^{\infty} (1 - \Gamma_n)^{a(p+p')} \Gamma_s^{p+p'+c} [\dots]_{\gamma, p, p'}. \quad (7)$$

To obtain (7), we paired trajectories by class, noting that for a given class, all trajectories have the same stability but differ only by the number of Andreev reflections at the superconductors and normal reflections at the normal leads, and by the different action phases they accumulate along their Andreev loop. The sum over classes is then represented by a sum over primitive trajectories, and the exponents $a = 1$ and $c = 1$ for class I are determined by the number of Andreev and normal reflections in Eq. (5). Reflection phases at the tunnel barriers do not appear because all trajectories are traveled as many times by an electron as by a hole. The evaluation of $\sum B_\gamma^2$ proceeds along the lines of Ref. [15], and details will be presented elsewhere. The resonant part of the conductance from class I and II contributions finally reads

$$\langle T_{LL}^{he} \rangle_r = \frac{\pi \Gamma_n^2 N_n}{4} \left(\frac{N_s}{2\Gamma_n N_n + 2\Gamma_s N_s} \right)^2 (1 - (1 + \tau_E/\tau_D) \exp[-\tau_E/\tau_D]) \frac{\Gamma_s}{1 - 2\Gamma_s(1 - \Gamma_n) \cos[\pi - \phi] + \Gamma_s^2(1 - \Gamma_n)^2}. \quad (8a)$$

$$\langle T_{RL}^{ee} \rangle_r = \frac{\pi^2 \Gamma_n^2 N_n^2}{8N_s} \left(\frac{N_s}{2\Gamma_n N_n + 2\Gamma_s N_s} \right)^3 (1 - (1 + \tau_E/\tau_D + \tau_E^2/2\tau_D^2) \exp[-\tau_E/\tau_D]) \times \frac{1 + \Gamma_s^2(1 - \Gamma_n)^2}{1 - 2\Gamma_s(1 - \Gamma_n)^2 \cos[\pi - \phi] + \Gamma_s^2(1 - \Gamma_n)^4}. \quad (8b)$$

The sum of Eqs. (8a) and (8b) gives the dominant semiclassical contribution to the conductance. It exhibits the functional dependence of resonant tunneling [compare to Eq. (1)], where the resonance is however always at the Fermi level, and is achieved by setting the phase difference between the two superconductors at $\phi = \pi$. This resonance condition is the same for all trajectories. This is why the resonance is macroscopic, $\propto N_n$, and not of order one, as is the case for standard resonant tunneling in chaotic systems. In most instances, $\langle T_{LL}^{he} \rangle_r \gg \langle T_{RL}^{ee} \rangle_r$. Then the resonance height at large τ_E/τ_D , small Γ_n , and $\Gamma_s = 1$ is given by $G(\pi) \approx \pi N_n/16$. Simultaneously, the sharpness of the resonance peak, measured by its width at half height, is proportional to Γ_n . We also note that the effect is damped for imperfectly connected superconductors $\Gamma_s < 1$ because the stability of resonant orbits decreases $\propto \Gamma_s^p$ with the number p of times the loop is travelled. For $\Gamma_s = 0$, the effect disappears completely.

The conductance is the sum of the semiclassical contributions, Eqs. (8), and of quantum universal contributions. We calculated the latter using Nazarov's circuit theory [21] and obtained $G_{\text{nct}}(\pi)/G_0 = \Gamma_n N_n/2$ [22]. In the tunneling regime, semiclassical contributions thus enhance the conductance by a factor $\Gamma_n^{-1} \gg 1$ at resonance.

To confirm our predictions numerically, we extend the open kicked rotator of Refs. [9,23] to take into account both transport between two normal leads and Andreev reflection at two superconducting terminals. We construct a four-terminal scattering matrix from the Floquet operator of the kicked rotator as in Refs. [9,23], and evaluate the

exact expression for the conductance [14],

$$G/G_0 = T_{RL}^{ee} + T_{RL}^{he} + 2 \frac{T_{LL}^{he} T_{RR}^{he} - T_{LR}^{he} T_{RL}^{he}}{T_{LL}^{he} + T_{RR}^{he} + T_{LR}^{he} + T_{RL}^{he}}. \quad (9)$$

In our numerics, we restrict ourselves to perfectly connected superconductors, $\Gamma_s = 1$. We average our data over ensembles of systems with fixed classical parameters—such as the width of the leads, the strength of the tunnel barriers, and the size and chaoticity of the cavity—but different cavity shape or lead positions. We focus our investigations on the semiclassical limit $k_F L_c \rightarrow \infty$.

In Fig. 2, we show a resonance in the semiclassical regime. We obtain very good agreement between the numerical data (circles) and the analytical prediction with $\tau_E/\tau_D \approx 0.79$. Without the semiclassical contribution, this agreement would break down close to resonance, where universal contributions give a prediction $G_{\text{nct}}(\pi) = 2$, too small by an order of magnitude. The left inset illustrates the increase of the peak height and narrowness as the semiclassical parameter $k_F L_c$ increases, all classical parameters being kept constant. The four sets of data in this inset correspond to a given classical configuration, with the electronic wavelength decreasing by factors of four from one curve to the next, starting from the bottommost (black) curve. The conductance increases at each step because the number of conduction channels scales linearly with $k_F L_c$. In the absence of semiclassical contributions, these four curves would exhibit the same peak-to-valley ratio. Here, however, this ratio increases with $k_F L_c$ due to the onset of semiclassical contributions.

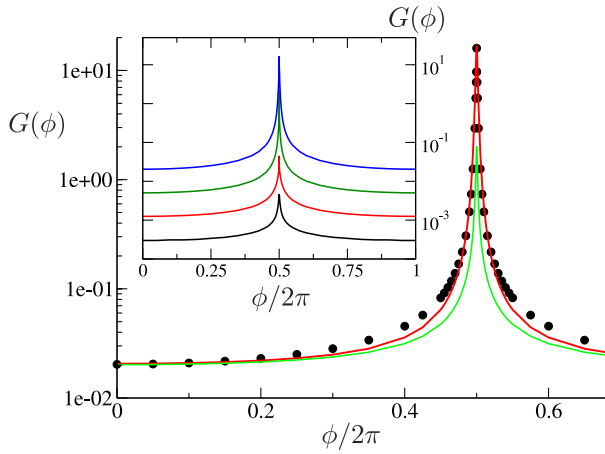


FIG. 2 (color online). Conductance through a chaotic Josephson junction vs the phase difference ϕ between the two superconductors. Circles are numerical results for $\Gamma_n = 0.01$, $k_F L_c = 8192$, $k_F L_c/N_n = 20$, $k_F L_c/N_s = 10$, and Lyapunov exponent $\lambda \approx 1.3$. The upper curve is the analytical prediction obtained by summing the semiclassical resonant contributions of Eqs. (8) with the quantum universal contributions. The lower curve gives the universal prediction obtained from circuit theory. Left inset: Numerical data for the same classical parameters $k_F L_c/N_n = 20$, $k_F L_c/N_s = 10$, and $K = 10$ as in the main plot, for $k_F L_c = 128, 512, 2048$, and 8192 (from bottom to top). Note the change in peak-to-valley ratio.

The connection can be made between the predicted and observed enhancement of conductance at $\phi = \pi$ and resonant tunneling through a macroscopic number of quasi-degenerate Andreev levels. In π -biased closed chaotic Andreev billiards, Bohr-Sommerfeld quantization predicts that all periodic orbits touching both superconductors contribute to a peak in the density of states at the Fermi energy with $\sim N_s$ states. Once electrodes are connected to the billiard, each $\epsilon = 0$ level that significantly overlaps with the tunnel barriers to both electrodes contributes one perfect transmission channel to transport via resonant tunneling, which therefore becomes macroscopic. We have numerically checked that the observed increase of conductance is accompanied by the emergence of a large peak around $\epsilon = 0$ in the corresponding Andreev billiard. This and other results will be presented elsewhere [22].

In summary, we investigated semiclassically the conductance through quantum chaotic Josephson junctions connected to two external normal leads. We found an order-of-magnitude enhancement of the conductance when the two superconductors have a phase difference of π . We identified the mechanism behind this enhancement as resonant tunneling through a macroscopic number of quasidegenerate levels at the Fermi energy.

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