Manipulating Electromagnetic Waves in Magnetized Plasmas: Compression, Frequency Shifting, and Release

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A new approach to manipulating the duration and frequency of microwave pulses using magnetized plasmas is demonstrated. The plasma accomplishes two functions: (i) slowing down and spatially compressing the incident wave, and (ii) modifying the propagation properties (group velocity and frequency) of the wave in the plasma during a uniform in space adiabatic in time variation of the magnitude and/or direction of the magnetic field. The increase in the group velocity results in the shortening of the temporal pulse duration. Depending on the plasma parameters, the frequency of the outgoing compressed pulse can either change or remain unchanged. Such dynamic manipulation of radiation in plasma opens new avenues for manipulating high power microwave pulses.

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Strong coupling between optical fields and atomic degrees of freedom can result in anomalous propagation modes of the electromagnetic fields. The best known examples include "slow" [1–3] and "fast" [4,5] light. Such dramatic alteration of light propagation is not only interesting from a fundamental standpoint, but can also result in new technological applications. For example, the concept of active control of slow light has led to advances that include all-optical storage and switching [6,7]. While plasma does not possess the complex internal structure of an atomic medium, it supports a rich variety of electromagnetic waves that can be externally coupled to each other (e.g., via external magnetic field) and change their propagation properties as a result. For example, an external helical magnetic field $\boldsymbol{B}_u$ of an undulator can couple the transverse and longitudinal waves in a uniformly magnetized plasma ($\boldsymbol{B}_0 = B_0\boldsymbol{e}_z$) to achieve both slow [8] and fast [9] light propagation at a frequency equal to the electron cyclotron frequency $\Omega_0 = eB_0/mc$. Thus, an electromagnetic pulse with a frequency $\omega = \Omega_0$ can enter the magnetized plasma, slow down considerably, and then emerge from the plasma without changing its duration or frequency. In this Letter, we pose and affirmatively answer the following question: is it possible to dynamically change plasma conditions during the time period when the pulse is traveling in the plasma in such a way that the pulse emerges from the plasma compressed and/or frequency shifted?

Specifically, we demonstrate that the simplest plasma geometries—electromagnetic (EM) wave propagating either at a small angle to the magnetic field in an infinite plasma [Fig. 1(a)], or inside a plasma-filled metallic waveguide as shown in [Fig. 1(b)]—enable such dynamic manipulation of a microwave pulse using the uniform in space adiabatic in time (USAT) variation of the external magnetic field $\boldsymbol{B}_0(t)$. The time scale of the magnetic field variation must be much longer than one wave period yet shorter than the transit time of the slowed-down pulse through the plasma. As schematically illustrated in Fig. 1, electromagnetic pulse manipulation is a three-step procedure. First, EM waves are coupled into the plasma and slowed down to a group velocity $v_{g0} \ll c$. This happens because a fast transverse electromagnetic wave can effectively couple [10] to an almost-electrostatic (slow) wave in magnetized plasma if the wave propagates at a small angle θ with respect to the magnetic field. Because the plasma profile is temporally stationary but spatially nonuniform, the pulse's frequency ω_0 and duration T are unaltered while its spatial length is compressed to $L_p = v_{g0}T \ll cT$. Second, the magnetic field $\boldsymbol{B}_0(t)$ is adiabatically varied in a USAT manner while the pulse is stored in the plasma, resulting in the group velocity change to $v_{g1} \gg v_{g0}$. Because plasma parameters remain uniform

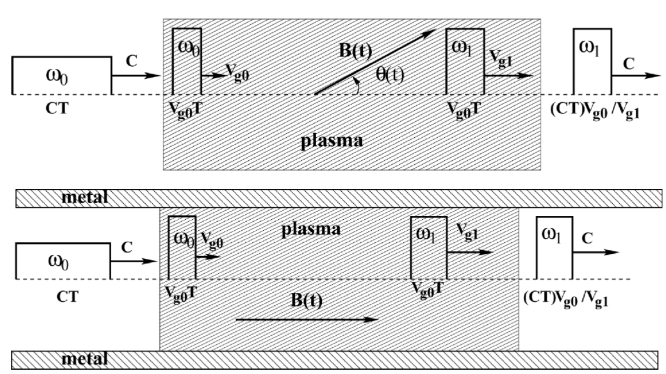

FIG. 1. Schematic of pulse compression in magnetized plasma. Radiation pulse with initial frequency ω_0 and duration T slows down in the plasma to a group velocity $v_{g0} \ll c$. USAT variation of the magnetic field changes the radiation frequency to ω_1 and increases the group velocity to $v_{g1} \gg v_{g0}$ of the radiation in the plasma. The emerging pulse is compressed to $T_1 = Tv_{g0}/v_{g1}$. One-dimensional (a) and waveguide (b) implementations.

in space while changing temporally, the pulse's wave number $\boldsymbol{k}$ and spatial extent L_p are unaltered while its frequency ω_1 and temporal duration $T_1 = Tv_{g0}/v_{g1}$ are changed. Third, because its temporal characteristics (T_1 and ω_1) are unaltered during the exit from a stationary plasma, the pulse emerges compressed by the factor v_{g1}/v_{g0}. We demonstrate that the pulse's frequency ω remains unchanged if $\omega = \omega_p$ [where $\omega_p = (4\pi e^2 n_0/m)^{1/2}$ is the electron plasma frequency]. Otherwise, the pulse emerging from the plasma is both compressed and frequency shifted. Pulse compression in plasma based on spatiotemporal variation of the plasma parameters and the resulting frequency chirping of the microwave pulse has been considered in the past [11]. Our approach is conceptually different in that it does not rely on frequency chirping or group velocity dispersion in the plasma.

We start by reviewing the classic problem [12] of a plane EM wave ($\boldsymbol{E}(\boldsymbol{x}, t) = \tilde{\boldsymbol{E}} \exp(i\boldsymbol{k}\cdot\boldsymbol{x} - i\omega t)$) propagating in a cold plasma at a small angle θ with respect to a static magnetic field $\boldsymbol{B}_0 = B_0\hat{\boldsymbol{e}}_z$. The cold plasma dielectric tensor is given by

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix}, \quad (1)$$

where $\varepsilon_1 = 1 - \omega_p^2/(\omega^2 - \Omega_0^2)$, $\varepsilon_2 = \omega_p^2\Omega_0/\omega(\omega^2 - \Omega_0^2)$, and $\varepsilon_3 = 1 - \omega_p^2/\omega^2$. Introducing the refractive index $n = ck/\omega$ (where $k \equiv |\boldsymbol{k}|$) and the propagation angle $\theta = \tan^{-1}(k_x/k_z)$, the dispersion relation is given by the roots of the quadratic equation in n^2:

$$\mathcal{A}\, n^4 - \mathcal{B} n^2 + \mathcal{C} = 0, \quad (2)$$

where $\mathcal{A} = \varepsilon_1 \sin^2\theta + \varepsilon_3\cos^2\theta$, $\mathcal{B} = (\varepsilon_1^2 - \varepsilon_2^2)\sin^2\theta + (1 + \cos^2\theta)\varepsilon_1\varepsilon_3$, and $\mathcal{C} = \varepsilon_3(\varepsilon_1^2 - \varepsilon_2^2)$. Note that for $\omega = \omega_p$ the following relations hold true: $\mathcal{C} = 0$, $\mathcal{A} = \mathcal{B}$. Therefore, the only propagating solution is $n^2 = 1$, or $|\boldsymbol{k}^2| = \omega^2/c^2$, independently of the magnitude of the magnetic field B_0 or the propagation direction θ. As shown below, this (rarely recognized) property of EM waves in the plasma can be utilized for compressing the duration of microwave pulses while preserving their frequency. For the $\theta = 0$ case (propagation parallel to the magnetic field) shown in Fig. 2(a), the longitudinal electrostatic plasma wave with the dispersion relation $\omega(k) \equiv \omega_p$ (shown as a dashed line) passes through the ($n = 1$, $\omega = \omega_p$) point. For the $\theta \neq 0$ case shown in Fig. 2(b), the electrostatic wave hybridizes with the nearby electromagnetic mode, yet retains its low group velocity at the ($n = 1$, $\omega = \omega_p$) point for $\theta \ll 1$. The low group velocity of this mode for small values of θ and Ω_0 is related to the large longitudinal component of the electric field. As Figs. 2(c) and 2(d) illustrate, the group velocity $\partial\omega/\partial k$ is slow for low values of θ and Ω_0 (as long as $\theta \neq 0$ and $\Omega_0 \geq \omega_p$) and rapidly increases with θ and Ω_0.

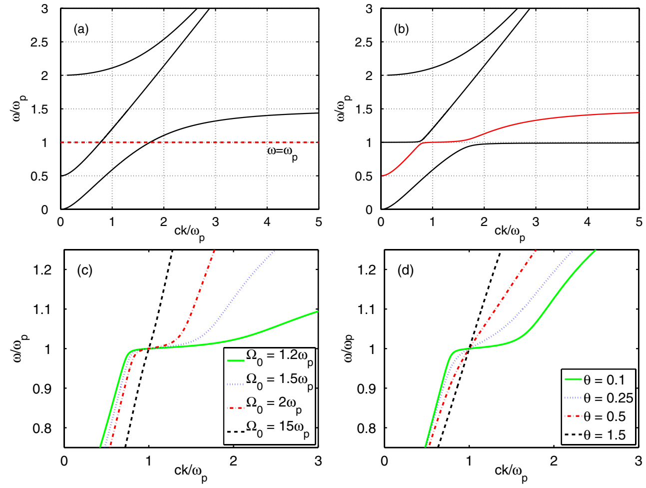

FIG. 2 (color online). The dispersion relation of a plane wave propagating at an angle θ to a static magnetic field. (a),(b) four propagation branches for $\Omega_0 = 1.5\omega_p$, with $\theta = 0$ in (a) and $\theta = 0.1$ in (b). (c),(d) dispersion curves for the second branch for different values of Ω_0 and θ, where in (c) $\theta = 0.1$ and Ω_0 is variable, and in (d) $\Omega_0 = 1.5\omega_p$ and θ is variable.

The three-step process of pulse compression without frequency shifting can now be realized by injecting waves at $\omega = \omega_p$. Because spatially uniform variation of the plasma parameters (in this case, $\boldsymbol{B}$-field) cannot change the wave number $\boldsymbol{k}$, and because the wave dispersion dictates that $\omega = \omega_p$ for $c^2\boldsymbol{k}^2/\omega^2 = 1$, the modified electromagnetic wave retains its frequency, its propagation direction, *and* its spatial extent, if the magnitude or direction of the magnetic field is changed in a USAT manner. If the group velocity changes to $v_{g1} \gg v_{g0}$ [compare, for example, the $\Omega_0 = 1.2\omega_p$ and $\Omega_0 = 15\omega_p$ curves in Fig.2(c)], then the temporal duration of thus transformed pulse becomes $T_1 = Tv_{g0}/v_{g1}$ and remains such after emerging from the plasma.

The concept of USAT variation of the plasma properties (magnetic field) for changing the duration (but not the frequency) of radiation pulses differs from several previously suggested approaches. Both rapid [13,14] and adiabatic [15] change of the plasma density have been proposed and later experimentally realized [16,17] for frequency shifting of radiation. Because plasma ionization results in the transfer of the electromagnetic energy to the kinetic energy of plasma electrons, it results in the *reduction* of the group velocity and, therefore, cannot be used for pulse compression. On the contrary, our approach of changing the amplitude and/or direction of the magnetic field results in the increase of the group velocity and pulse compression. Choosing plasma parameters such that $\omega_p = \omega$ enables pulse compression without frequency shifting. To validate the concept of USAT variation of the magnetic field, we have numerically solved the Maxwell's equations combined with the linearized equation for the plasma current:

$$\partial_t \boldsymbol{E} = c\nabla \times \boldsymbol{B} - 4\pi(\boldsymbol{J}_p + \boldsymbol{J}_{\rm ext}), \qquad \partial_t \boldsymbol{B} = -c\nabla \times \boldsymbol{B},$$
$$\partial_t \boldsymbol{J}_p = \omega_p^2 \boldsymbol{E}/4\pi - \boldsymbol{J}_p \times \boldsymbol{\Omega}_0, \tag{3}$$

assuming that $\nabla \equiv i\boldsymbol{k}$ and choosing the external current $\boldsymbol{J}_{\rm ext}$ as a finite duration pulse spectrally centered at a frequency ω_0. The value of k was taken to match ω_0 according to the dispersion relation obtained from Eq. (2). Eqs. (3) are thus reduced to a set of ordinary differential equations (ODE)s that were solved numerically. The numerical solution was checked to be self-consistent by verifying that a constant-amplitude electromagnetic field with the correct frequency $\omega = \omega_0$ persists after $\boldsymbol{J}_{\rm ext} = 0$.

With $\boldsymbol{J}_{\rm ext}$ off, the external magnetic field is adiabatically varied. Simulation results are shown in Fig. 3 for two initial radiation frequencies: frequency preserving, with $\omega_0 = \omega_p$ [Figs. 3(a) and 3(c)] and frequency shifting, with $\omega_0 = 1.1\omega_p$ [Figs. 3(b) and 3(d)]. As the angle θ [Figs. 3(a) and 3(b)] or magnitude Ω_0 [Figs. 3(c) and 3(d)] increase, the ratio of $E_\perp/E_\parallel$ increases, and so does the group velocity as the energy is transferred from longitudinal to transverse fields. Plots of $\omega(t)$ shown in Figs. 3(a) and 3(d) also confirm that the radiation frequency is unchanged by the magnetic field variation whenever $\omega = \omega_p$, and changes otherwise.

With the concept of USAT firmly established, we consider a more practical realization of pulse compression

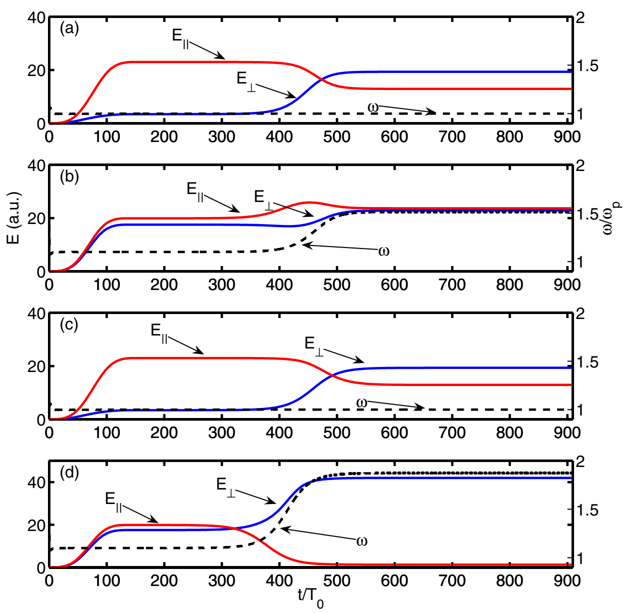

FIG. 3 (color online). Evolution of the longitudinal $\boldsymbol{E}_\parallel = \boldsymbol{n} \cdot \boldsymbol{E}$ and transverse $\boldsymbol{E}_\perp = \boldsymbol{n} \times \boldsymbol{E}$ electric fields upon changing the magnetic field angle $\theta(t) = \theta_0 + \Delta\theta f(t)$ [(a) and (b)] and amplitude $\Omega_0(t) = \Omega_0(0) + \Delta\Omega_0 f(t)$ [(c) and (d)], where $f(t) = (1 + \tanh[\pi(t - t_0)/\tau])/2$ with $t_0/T_0 = 480$, $\tau/T_0 = 160$, and $T_0 = 2\pi/\omega_0$. Initial radiation frequency $\omega_0 = \omega_p$ [(a) and (c)] and $\omega_0 = 1.1\omega_p$ [(b) and (d)]. Dashed lines: radiation frequency variation $\omega(t)$. Magnetic field parameters: $\theta_0 = 0.1$, $\Delta\theta = 1.4$, $\Omega_0(0) = 1.5\omega_p$, and $\Delta\Omega_0 = 13.5\omega_p$.

schematically shown in Fig. 1(b): plasma is placed in a wide metallic waveguide and confined by an external magnetic field. For simplicity, assume that the waveguide and plasma are infinite in the y direction and that the metallic plates are placed at $x = 0$ and $x = L$. Qualitatively, having an electromagnetic wave confined between two metal plates is equivalent to propagating it for an equivalent angle $\theta_m = \tan^{-1}(k_m/k_z)$, where $k_m = m\pi/L$ and m is an integer number labeling the specific propagation mode. Quantitatively, however, the situation is more complicated as found below both analytically and numerically: the spatial spectrum of a given mode contains a spectrum of wave numbers k_m. Assuming $\boldsymbol{E}(\boldsymbol{x}, t) = \tilde{\boldsymbol{E}}(x)\exp(ik_z z - i\omega t)$ and defining $k_0 \equiv \omega/c$, a generalized eigenvalue equation for $\tilde{\boldsymbol{E}}$ is obtained:

$$\left\{\begin{bmatrix} k_z^2 & 0 & ik_z\partial_x \\ 0 & -\partial_x^2 + k_z^2 & 0 \\ ik_z\partial_x & 0 & -\partial_x^2 \end{bmatrix} - k_0^2\boldsymbol{\varepsilon}\right\} \cdot \tilde{\boldsymbol{E}} = 0, \tag{4}$$

Eliminating $\tilde{E}_x$ results in the two coupled ODEs:

$$[\partial_x^2 + A]\tilde{E}_y = B\partial_x\tilde{E}_z \quad \text{and} \quad [\partial_x^2 + C]\tilde{E}_z = D\partial_x\tilde{E}_y, \tag{5}$$

where $A = \gamma - \varepsilon_2^2 k_0^4/\gamma$, $B = -k_z\varepsilon_2 k_0^2/\gamma$, $C = \gamma\varepsilon_3/\varepsilon_1$, $D = k_z\varepsilon_2/\varepsilon_1$, and $\gamma = \varepsilon_1 k_0^2 - k_z^2$. Expanding $\tilde{E}_{y,z}$ as

$$\begin{bmatrix} \tilde{E}_y \\ \tilde{E}_z \end{bmatrix} = \sum_{n=1}^{\infty} \begin{bmatrix} a_n \\ b_n \end{bmatrix} \sin k_n x, \tag{6}$$

and inserting Eq. (6) into Eq. (5), the following implicit dispersion relation is obtained:

$$\begin{aligned} a_m(A - k_m^2) &= \frac{4B}{L}\sum_{n+m\,\mathrm{odd}} \frac{k_m k_n}{k_m^2 - k_n^2} b_n, \\ b_m(C - k_m^2) &= \frac{4D}{L}\sum_{n+m\,\mathrm{odd}} \frac{k_m k_n}{k_m^2 - k_n^2} a_n. \end{aligned} \tag{7}$$

Even though Eqs. (7) must be solved numerically by truncating the number of spatial modes to $1 \leq m, n \leq N$, simple inspection reveals several features of the solution. First, there are no single-mode solutions. Second, if, for example, the E_y component is dominated by the k_1 mode, then the E_z component is dominated by the k_2 mode, and vice versa; i.e., the dominant mode is *different* for the E_z and E_y components of the field. Truncated Eqs. (7) were numerically solved as a generalized nonlinear eigenvalue problem of the 4th order for k_z. It is found from the numerical solution that the E_y and E_z field components of the slowest mode propagating in the magnetized plasma in a metal waveguide (MPMW) mostly correspond to the $m = 2$ and $m = 1$ modes, respectively. Mode profiles of E_y and E_z are plotted in Fig. 4(b) for $\omega = \omega_p$ and $\Omega_0 = 1.5\omega_p$. The dispersion curves ω vs k_z for the waves propagating in a MPMW plotted in Fig. 4(a) for different values of Ω_0 look remarkably similar to those plotted for the waves propagating in an infinite plasma in Fig. 2(c) despite their complicated nature. Figure 4(a) demonstrates that

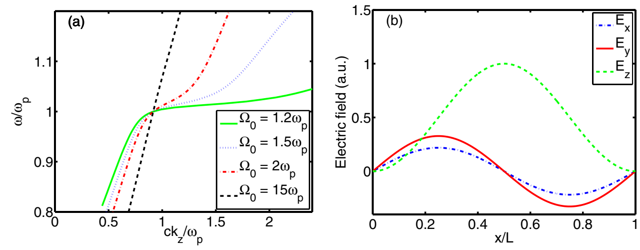

FIG. 4 (color online). (a) Dispersion curves (for various values of Ω_0) and (b) electric field profiles (for $\omega = \omega_p$ and $\Omega_0 = 1.5\omega_p$) of the slow mode of a plasma-filled waveguide of width $L = 5\pi c/\omega_p$.

(i) group velocity can be controlled by controlling the external magnetic field, and (ii) a "pivot point" exists at $\omega = \omega_p$, corresponding to the wave whose frequency (but not the group velocity) remains invariant with Ω_0.

To demonstrate the USAT compression of microwave pulses in a MPMW configuration, the time-dependent Eqs. (3) have been solved numerically for Ω_0 increasing adiabatically from $\Omega_0 = 1.5\omega_p$ to $\Omega_0 = 15\Omega_p$. We assumed $\partial_y = 0$, $\partial_z = ik_z$ and solved the one-dimensional time-dependent PDE using the standard leapfrog method. The system is again driven initially by an external current pulse at a fixed frequency ω_0, and the appropriate k_z is determined by the numerical solution of Eqs. (7). Simulation results presented in Fig. 5 demonstrate that in the MPMW configuration (i) group velocity can be increased by an order of magnitude by increasing the magnetic field in the same proportion, (ii) that the spatial field profile remains essentially unchanged in the course of USAT, and (iii) the frequency of the compressed pulse can either remain the same (for $\omega_0 = \omega_p$) or increase otherwise. As a practical example, consider the compression of an $\omega/2\pi = 3$ GHz pulse in a magnetized plasma of density $n_0 = 10^{11}$ cm^{-3} placed inside a waveguide of width $L = 25$ cm using the USAT approach. By adiabatically increasing the magnetic field from $B_0 = 0.15$ T to 1.5 T, pulse compression by an order of magnitude can be accomplished.

In conclusion, we have numerically demonstrated that microwave pulses can be compressed (with or without a frequency shift) inside a magnetized plasma by changing the magnitude or direction of the magnetic field uniformly in space and adiabatically in time. Various applications, including radars and particle accelerators, could benefit from this pulse compression technique.

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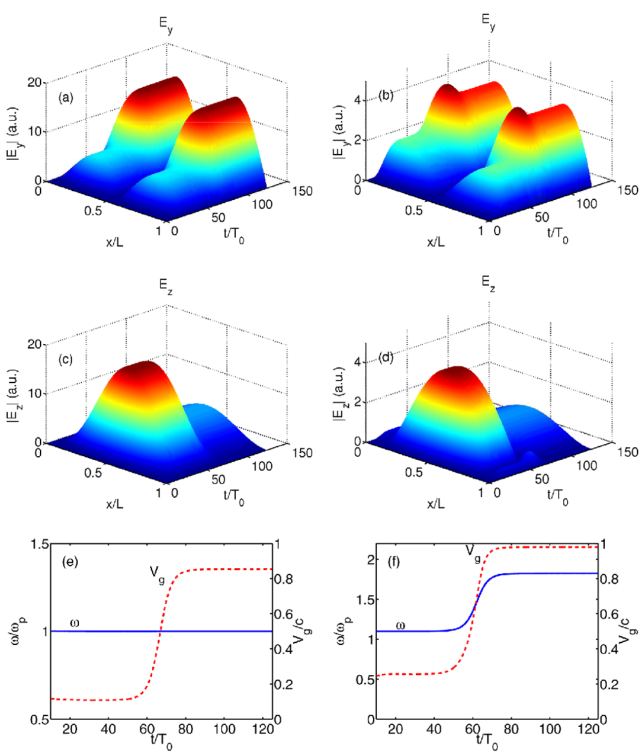

FIG. 5 (color online). Time evolution of the slow mode of the MPMW ($L = 5\pi c/\omega_p$) during the variation of the magnetic field: $1.5 < \Omega_0/\omega_p < 15$. Left column [(a),(c), and (e)]: $\omega_0 = \omega_p$, right column [(b),(d) and (f)]: $\omega_0 = 1.1\omega_p$. Group velocities calculated numerically from the Poynting flux.

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