# Kinetic Simulations of Magnetized Turbulence in Astrophysical Plasmas

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This Letter presents the first *ab initio*, fully electromagnetic, kinetic simulations of magnetized turbulence in a homogeneous, weakly collisional plasma at the scale of the ion Larmor radius (ion gyroscale). Magnetic- and electric-field energy spectra show a break at the ion gyroscale; the spectral slopes are consistent with scaling predictions for critically balanced turbulence of Alfvén waves above the ion gyroscale (spectral index $-5/3$) and of kinetic Alfvén waves below the ion gyroscale (spectral indices of $-7/3$ for magnetic and $-1/3$ for electric fluctuations). This behavior is also qualitatively consistent with *in situ* measurements of turbulence in the solar wind. Our findings support the hypothesis that the frequencies of turbulent fluctuations in the solar wind remain well below the ion cyclotron frequency both above and below the ion gyroscale.

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*Introduction.*—A wide variety of astrophysical plasmas—the solar wind and corona, the interstellar and intracluster medium, accretion disks around compact objects—are magnetized and turbulent. The turbulence is damped at small scales where the plasma is weakly collisional, requiring a kinetic description. It is often a good approximation to consider small fluctuations about an equilibrium state with a uniform (or large-scale) dynamically strong mean magnetic field (the Kraichnan hypothesis [1]). The resulting (subsonic) magnetohydrodynamic (MHD) turbulence is believed to be a Kolmogorov-like cascade of spatially anisotropic Alfvénic fluctuations [2]. Such anisotropy is observed in laboratory plasmas [3], the solar wind [4], and numerical simulations [5]. Assuming a critical balance between the linear frequencies and nonlinear decorrelation rates [2,6], the anisotropy is scale dependent with wave numbers parallel and perpendicular to the local mean field related by $k_\parallel \propto k_\perp^{2/3}$. This implies that in most astrophysical plasmas, the frequencies of the Alfvénic fluctuations remain below the ion cyclotron frequency, $\omega = k_\parallel v_A \ll \Omega_i$, even as the perpendicular wavelength reaches the ion gyroscale, $k_\perp \rho_i \sim 1$.

Such fluctuations are described by gyrokinetics (GK), a rigorous low-frequency anisotropic limit of kinetic theory [7–10], which systematically averages out the particle cyclotron motion. GK orders out the MHD fast wave and cyclotron resonances but retains finite Larmor radius effects and the collisionless Landau resonance. GK enables numerical studies of kinetic turbulence with today's computational resources because the gyroaveraging eliminates fast time scales and reduces the dimensionality of phase space from six to five. GK has been used to study electrostatic turbulence in fusion plasmas for decades, but there have been few GK treatments of astrophysical turbulence. GK is not applicable to large-scale, roughly isotropic fluctuations, such as are directly excited in the interstellar medium by supernovae. However, the fluctuations in magnetized turbulence become lower amplitude and more anisotropic at smaller scales. GK theory and simulations are thus appropriate, and hold considerable promise, for studies of microscopic phenomena such as turbulent heating and magnetic reconnection, and for interpreting observations of short-wavelength turbulent fluctuations. This Letter reports the first *ab initio*, fully electromagnetic, kinetic simulations of turbulence in a magnetized weakly collisional astrophysical plasma.

The study of turbulence in weakly collisional plasmas benefits from *in situ* measurements of the near-Earth solar wind, illuminating the properties of turbulence from the large (energy-containing) scales to the small, kinetic scales at which fluctuations are damped. The one-dimensional frequency spectrum of magnetic fluctuations typically shows a power-law behavior with a $-5/3$ slope at low frequencies [11], a break at a few tenths of a Hz, and a steeper power law at higher frequencies with a slope that varies between $-2$ and $-4$ [12]. It is generally agreed that the $-5/3$ range is an MHD inertial range, while the break and the dissipation-range slope have been attributed to proton cyclotron damping [13], Landau damping of kinetic Alfvén waves (KAW) [14], or dispersion of whistler waves [15]. Recent simultaneous magnetic- and electric-field measurements found an increase in the wave phase velocity above the spectral break [16], consistent with the conversion to a KAW cascade but inconsistent with cyclotron damping [10]. The GK simulations presented below capture all of these spectral features with magnetic- and electric-energy spectra similar to those reported empirically in [16]. Our simulation results suggest that the turbulent spectra observed in the solar wind are a consequence of the transition from an Alfvén-wave to a KAW cascade.

*The code.*—We have used the new code, ASTROGK, developed to study astrophysical turbulence. ASTROGK is essentially a slab version of the publicly available code GS2, used to study plasma turbulence in fusion devices [17]. We now give a brief overview of the code.

The simulation domain is a periodic flux tube with a straight uniform mean magnetic field $B_0$ and no equilibrium gradients. All particle species have Maxwellian equilibrium distributions. The code solves the GK equation [8], evolving the perturbed gyroaveraged distribution function $h_s(x, y, z, \varepsilon_s, \xi)$ of the guiding centers for each species $s$—ions (protons) and electrons with the correct mass ratio $m_i/m_e = 1836$. Spatial dimensions perpendicular to the mean field $(x, y)$ are treated pseudospectrally; a conservative finite-difference scheme is used in the parallel direction $z$. A gyroaveraged pitch-angle-scattering collision operator [9] is used, with pitch-angle derivatives computed from second-order finite differences. The electromagnetic field is represented by the scalar potential $\varphi$, parallel vector potential $A_\parallel$, and parallel magnetic field perturbation $\delta B_\parallel$. These are determined from the quasineutrality condition and Ampère's law [8], where the charge densities and currents are calculated as velocity-space moments of the distribution function. These velocity-space integrals (over particle energies $\varepsilon_s = m_s v^2/2$ and pitch angles $\xi = v_\parallel/v$) are done with spectral accuracy, using high-order Gaussian-Legendre integration rules. The linear terms in the GK system, including the field equations, are advanced implicitly in time; for the nonlinear terms, an explicit, third-order Adams-Bashforth scheme is used.

*Linear benchmarks.*—GS2 has been verified to describe correctly the linear kinetic physics in parameter regimes relevant to astrophysical plasmas [8]. ASTROGK has been checked to agree with GS2 exactly and benchmarked against linear kinetic theory, as illustrated by Fig. 1: for $k_\perp \rho_i \ll 1$, we find Alfvén waves with $\omega = \pm k_\parallel v_A$ and little damping; for $k_\perp \rho_i \gg 1$, these become kinetic Alfvén waves with $\omega = \pm k_\parallel v_A k_\perp \rho_i / \sqrt{\beta_i + 2/(1 + T_e/T_i)}$ and stronger damping, in agreement with linear theory [8,18]. Here $v_A = B_0/\sqrt{4\pi m_i n_i}$ is the Alfvén speed, $n_i$ the ion number density, $T_e$ and $T_i$ the ion and electron temperatures, and $\beta_i = 8\pi n_i T_i / B_0^2$.

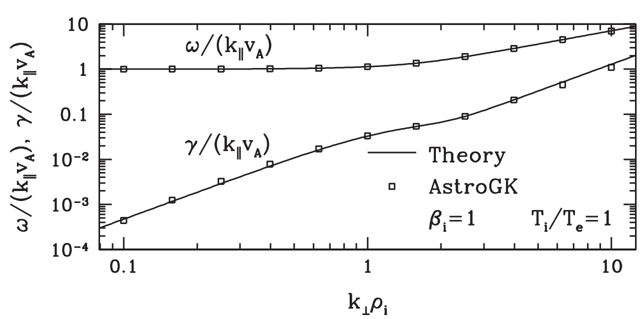


FIG. 1. Normalized frequencies $\omega/k_\parallel v_A$ and damping rates $\gamma/k_\parallel v_A$ vs normalized perpendicular wave number $k_\perp \rho_i$ for a plasma with $\beta_i = 1$ and $T_i/T_e = 1$. ASTROGK (squares) correctly reproduces the analytic results from the linear collisionless gyrokinetic dispersion relation (line) [8].

*Driving.*—The driving and dissipation scales in astrophysical turbulence are widely separated—in the solar wind, the effective driving scale is $L \sim 10^{11}$ cm and the ion gyroscale is $\rho_i \sim 10^6$ cm [10]—beyond the range accessible numerically. Our simulation domain is understood to be much smaller than the driving scale. We model the energy influx from larger scales by adding to Ampère's law a parallel "antenna" current $j^a_{\parallel,\mathbf{k}}$. For each chosen driving wave vector $\mathbf{k}_a$, the antenna amplitude is calculated from a Langevin equation whose solutions are Alfvén waves with wave vector $\mathbf{k}_a$, frequency $\omega = \pm k_{a\parallel} v_A$, and a decorrelation rate comparable to $\omega$. This driving is motivated by the expectation that turbulence in the inertial range (at scales $\rho_i \ll k^{-1} \ll L$) is Alfvénic and critically balanced [2].

*Dissipation.*—The driving injects power into the system; in steady state, this power must be dissipated into heat. By Boltzmann's $H$ theorem, no entropy increase, and therefore no heating, is possible in a kinetic system without collisions. If the collision rate is low, the distribution function develops a small-scale structure in velocity space [8,9]. This makes the velocity derivatives in the collision integral large so the collisions can act, a situation analogous to the emergence of small spatial scales in neutral fluids with small viscosity (Kolmogorov cascade). In GK turbulence, the cascades in position and velocity space are linked, creating a kinetic cascade in five-dimensional phase space [9]. Collisionless Landau damping of the electromagnetic fluctuations leads to particle heating in the sense that it transfers the electromagnetic fluctuation energy into fluctuations of the particle distribution function (the kinetic entropy cascade [9]), which are then converted into heat by collisions.

A detailed analysis of the kinetic cascade will be presented in a separate study, but the lesson is that kinetic turbulence simulations must include collisions and have sufficient velocity-space resolution for the correct relationship to be established between small-scale structures in velocity and position space. Accomplishing this with a physical collision operator simultaneously for ions and electrons is difficult. To ease the resolution requirements, we employ a hypercollisionality (analogous to hyperviscosity in fluid simulations) with the form of a pitch-angle-scattering operator with a wave-number-dependent collision rate $\nu_h (k_\perp / k_{\perp \max})^8$, where $k_{\perp\max}$ is the grid-scale wave number. This artificially enhanced collision term terminates the cascade and produces positive-definite heating close to the grid scale, but leaves essentially collisionless physics at larger scales. For the ions, the importance of the hypercollisionality is marginal, while for the electrons we need a large value of $\nu_h$. As a result, electron heating (at the electron gyroscale $\rho_e$) is not well modeled, but this is an acceptable sacrifice because our focus is on the turbulent cascade through the ion gyroscale at $\rho_i \gg \rho_e$.

*Results.*—The physical parameters in GK simulations of plasma turbulence are $\beta_i$ and $T_i/T_e$. Here both are set to 1, sensible characteristic values for the solar wind at 1 AU; a full parameter scan is desirable in the future. By varying the driving wave number $k_a$ and the (hyper)collision rate, we may focus on various scale ranges. Here we present results obtained for the inertial range ($k_\perp \rho_i \ll 1$) and the ion gyroscale ($k_\perp \rho_i \sim 1$). The normalized magnetic-energy spectrum is defined $E_{B_\perp}(k_\perp) = (L_z/L_\perp^2)2\pi k_\perp^3 \int dz \langle |A_{\|,\mathbf{k}_\perp}(z)|^2\rangle / 8\pi n_i T_i$, where $k_\perp$ is measured in units of $\rho_i^{-1}$, $L_z$ and $L_\perp$ are parallel and perpendicular box dimensions, and the angle brackets denote angle averaging over a wave number shell centered at $|\mathbf{k}_\perp| = k_\perp$ with width $2\pi/L_\perp$. The normalized electric-energy spectrum $E_{E_\perp}(k_\perp)$ is defined similarly in terms of $\varphi_{\mathbf{k}_\perp}$, with an extra factor of $(c/v_A)^2$, where $c$ is the speed of light.

In the inertial range, $k_\perp \rho_i \ll 1$, the reduced MHD equations are the rigorous limit of GK for Alfvénic fluctuations [9], so kinetic turbulence in this regime must be consistent with the numerical results obtained in MHD simulations [5]. Figure 2 shows the normalized magnetic and electric-energy spectra calculated gyrokinetically in this regime. As expected for critically balanced Alfvénic turbulence [2], these spectra are coincident, showing a scaling consistent with $k_\perp^{-5/3}$. This is the first demonstration of an MHD turbulence spectrum in a *kinetic* simulation. While not surprising, this result can be viewed as a nonlinear benchmark.

Our main focus is on scales near $k_\perp \rho_i \sim 1$, a regime that cannot be simulated by any fluid model. We know, however, that low-frequency Alfvénic turbulence is rigorously described by reduced MHD equations for $k_\perp \rho_i \ll 1$ and by a similarly reduced version of the electron MHD equations for $k_\perp \rho_i \gg 1$ [9]. The latter system supports kinetic Alfvén waves (see Fig. 1). If one assumes a turbulent cascade of KAW-like fluctuations decorrelating on a time scale comparable to the linear KAW period (critical balance), scaling arguments predict that the magnetic-energy spectrum steepens from $k_\perp^{-5/3}$ to $k_\perp^{-7/3}$, while the electric-energy spectrum flattens to $k_\perp^{-1/3}$ [9,10,19]. A spectral break at the transition between Alfvén-wave and KAW turbulence is expected at $k_\perp \rho_i \sim 1$. Figure 3 shows the energy spectra in our simulation around this transition. Both the spectral break (at $k_\perp \rho_i \simeq 2$) and the steepening (flattening) of the magnetic-(electric-)energy spectra are observed. The spectra at wave numbers below and above the transition are consistent with the predictions for critically balanced Alfvén-wave and KAW cascades [2,9,10,19].

There is a striking similarity between the simulated spectra shown in Fig. 3 and the magnetic- and electric-energy spectra in the solar wind reported in [16]. The increase in phase velocity in the dissipation range ($k_\perp \rho_i > 1$), shown by both measurement and simulation, is compelling evidence that the observed breaks in the spectra are caused by a transition to a KAW cascade, not by the onset of ion cyclotron damping [10].

The scaling predictions for KAW turbulence are made assuming negligible Landau damping. In our simulations, the damping is weak, so it is reasonable that the scaling predictions are well satisfied. However, this will not be true in all real astrophysical situations. We have argued [10] that the spectra much steeper than $k_\perp^{-7/3}$ often observed in

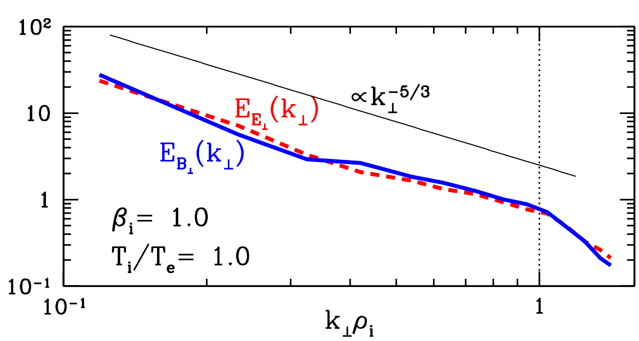


FIG. 2 (color online). Magnetic (solid line) and electric (dashed line) energy spectra in the MHD regime ($k_\perp \rho_i < 1$). The box size is $L_\perp/2\pi = 10\rho_i$. Electron hypercollisionality is dominant for $k_\perp \rho_i \geq 1$ (dotted line).

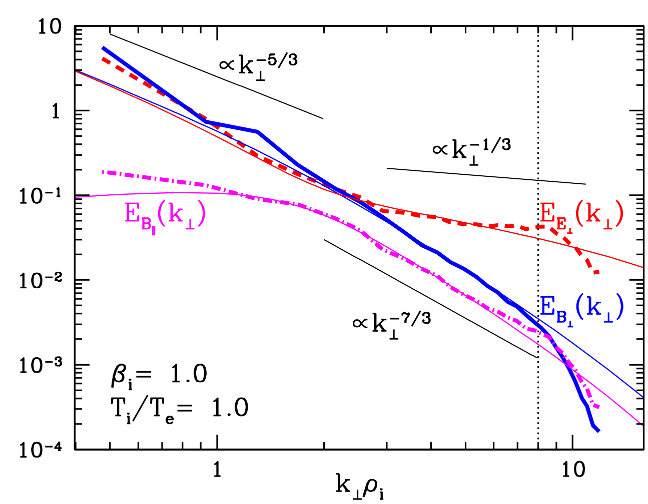


FIG. 3 (color online). Bold lines: normalized energy spectra for $\delta B_\perp$ (solid line), $\delta B_\|$ (dash-dotted line), and $E_\perp$ (dashed line). Thin lines: solution of the turbulent cascade model of [10]. Dimensions are $(N_x, N_y, N_z, N_\varepsilon, N_\xi, N_s) = (64, 64, 128, 8, 64, 2)$, requiring $\simeq 0.5 \times 10^9$ computational mesh points, with box size $L_\perp/2\pi = 2.5\rho_i$. Electron hypercollisionality is dominant for $k_\perp \rho_i \geq 8$ (dotted line).

the solar wind [12] may be due to significant Landau damping. The effect of this damping on the energy spectra can be estimated, as proposed in [10] (see also [18]), using a spectral model of the turbulent cascade based on three assumptions: (i) spectrally local energy transfer, (ii) critical balance, (iii) the applicability of the linear damping rates. Using this model, the energy spectrum $E_{B_\perp}(k_\perp)$ can be predicted over the entire simulation range, given one "Kolmogorov" constant, which quantifies the linear damping rate relative to the nonlinear cascade rate. In Fig. 3, we show that this analytical model reproduces the entire shape of the numerical spectrum. Using linear GK eigenfunctions for KAWs enables the determination of energy spectra for the electric-field fluctuations ($E_\perp$) and for the fluctuations of the magnetic field strength ($\delta B_\parallel$). The model works well without fine-tuning, for a range of values of the constant; this is because the damping is small in this simulation and our model captures the transition from Alfvénic to KAW turbulence. The agreement between the analytical model and the simulations is a nontrivial result: it suggests that the linear damping rate does not significantly underestimate the rate at which the electromagnetic energy is dissipated in the nonlinear simulations. Future simulations will determine whether stronger linear damping can account for the steeper spectra often observed in the solar wind.

*Conclusions.*—We have presented first-of-a-kind kinetic simulations of turbulence in a weakly collisional, magnetized plasma. The ion-gyroscale turbulent fluctuations simulated here represent the fate of a larger-scale MHD cascade. The qualitative agreement between our simulations and solar-wind measurements [16] supports theoretical models in which the turbulent fluctuations in the solar wind have frequencies well below the ion cyclotron frequency even when the cascade reaches the (perpendicular) scale of the ion Larmor radius. The observed break in the magnetic-energy spectrum in the solar wind is inferred to correspond to a transition to kinetic-Alfvén-wave turbulence, not to the onset of ion cyclotron damping. Although half a billion mesh points were used in the case of Fig. 3, the resolution in velocity space is still not fully sufficient to draw detailed conclusions about the turbulent heating. Nonetheless, the agreement between the simulations and an analytical cascade model based on linear damping rates implies that the latter do not significantly underestimate the true damping in a turbulent collisionless plasma. Future simulations will probe a range of plasma parameters, including more heavily damped regimes, that will allow a more quantitative study of the role of collisionless damping in turbulent plasmas. The first results reported in this Letter demonstrate that such kinetic simulations of plasma turbulence may be undertaken with some confidence, using existing computational resources.

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