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Shell Model Description of the ¹⁴C Dating β Decay with Brown-Rho-Scaled NN Interactions

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We present shell model calculations for the beta decay of ¹⁴C to the ¹⁴N ground state, treating the states of the $A = 14$ multiplet as two $0p$ holes in an ¹⁶O core. We employ low-momentum nucleon-nucleon (NN) interactions derived from the realistic Bonn-B potential and find that the Gamow-Teller (GT) matrix element is too large to describe the known lifetime. By using a modified version of this potential that incorporates the effects of Brown-Rho scaling medium modifications, we find that the GT matrix element vanishes for a nuclear density around 85% that of nuclear matter. We find that the splitting between the $(J^{\pi}, T) = (1^+, 0)$ and $(J^{\pi}, T) = (0^+, 1)$ states in ¹⁴N is improved using the medium-modified Bonn-B potential and that the transition strengths from excited states of ¹⁴C to the ¹⁴N ground state are compatible with recent experiments.

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The beta decay of ¹⁴C to the ¹⁴N ground state has long been recognized as a unique problem in nuclear structure. Its connection to the radiocarbon dating method, which has had a significant impact across many areas of science, makes the decay of broad interest even beyond nuclear physics. But *a priori* one would not expect the beta decay of ¹⁴C to be a good transition for radiocarbon dating over archaeological times, because the quantum numbers of the initial state $(J^{\pi}, T) = (0^+, 1)$ and final state $(J^{\pi}, T) = (1^+, 0)$ satisfy the selection rules for an allowed Gamow-Teller transition. The expected half-life would therefore be on the order of hours, far from the unusually long value of 5730 years [1] observed in nature. The corresponding nuclear transition matrix element is very small ($\simeq 2 \times 10^{-3}$) and is expected to result from an accidental cancellation among the different components contributing to the transition amplitude. This decay has therefore been used to investigate phenomena not normally considered in studies of allowed transitions, such as meson exchange currents [2,3], relativistic effects [4], and configuration mixing [5,6]. Of broader importance, however, is that this decay provides a very sensitive test for the in-medium nuclear interaction and in particular for the current efforts to extend the microscopic description of the nuclear force beyond that of a static two-body potential fit to the experimental data on two-nucleon systems. One such approach is to include hadronic medium modifications, in which the masses of mesons and nucleons are altered at finite density due to the partial restoration of chiral symmetry [7–9] or many-body interactions with either intermediate nucleon-antinucleon excitations [10] or resonance-hole excitations [11]. These effects are traditionally incorporated in models of the three-nucleon force, which have been well-tested in *ab initio* nuclear structure calculations of light nuclei [12,13].

In this Letter we suggest that a large part of the observed ¹⁴C beta decay suppression arises from in-medium modifications to the nuclear interaction. We study the problem from the perspective of Brown-Rho scaling (BRS) [14,15], which was the first model to make a comprehensive prediction for the masses of hadrons at finite density. In BRS the masses of nucleons and most light mesons (except the pion whose mass is protected by its Goldstone boson nature) decrease at finite density as the ratio of the in-medium to free-space pion decay constant:

$$\sqrt{\frac{g_A}{g_A^*}}\frac{m_N^*}{m_N} = \frac{m_\sigma^*}{m_\sigma} = \frac{m_\rho^*}{m_\rho} = \frac{m_\omega^*}{m_\omega} = \frac{f_\pi^*}{f_\pi} = \Phi(n), \qquad (1)$$

where g_A is the axial-vector coupling constant, Φ is a function of the nuclear density n with $\Phi(n_0) \simeq 0.8$ at nuclear matter density, and the star indicates in-medium values of the given quantities. Since all realistic models of the NN interaction are based on meson exchange and fit to only free-space data, Eq. (1) prescribes how to construct a density-dependent nuclear interaction that accounts for hadronic medium modifications. This program has been carried out in several previous studies of symmetric nuclear matter [16,17], where it was found that one could well describe saturation and several bulk equilibrium properties of nuclear matter using such Brown-Rho-scaled NN interactions.

The case of the ¹⁴C beta decay provides a nearly ideal situation in nuclear structure physics for testing the hypothesis of Brown-Rho scaling. Just below a double shell closure, the valence nucleons of ¹⁴C inhabit a region with a large nuclear density. But more important is the sensitivity of this GT matrix element to the nuclear tensor force, which as articulated by Zamick and collaborators [18,19] is one of the few instances in nuclear structure where the role of the tensor force is clearly revealed. In fact, with a

residual interaction consisting of only central and spin-orbit forces it is not possible to achieve a vanishing matrix element in a pure p^{-2} configuration [20]. Jancovici and Talmi [21] showed that by including a strong tensor force one could construct an interaction which reproduces the lifetime of ^{14}C as well as the magnetic moment and electric quadrupole moment of ^{14}N, although agreement with the known spectroscopic data was unsatisfactory.

The most important contributions to the tensor force come from π and ρ meson exchange, which act opposite to each other:

$$
\begin{aligned}
V_\rho^T(r) &= \frac{f_{N\rho}^2}{4\pi} m_\rho \tau_1 \tau_2 \left(-S_{12}\left[\frac{1}{(m_\rho r)^3} + \frac{1}{(m_\rho r)^2} \right.\right. \\
&\quad \left.\left.+ \frac{1}{3 m_\rho r}\right] e^{-m_\rho r}\right), \\
V_\pi^T(r) &= \frac{f_{N\pi}^2}{4\pi} m_\pi \tau_1 \tau_2 \left(S_{12}\left[\frac{1}{(m_\pi r)^3} + \frac{1}{(m_\pi r)^2} \right.\right. \\
&\quad \left.\left.+ \frac{1}{3 m_\pi r}\right] e^{-m_\pi r}\right).
\end{aligned}
\tag{2}
$$

Since the ρ meson mass is expected to decrease substantially at nuclear matter density while the π mass remains relatively constant, an unambiguous prediction of BRS is the decreasing of the tensor force at finite density, which should be clearly seen in the GT matrix element. In fact, recent shell model calculations [22] performed in a larger model space consisting of $p^{-2} + 2\hbar\omega$ excitations have shown that the beta decay suppression requires the in-medium tensor force to be weaker and the in-medium spin-orbit force to be stronger in comparison to a typical G-matrix calculation starting with a realistic NN interaction. We show in Fig. 1 the radial part of the tensor interaction $V^T(r) = V_\pi^T(r) + V_\rho^T(r)$ at zero density and nuclear matter density assuming that $m_\rho^*(n_0)/m_\rho = 0.80$.

Experiments to determine the properties of hadrons in medium have been performed for all of the light mesons important in nuclear structure physics. Studies of deeply bound pionic atoms [23] find only a small increase in the π^- mass at nuclear matter density and a related decrease in
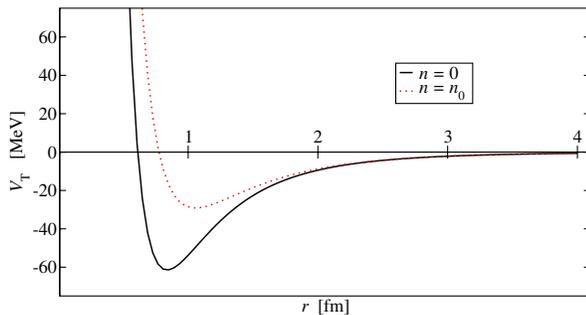

FIG. 1 (color online). The radial part of the nuclear tensor force given in Eq. (2) from π and ρ meson exchange at zero density and nuclear matter density under the assumption of BRS.

the π^+ mass. Experimental information on the scalar and vector particles comes from mass distribution measurements of in-medium decay processes. Recent photoproduction experiments [24] of correlated pions in the $T = J = 0$ channel (σ meson) have found that the distribution is shifted to lower masses in medium. The vector mesons have been the most widely studied. Whereas the situation is clear with the ω meson, the mass of which drops by $\sim 14\%$ at nuclear matter density [25], with the ρ meson it is still unclear [26,27]. We believe that our present study tests the decrease in ρ mass more simply.

Today there are a number of high precision NN interactions based solely on one-boson exchange. In the present work we use the Bonn-B potential [28] which includes the exchange of the π, η, σ, a_0, ρ, and ω mesons. In [16] the consequences of BRS on the free-space NN interaction were incorporated into the Bonn-B potential and shown to reproduce the saturation properties of nuclear matter in a Dirac-Brueckner-Hartree-Fock calculation. The masses of the pseudoscalar mesons were unchanged, and the vector meson masses as well as the corresponding form factor cutoffs were decreased according to

$$
\frac{m_\rho^*}{m_\rho} = \frac{m_\omega^*}{m_\omega} = \frac{\Lambda^*}{\Lambda} = 1 - 0.15\frac{n}{n_0}.
\tag{3}
$$

The medium-modified (MM) Bonn-B potential is unique in its microscopic treatment of the scalar σ particle as correlated 2π exchange. Finite density effects arise through medium modifications to the exchanged ρ mesons in the pionic s-wave interaction as well as through the dressing of the in-medium pion propagator with Δ-hole excitations. These modifications to the vector meson masses and pion propagator would traditionally be included in the chiral three-nucleon contact interaction and the three-nucleon force due to intermediate Δ states, respectively.

Using realistic NN interactions in many-body perturbation theory is problematic due to the strong short distance repulsion in relative S states. The modern solution is to integrate out the high momentum components of the interaction in such a way that the low energy physics is preserved. The details for constructing such a low momentum interaction, $V_{\text{low-k}}$, are described in [29,30]. We define $V_{\text{low-k}}$ through the T-matrix equivalence $T(p', p, p^2) = T_{\text{low-k}}(p', p, p^2)$ for $(p', p) \leq \Lambda$, where T is given by the full-space equation $T = V_{NN} + V_{NN}gT$ and $T_{\text{low-k}}$ by the model-space (momenta $\leq \Lambda$) equation $T_{\text{low-k}} = V_{\text{low-k}} + V_{\text{low-k}}gT_{\text{low-k}}$. Here V_{NN} represents the Bonn-B NN potential and Λ is the decimation momentum beyond which the high momentum components of V_{NN} are integrated out. Since pion production starts around $E_{\text{lab}} \simeq 300$ MeV, the concept of a real NN potential is not valid beyond that energy. Consequently, we choose $\Lambda \approx 2.0$ fm^{-1}, thereby retaining only the information from a given potential that is constrained by experiment. In fact for this Λ, the $V_{\text{low-k}}$ derived from various NN potentials are all nearly identical [30].

TABLE I.   The coefficients of the LS-coupled wave functions defined in Eq. (5) and the associated GT matrix element as a function of the nuclear density n.

n/n_0	x	y	a	b	c	M_{GT}
0	0.844	0.537	0.359	0.168	0.918	-0.615
0.25	0.825	0.564	0.286	0.196	0.938	-0.422
0.5	0.801	0.599	0.215	0.224	0.951	-0.233
0.75	0.771	0.637	0.154	0.250	0.956	-0.065
1.0	0.737	0.675	0.103	0.273	0.956	0.074

We use the folded diagram formalism to reduce the full-space nuclear many-body problem $H\Psi_n = E_n\Psi_n$ to a model space problem $H_{\text{eff}}\chi_m = E_m\chi_m$ as detailed in [31]. Here $H = H_0 + V$, $H_{\text{eff}} = H_0 + V_{\text{eff}}$, $E_n = E_n(A = 14) - E_0(A = 16, \text{core})$, and V denotes the bare NN interaction. The effective interaction V_{eff} is derived following closely the folded diagram method detailed in [32]. A main difference is that in the present work the irreducible vertex function ($\hat{Q}$ box) is calculated from the low-momentum interaction $V_{\text{low-k}}$, while in [32] from the Brueckner reaction matrix (G matrix). In the $\hat{Q}$ box we include hole-hole irreducible diagrams of first and second order in $V_{\text{low-k}}$. Previous studies [33–35] have found that $V_{\text{low-k}}$ is suitable for perturbative calculations; in all of these references satisfactory converged results were obtained including terms only up to second order in $V_{\text{low-k}}$.

Our calculation was carried out in jj coupling, where in the basis $\{p_{3/2}^{-2}, p_{3/2}^{-1}p_{1/2}^{-1}, p_{1/2}^{-2}\}$ one must diagonalize

$$[V_{\text{eff}}^{ij}] + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & 2\epsilon \end{bmatrix} \quad (4)$$
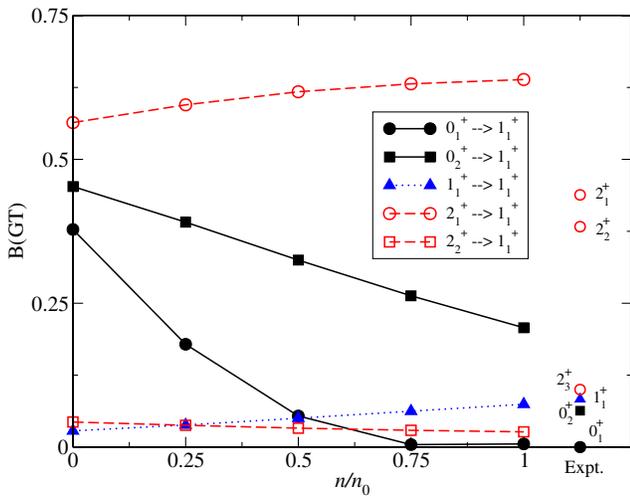
FIG. 2 (color online).   The $B(GT)$ values for transitions from the states of ^{14}C to the ^{14}N ground state as a function of the nuclear density and the experimental values from [36]. Note that there are three experimental low-lying 2^+ states compared to two theoretical 2^+ states in the p^{-2} configuration.
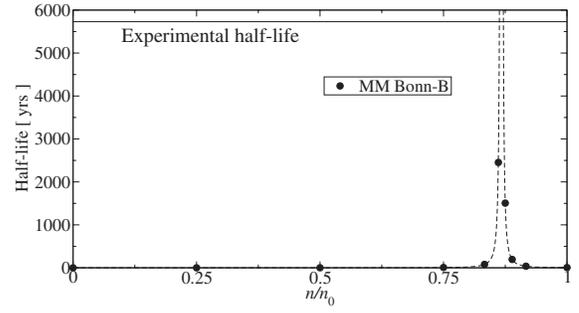
FIG. 3.   The half-life of ^{14}C, as a function of the nuclear density, calculated from the MM Bonn-B potential.

to obtain the ground state of ^{14}N (and a similar 2×2 matrix for ^{14}C). We used $\epsilon = E(p_{1/2}^{-1}) - E(p_{3/2}^{-1}) = 6.3$ MeV, which is the experimental excitation energy of the first $\frac{3}{2}^-$ state in ^{15}N. One can transform the wave functions to LS coupling, where the ^{14}C and ^{14}N ground states are

$$\psi_i = x|^1S_0\rangle + y|^3P_0\rangle$$
$$\psi_f = a|^3S_1\rangle + b|^1P_1\rangle + c|^3D_1\rangle \quad (5)$$

and the Gamow-Teller matrix element M_{GT} is given by [21]

$$\sum_k \langle\psi_f\|\sigma(k)\tau_+(k)\|\psi_i\rangle = -\sqrt{6}(xa - yb/\sqrt{3}). \quad (6)$$

Since x and y are expected to have the same sign [20], the GT matrix element can vanish only if a and b have the same sign, which requires that the $\langle^3S_1|V_{\text{eff}}|^3D_1\rangle$ matrix element furnished by the tensor force be large enough [21]. In Table I we show the ground state wave functions of ^{14}C and ^{14}N, as well as the GT matrix element, calculated with the MM Bonn-B interaction.

In Fig. 2 we plot the resulting $B(GT) = \frac{1}{2J_i+1}|M_{GT}|^2$ values for transitions between the low-lying states of ^{14}C and the ^{14}N ground state for the in-medium Bonn-B NN interaction taken at several different densities. Recent experiments [36] have determined the GT strengths from the ^{14}N ground state to excited states of ^{14}C and ^{14}O using the charge exchange reactions ^{14}N$(d, {}^2\text{He})^{14}$C and ^{14}N$(^3\text{He}, t)^{14}$O, and our theoretical calculations are in
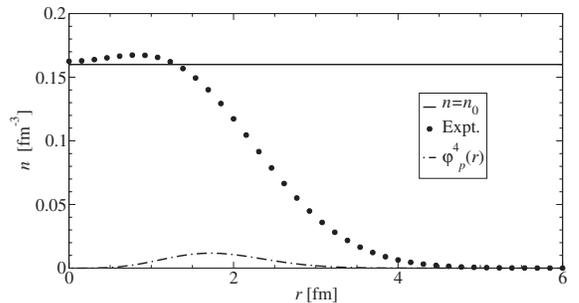
FIG. 4.   Twice the charge distribution of ^{14}N taken from [37,38] and the fourth power of the p-shell wave functions.
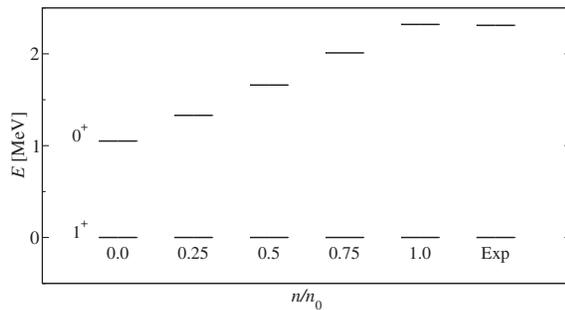
FIG. 5.   The splitting between the 1_1^+ and 0_1^+ levels in ^{14}N for different values of the nuclear density. Also included is the experimental value.

good overall agreement. The most prominent effect we find is a robust inhibition of the ground state to ground state transition for densities in the range of 0.75–1.0n_0. In contrast, the other transition strengths are more mildly influenced by the density dependence in BRS. In Fig. 3 we show the resulting half-life of ^{14}C calculated from the MM Bonn-B potential.

We emphasize that the nuclear density experienced by p-shell nucleons is actually close to that of nuclear matter; in Fig. 4 we compare twice the charge distribution of ^{14}N obtained from electron scattering experiments [37,38] with the radial part of the $0p$ wave functions, indicating clearly that the nuclear density for $0p$ nucleons is $\sim 0.8n_0$. The first excited 0^+ state of ^{14}N together with the ground states of ^{14}O and ^{14}C form an isospin triplet. We have calculated the splitting in energy between this state and the ground state of ^{14}N for a range of nuclear densities. Our results are presented in Fig. 5, where the experimental value is 2.31 MeV.

In summary, we have shown that by incorporating hadronic medium modifications into the Bonn-B potential the decay of ^{14}C is strongly suppressed at densities close to that experienced by valence nucleons in ^{14}C. In a more traditional approach such medium modifications would be built in through three-nucleon forces, and we suggest that calculations with free-space two-nucleon interactions supplemented with three-nucleon forces should also inhibit the GT transition.

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