

Measuring Overlaps in Mesoscopic Spin Glasses via Conductance Fluctuations

David Carpentier and Edmond Orignac

CNRS UMR 5672 - Laboratoire de Physique de l'École Normale Supérieure de Lyon, 46, Allée d'Italie, 69007 Lyon, France

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We consider the electronic transport in a mesoscopic metallic spin glass. We show that the distribution of overlaps between spin configurations can be inferred from the reduction of the conductance fluctuations by the magnetic impurities. Using this property, we propose new experimental protocols to probe spin glasses directly through their overlaps.

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Understanding the physics of glasses remains one of the deepest experimental and theoretical challenge in condensed matter. Considered as the simplest glassy phases, spin glasses have attracted considerable attention both experimental and theoretical during more than 30 years [1,2]. In spin glasses (SG), magnetic moments occupying random positions in a host lattice get frozen with random orientations below a spin glass transition temperature T_{SG} . The initial theoretical efforts were devoted to the solution of equilibrium lattice models. This culminated with the mean-field solution of the fully connected Sherrington-Kirkpatrick (SK) model [3], now proven to provide the exact free energy [4,5]. However, an intense theoretical debate remains around the relevance of the SK model to the thermodynamic properties of three dimensional spin glasses. Although the initial theoretical studies were focused on the thermodynamics, it was long known that spin glass materials drop out of equilibrium below T_{SG} , and never reach a steady state. This has led to the development of various models of nonequilibrium spin glass dynamics including a scaling approach [6,7], phenomenological trap models [8], and more recently aging studies of the mean-field models (see [9] and references therein). In the simplest scaling approach to spin glasses, the thermodynamics below T_{SG} consists, in contrast to mean-field solutions, of a doubly degenerate broken Z_2 symmetry ground state (for Ising spins). All the nontrivial properties of the spin glass state are then assumed to be consequences of the extremely slow relaxation toward this ground state resulting from the slow growth of small domains (droplets) of equilibrium phase, similar to ferromagnet quench.

In view of the current understanding of the spin glass state, it is a worthwhile goal to propose new probes of their properties. A central quantity in current theoretical descriptions of spin glasses is the overlap between two spin configurations defined as (N_{imp} being the number of spins and $\langle \rangle_{th}$ the thermal average)

$$Q_{12} = \frac{1}{N_{imp}} \sum_{i=1}^{N_{imp}} \langle \vec{S}_i^{(1)} \cdot \vec{S}_i^{(2)} \rangle_{th}. \quad (1)$$

Configurations overlaps gives access to distances between spins configurations, as opposed to conventional probes

like spin susceptibility. For configurations corresponding to equilibrium states in the same sample, overlap distribution is the equilibrium order parameter [2]. For configurations taken in the same quench but at different times, overlaps characterize the glass aging [9]. Besides, other physical effects such as temperature or disorder chaos [10] are described in terms of spins overlaps. It is the purpose of this Letter to propose the first experimental probe of these central quantities *via* the study of conductance fluctuations in mesoscopic metallic spin glasses.

The dependence of the average conductance fluctuations on magnetic impurities was first analyzed by Al'tshuler and Spivak [11]. Soon after, Feng *et al.* [12], building on these results, predicted *within the scaling approach* a chaotic behavior of conductance as a function of temperature in a spin glass. Parallel to these theoretical developments, experimental measurements of conductance fluctuations in metallic spin glasses by de Vegvar *et al.* [13] (see also [14]) demonstrated for the first time a clear signature of the spin glass freezing in the time-reversal antisymmetric part of the four terminal conductance. Later, several experiments focused on noise measurements in Cu:Mn [15] and Au:Fe [16] (see also [17] for similar studies in the doped semiconductor).

However, a connection between these theoretical and experimental analyzes and spin glass theories was difficult as none of these approaches allowed a simple interpretation within the spin glass theoretical framework. This motivates a reexamination of the description of conductance fluctuations in a spin glass in relation with spin overlaps.

We consider a mesoscopic metallic sample of size L containing magnetic impurities. These impurities provide three different contributions to the scattering potential for the conduction electrons: (i) a scalar potential $V(\mathbf{r}) = \sum_i v_o \delta(\mathbf{r} - \mathbf{r}_i)$, where \mathbf{r}_i denotes the positions of impurities, (ii) a spin coupling $V_S = J(T) \sum_{i=1} \vec{S}_i \cdot \vec{\sigma}_e$, and (iii) a spin-orbit contribution $V_{so}(\vec{k}_1, \vec{k}_2) = iV_{so}(\hat{k}_1 \times \hat{k}_2) \cdot \vec{\sigma}_{s_1 s_2}$. The magnetic impurities interact with each other *via* a RKKY interaction [18] $\sum_{i \neq j} J_{ij} \vec{S}_i \cdot \vec{S}_j$, and at high enough impurity concentrations, the corresponding spin glass transition temperature T_{SG} is larger the Kondo temperature T_K . In this regime, the local moments remain unscreened at

T_{SG} and as a result, due to the random signs of the couplings, they freeze into the spin glass state for $T < T_{\text{SG}}$ [18]. In the spin glass state, the frozen spins act on the conduction electrons just like a classical random magnetic field. In the rest of the letter, we will focus on this regime ($T_{\text{SG}} > T \gg T_K$) where transport properties simply result from coherent diffusion of electrons by both a classical random magnetic field and the associated scalar potential.

Let us start by recalling known results about the fluctuations of conductance induced by a scalar random potential $V(\mathbf{r})$. We denote by L_ϕ the dephasing or inelastic scattering length, which is considered larger than the sample size L (mesoscopic regime). The associated inelastic scattering rate is $\gamma_\phi = \hbar/(2\tau_\phi) = D\hbar/(2L_\phi^2)$, D being the diffusion constant in the sample. For the sake of clarity, we focus on the longitudinal $G = G_{xx}$, although the following discussion extends naturally to other components of the conductance [19]. In this mesoscopic regime, the conductance G is a function of the random scattering potential $V(\mathbf{r})$, and in the weak disorder limit, its distribution is approximately gaussian (see [20] for a recent discussion). Its average incorporates weak-localization corrections [21,22], and its variance, describing the sample-to-sample fluctuations, contains contributions from both fluctuations of the diffusion coefficient and of the density of states (see [23] for a pedagogical introduction). With only a scalar potential $V(\mathbf{r})$ and in the so-called diffusion limit, the fluctuations of the conductance read for weak disorder:

$$\langle(\delta G)^2\rangle_V = F(\gamma_\phi) = 6\left(\frac{e^2 D}{hL^2}\right)^2 \sum_{\vec{q}} (Dq^2 + \gamma_\phi)^{-2}, \quad (2)$$

where $\delta G = G - \langle G \rangle_V$. For a wire where diffusion takes place in one-dimension (1D) between two absorbing reservoirs, the variance (2) reduces to $\langle(\delta G)^2\rangle_V = 8/15(e^2/h)^2$, the so-called universal conductance fluctuations in the limit $L \ll L_\phi$. In the other case $L_\phi < L$, these fluctuations reduce to $\langle(\delta G)^2\rangle_V \approx (e^2/h)^2(L_\phi/L)^{4-d}$ with a geometrical factor.

How are these results modified in the presence of the random field component V_S of the scattering potential? First, those spins that can flip during the electron diffusion time (either weakly connected or maximally frustrated) will contribute to the enhancement of the inelastic scattering rate γ_ϕ . We assume that the inelastic coherence length of the sample, including the effects of these quasifree spins, is still larger than the system size at low enough temperatures. The remaining spins are considered as classical random fields, frozen on the electrons diffusion time scale. These random fields “flip” the electron spin, and thus provide a finite lifetime to different diffusion spin states. Using a semiclassical approach allows us to consider a given realization of spins, without averaging. A diffusion path is labeled by the sequence of encountered impurities, ordered chronologically. At each impurity j , the

electron’s spin is rotated according to $\mathcal{R}_j = e^{iJ\vec{S}_j \cdot \vec{\sigma}} = \cos(JS) + i \sin(JS)\hat{S} \cdot \vec{\sigma}$. The end action of the random fields along the path is encoded by the chronological product $\prod_j \mathcal{R}_j$. Expanding this product in the limit of weak J , and using the central-limit theorem we obtain the *typical* magnetic dephasing rate of an electron state as $\gamma_m = 2\pi\rho_0 n_{\text{imp}} J^2 \langle S^2 \rangle_{\text{th}}$, where ρ_0 is the density of state, n_{imp} the concentration of impurities and $\langle \rangle_{\text{th}}$ means an average over thermal fluctuations. Note that in doing so, we have neglected all spatial spins correlations, a coherent assumption in a spin glass state. Moreover, we have assumed a good impurities sampling by typical diffusion path, approximating the number of impurities along typical path by N_{imp} .

Coming back to the conductance fluctuations, we assume that both sources of disorder $V(\mathbf{r})$ and $V_S(\mathbf{r})$ can be treated as independent from each other (the orientation of the frozen moment is not directly correlated with the position of the single impurity). We focus on the average (over V) correlations between conductances in a given sample $V(\mathbf{r})$ with two different spin configurations $\{S_j^{(1)}\}$ and $\{S_j^{(2)}\}$

$$\langle(\Delta G)_{S^{(1)}S^{(2)}}^2\rangle = \langle\delta G(V, \{S_j^{(1)}\})\delta G(V, \{S_j^{(2)}\})\rangle_V \quad (3)$$

where $\delta G(V, \{S_j^{(1)}\}) = G(V, \{S_j^{(1)}\}) - \langle G(V, \{S_j\}) \rangle_V$. The weak disorder expression of this average correlation is obtained similarly to (2) using standard diagrammatic techniques (see [11,23,24]). In doing so, one formally considers the diffusion of the so-called Cooperons and diffusons. They can be viewed as the coherent propagation of pairs of electrons along a path in the same chronological order (diffuson), or in reversed orders (Cooperon). In the correlations (3) the two components of the diffuson or Cooperon see *a different spin configuration*. The action of the magnetic impurities on their diffusion is

$$\prod_j e^{i(JS)\vec{S}_j^{(1)} \cdot \vec{\sigma}^{(1)}} e^{\pm i(JS)\vec{S}_j^{(2)} \cdot \vec{\sigma}^{(2)}}, \quad (4)$$

where the \pm sign depends on the nature of the diffusive object. The diffuson or Cooperon states carry a pair of spins 1/2, and are naturally decomposed into a singlet and triplet states. In a given spin configuration ($\vec{S}_j^{(1)} = \vec{S}_j^{(2)}$), only the triplet states couple to these random magnetic fields while for two different spin configurations $\{S_j^{(1)}\}$, $\{S_j^{(2)}\}$, both the singlet and the triplet states acquire a finite diffusion lifetime. From Eq. (4), we obtain the *typical* dephasing rate of these composite diffusons in the limit of weak J : $\gamma_m^{D,S} = \gamma_m(1 - Q_{12})$, $\gamma_m^{D,T} = \gamma_m(1 + \frac{1}{3}Q_{12})$ where Q_{12} is defined in (1). The scattering rates for the Cooperons follow by $Q_{12} \rightarrow -Q_{12}$. Now plugging these scattering rates back into the diffusion propagators, we obtain the following expression for the average correla-

tions:

$$(\Delta G)^2 = \frac{1}{4}F(\gamma_m^{D,S} + \gamma_\phi) + \frac{3}{4}F(\gamma_m^{D,T} + \gamma_{so} + \gamma_\phi) + \frac{1}{4}F(\gamma_m^{C,S} + \gamma_\phi) + \frac{3}{4}F(\gamma_m^{C,T} + \gamma_{so} + \gamma_\phi), \quad (5)$$

where we have included the spin-orbit and inelastic dephasing rates. When (some) magnetic dephasing lengths are smaller than L ($<L_\phi, L_{so}$), the fluctuations are dominated by the smallest dephasing rate (one of the singlets). For $\gamma_\phi \ll E_c \ll \gamma_m(1 - Q_{12})$, we obtain for $D = 2$:

$$\langle \delta G(\{S_j^{(1)}\}) \delta G(\{S_j^{(2)}\}) \rangle_V \propto \left(\frac{e^2}{h}\right)^2 \frac{E_c}{\gamma_m(1 - Q_{12})}, \quad (6)$$

where E_c is the Thouless energy. Crucially, these correlations depends on the quantity Q_{12} , which is called the spin *overlap* and plays a central role in the description of spin glasses. Indeed, the distribution of Q_{12} is the spin-glass order parameter in the mean-field theory.

Before pursuing further into the spin glass considerations, we first discuss the disorder averaging in experiments. In the above analysis, we have carefully avoided to average over spin configurations while at the same time averaging over the scalar potential. However, in experiments, both disorder originates from the same source, i.e., the random positions of magnetic impurities. We thus need to propose an experimental setup that simulates an average over a scalar potential while keeping the distribution of spins (quasi-) fixed. Experiments usually rely on the ergodic hypothesis and probe these fluctuations by varying the magnetic field or the Fermi energy. Physically the origin of conductance fluctuations are phase-coherent contributions, encoded into the Cooperon and diffuson. A change in disorder changes the various diffusion paths, and the corresponding interferences. However, the various phases can also be modified either by the orbital effect of a uniform magnetic field or a change of the Fermi energy, hence the ergodic hypothesis. Averaging over the Fermi energy can be achieved in doped semiconductor spin glasses by applying a gate potential [17,25]. In metallic spin glasses, one has to resort to the magnetic field sampling and the average in Eq. (3) is replaced by $\langle \delta G(V, \{S_j^{(1)}\}) \delta G(V, \{S_j^{(2)}\}) \rangle_B$ defined by

$$\frac{1}{B_{\max} - B_\phi} \int_{B_\phi}^{B_{\max}} \delta G(V, \{S_j^{(1)}\}, B) \delta G(V, \{S_j^{(2)}\}, B) dB.$$

The decorrelation field B_ϕ corresponds to two flux quanta through the sample, and for the variance of the conductance, relatively weak field amplitude B_{\max}/B_ϕ are necessary for a correct sampling [20]. Note that with a magnetic field, Cooperons are dephased, and Eq. (5) simplifies to its first line. A crucial step for spins glasses, is to be able to find a magnetic field B_{\max} at low enough temperature such that conductance fluctuations are enough sampled, while at the same time the magnetic response of the spins *can be*

neglected. In the experimental work of Ref. [13], the authors carefully checked that this condition was fulfilled. On the theoretical side, we are not aware of any study of these effects in the limit of weak magnetic field [see, however, [26] for a study at fields large and comparable to $T_{SG}/(g\mu_B)$ and at the mean-field level].

Having access to overlaps of the spin configurations opens new perspectives in probing the metallic spin glasses that we illustrate here by discussing three possible experimental protocols. (a) the distribution of overlaps between spin configurations corresponding to different quench in the same sample can be obtained from the previous ideas, as described in Fig. 1. In the limit of long times t_w , this distribution provides information on the phase space structure of the spin glass. Obtaining this quantity for the first time in an experimental spin glass would be of major importance as current theoretical proposals differ on its expected behavior. (b) Direct access to spin overlaps at different times allows for an unprecedented analysis of the aging of experimental spin glasses [9]. In a canonical experimental scheme, the sample is cooled down below T_{SG} under a small magnetic field. This field is kept constant for a time t_w and then switched off. Magnetic field sweeps provide $\langle \delta G(t_w) \delta G(t_w + t) \rangle_B$ and thus the overlap $Q(t_w, t_w + t)$. By repeated cool down, both the average and crucially the statistics of this quantity can be determined. (c) a procedure similar to (a) allows to probe the

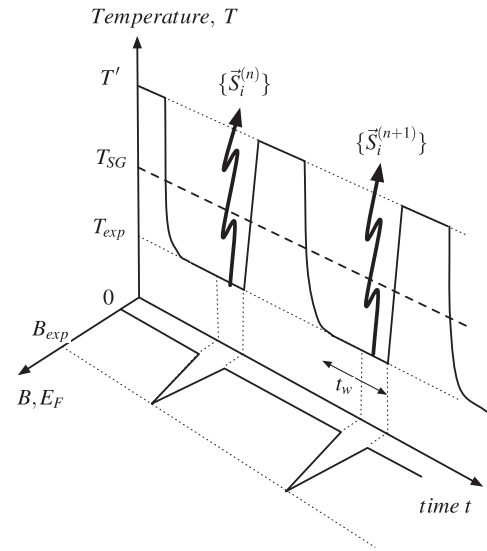


FIG. 1. Proposed experimental protocol: the temperature is cycled between T_{exp} and $T' > T_g$ in successive runs labeled by n . Measurements of the magneto-conductance are done at T_{exp} after some equilibration time t_w . Different runs n and m correspond to different spins states $\{\vec{S}_i^{(n)}\}$ and $\{\vec{S}_i^{(m)}\}$ at the same temperature T_{exp} . The average correlations $\langle \delta G_n \delta G_m \rangle_B$ between the corresponding magnetoconductance provides a determination of the overlap Q_{nm} between the two spin configurations. Repeating this process gives access to the (nonaverage) overlap distribution for the waiting time t_w .

temperature chaos of the spin glass [12], and its relation with the rejuvenation ($T' < T_{\text{exp}}$) and memory ($T' > T_{\text{exp}}$) phenomena [27]. The sample is kept at T_{exp} for a time t_w before applying a small field sweep. The temperature is then switched to $T' = T \pm \Delta T$ which now remains below T_{SG} . Successive magnetic sweep at times $t_n = t_w + n\tau$ are then applied at T' . Conductance correlations provides the overlap $Q(T_{\text{exp}}, t_w; T', t_n)$ which characterizes the temperature chaos of the spin glass, and determine, in particular, the dependence of the overlap length L_c on both temperature variation ΔT and time t_n . The behavior of L_c in relation with predictions from the droplet theory are currently theoretically investigated (see, e.g., [10,27]). The goal is a characterization of low energy excitations of a spin glass state, and their slow evolution.

Let us end by commenting on the dimensionality of a mesoscopic spin glass. To remain in the coherent transport regime, sample size L has to be of the order of (or lower than) the inelastic coherence length L_ϕ [28]. Often quantum diffusion takes place in an effective space of dimension $D = 1$. On the other hand, the dimensionality of the spin glass is determined by the dynamical correlation length. Values of the correlation length $\xi_{\text{SG}} = N_{\text{SG}} n_{\text{imp}}^{1/3}$ (n_{imp} the density of spins) extracted from field change experiments for various spin glasses [29] and extrapolation from recent numerical simulations [30] are of the order of $N_{\text{SG}} \sim 30\text{--}50$ spins after a waiting time $t_w = 1000$ s. For samples with transverse dimensions L_y, L_z larger than ξ_{SG} , the proposed conductance measurements will probe properties of an effective 3D spin glass. Reconsidering the experiments of [13], we obtain approximately ≈ 40 spins in the transverse dimensions $L_y \approx 900$ Å, implying a 3D spin glass behavior. Moreover, this opens the perspective of studying a possible 3D to 1D crossover for the spin glass dynamics, and possibly determining the associated dynamical correlation length.

To conclude, we have shown how new experimental protocols to determine mesoscopic conductance fluctuations in spin glasses can provide access to the spin configurations overlaps. We believe that these protocols can open completely new ways of characterizing the spin glass physics, thus allowing progress in their understanding.

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