Increase of the Backward Raman Reflectivity Caused by the Langmuir Decay Instability in an Inhomogeneous Plasma: The Loss of Gradient Stabilization

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We investigate the nonlinear evolution of the backward stimulated Raman scattering (BSRS) in the regime where the nonlinear saturation mechanism is the Langmuir decay instability resulting from the coupling of the BSRS-generated Langmuir wave with the ion acoustic waves. We present numerical results obtained with a fluid-type code in one- and two-dimensional spatial dimensions, in the case of an inhomogeneous plasma. The plasma density is under quarter-critical and depends linearly on the longitudinal spatial coordinate, in the regime where the Rosenbluth gain factor for the amplitude, denoted as G_{Ros}, is in the range $\pi/2 \leq G_{\mathrm{Ros}} \leq 6$. We observe that the Langmuir decay instability is able to suppress the gradient stabilization and restore the absolute nature of BSRS, thus leading to a significantly increased BSRS reflectivity.

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The stimulated Raman scattering is a well-known scattering process that may occur when a transverse electromagnetic wave interacts with a nonlinear dielectric. This scattering process plays a very important role in the case of a laser wave propagating in a plasma. The backward stimulated Raman scattering (BSRS) corresponds to the parametric instability by which an incident laser wave couples to a Langmuir wave to give rise to a backscattered electromagnetic wave. In the scheme of laser fusion, BSRS may lead to a net energy loss of the incident laser beams and to the generation of fast electrons able to preheat the fusion fuel. For these reasons, BSRS may be detrimental for the fusion gain, and many fundamental studies have been carried out experimentally, aimed at checking the theoretical predictions concerning the evolution of this instability.

In the laser fusion plasmas, the plasma density is spatially inhomogeneous, so that this situation enters into a generic case where the matching conditions for three wave resonance can only be satisfied locally. Such a generic case has been investigated by Rosenbluth in his 1972 seminal work [1]. He established the surprising result that in an infinitely long plasma, a parametric instability resulting from a resonant three wave coupling is stabilized by the plasma inhomogeneity, however small it might be, as long as it gives rise to a wave number mismatch that varies linearly with the propagation coordinate. Quite generally [2], in the inhomogeneous cases characterized by a resonance mismatch that is a monotonic function of the propagation coordinate, and for densities below quarter-critical, Rosenbluth's analysis predicts that the decay waves amplitudes $|a_\alpha|^2$ grow spatially, starting from the thermal level $|a_\alpha|^2_{\mathrm{th}}$ and reaching the level $|a_\alpha|^2 = |a_\alpha|^2_{\mathrm{th}} \exp 2G_{\mathrm{Ros}}$, where G_{Ros} denotes the so-called Rosenbluth gain factor for the amplitude. For this reason, the potential development of BSRS is usually considered to be controlled by the value of the Rosenbluth gain factor, and BSRS is expected to be negligible for $G_{\mathrm{Ros}} \leq 6$. On the other hand, the Rosenbluth result is also known to be nonrobust [3,4]: in particular, the absolute nature of BSRS could be restored when either (i) fast spatial variations of one of the plasma parameters, or (ii) small sinusoidal modulations, or (iii) low frequency fluctuations are superimposed to a linear density profile. This result holds in the regimes where the parametric coupling would give rise to an absolute instability if the plasma were homogeneous. An absolute instability corresponds to an infinite temporal growth of the decay waves within the linear stability analysis framework. In this case, the parametric instability develops to a level determined by the nonlinear saturation mechanisms, and this level can be significantly higher than the spatial amplification prediction $|a_\alpha|^2 = |a_\alpha|^2_{\mathrm{th}} \exp 2G_{\mathrm{Ros}}$.

In this Letter, we demonstrate that the BSRS nonlinear saturation mechanism is itself capable of generating low frequency fluctuations which are able to restore the absolute nature of BSRS. Thus, BSRS may behave as a nonlinear absolute instability, as already shown by Rose in Ref. [5] in the context where it is the Langmuir wave turbulence which nonlinearly self-sustains the stimulated Raman scattering. More precisely, we find that there exists a threshold G_T for the Rosenbluth gain factor, such that for $G_{\mathrm{Ros}} < G_T$ the decay waves amplitudes remain controlled by the Rosenbluth gain factor according to the above expression $|a_\alpha|^2 = |a_\alpha|^2_{\mathrm{th}} \exp 2G_{\mathrm{Ros}}$; on the other hand, for $G_{\mathrm{Ros}} > G_T$ the decay waves amplitudes are large enough to make it possible for the nonlinear effects to give rise to low frequency fluctuations able to restore the absolute nature of BSRS, so that the decay waves amplitudes are then well above this expression.

Specifically, we consider the BSRS nonlinear evolution in an inhomogeneous plasma, in the regimes for which the nonlinear BSRS saturation results from the coupling of the Langmuir waves (LW) with the ion acoustic waves (IAW). We restrict ourselves to the regime $k_L \lambda_{\mathrm{De}} \leq 0.25$ for which the saturation mechanism is the Langmuir decay instability

(LDI) cascade. Here k_L denotes the characteristic wave number of the BSRS-generated Langmuir wave, and $\lambda_{\rm De}$ is the electron Debye length. LDI is the process by which the LW generated by BSRS couples to an IAW and gives rise to a counterpropagating LW. This process can repeat itself, leading to the so-called LDI cascade. The BSRS nonlinear saturation then results from the transfer of the LW energy in a spectral domain which is off resonance with BSRS [6]. The condition $k_L\lambda_{\rm De} \le 0.25$ makes it possible to ignore nonlinear wave-particle interaction and consequently to use a fluid-type description in which the Langmuir wave damping is modeled by the usual Landau damping operator [7,8].

In our fluid-type model, the coupling of the Langmuir waves to the ion acoustic waves is described by the Zakharov equations [7,9] in which the plasma inhomogeneity is kept into account. The electromagnetic waves amplitudes $\tilde{E}_\alpha$ are decomposed into their fast and slow variations in space and time as $\tilde{E}_\alpha = E_\alpha \exp -i(\omega_\alpha^{\rm ref} t - k_\alpha^{\rm ref} x) + {\rm c.c.}$, where E_α are the slowly varying envelope amplitudes. The subscript α refers to the incident laser wave for $\alpha = 0$ and to the backscattered wave for $\alpha = R$. The frequency $\omega_\alpha^{\rm ref}$ and wave number $k_\alpha^{\rm ref}$ satisfy the transverse wave dispersion relation corresponding to an electron density called "reference," denoted as $n_0^{\rm ref}$, and arbitrarily chosen to be the density at the center of the plasma. They also satisfy the frequency matching condition $\omega_0^{\rm ref} = \omega_R^{\rm ref} + \omega_{\rm pe}^{\rm ref}$, where $\omega_{\rm pe}^{\rm ref}$ denotes the electron plasma frequency calculated for the reference density $n_0^{\rm ref}$. The Langmuir waves amplitude $\tilde{E}_L$ is similarly decomposed into its fast and slow variation in time as $\tilde{E}_L = E_L \exp -i\omega_{\rm pe}^{\rm ref} t + {\rm c.c.}$, where E_L is the slowly varying envelope amplitude. No envelope approximation is made for the IAW evolution. The plasma inhomogeneity is modeled by the equilibrium density n_0, which is a linear function of the propagation coordinate. The coupled mode equations describing BSRS coupled with LDI are the following:

$$\left(D_0 + \frac{i}{2\omega_0}\left[\frac{n_e^{bf} - n_0^{\rm ref}}{n_c}\right]\right)E_0 = -\frac{i}{2}C_{L,R}, \qquad \left(D_R + \frac{i}{2\omega_R}\left[\frac{n_e^{bf} - n_0^{ref}}{n_c}\right]\right)E_R = -\frac{i}{2}C_{L,0},$$
$$\left(D_L + \frac{i}{2\omega_{\rm pe}^{\rm ref}}\left[\frac{n_e^{bf} - n_0^{\rm ref}}{n_c}\right]\right)E_L = -\frac{i}{2}n_e^{bf}C_{0,R}/\sqrt{n_0^{\rm ref}}, \qquad D_s \ln\left(\frac{n_e^{bf}}{n_0}\right) = \frac{Zm_e}{m_i}\nabla^2\left\{\epsilon_L\frac{|E_L|^2}{\omega_{\rm pe}^2} + \epsilon_S\left[\frac{|E_0|^2}{\omega_0^2} + \frac{|E_R|^2}{\omega_R^2}\right]\right\}, \tag{1}$$

where $n_e^{bf} \equiv n_0 + n_s$ denotes the total low frequency density, n_0 being the background electron density and n_s the density perturbation associated to the IAW. The symbols D_α with $\alpha = 0, R$ denote the paraxial propagators for the transverse waves, $D_\alpha \equiv [\partial_t + \nu_\alpha + v_{g,\alpha}\partial_x - i(c_\alpha^2/2\omega_\alpha^{\rm ref})\nabla_\perp^2]$, where $v_{g,\alpha}$ is the group velocity of wave α at the reference density, and ν_α is its damping coefficient. D_L denotes the propagator for the Langmuir waves: $D_L \equiv [\partial_t + \nu_L - i(3v_{\rm the}^2/2\omega_{\rm pe}^{\rm ref})\nabla^2]$, where $v_{\rm the} = [T_e/m_e]^{1/2}$ is the electronic thermal velocity, ν_L is the Langmuir wave damping defined in Fourier space, T_e is the electron temperature, m_e is the electron mass. The IAW propagator is given by $D_s \equiv (\partial_t^2 + 2\nu_s\partial_t - c_s^2\nabla^2)$, where $c_s = [(ZT_e + 3T_i)/m_i]^{1/2}$ is the sound velocity, ν_s is the IAW damping defined in Fourier space, T_i is the ion temperature, m_i is the ion mass. The mode coupling terms are $C_{L,R} \propto f_{\rm win}(\nabla \cdot E_L)E_R \exp(-ik_L^{\rm ref}x)$, $C_{L,0} \propto f_{\rm win}(\nabla \cdot E_L^*)E_0 \exp(ik_L^{\rm ref}x)$, and $C_{0,R} \propto f_{\rm win}\nabla[E_0E_R^* \exp(ik_L^{\rm ref}x)]$ where $k_L^{\rm ref} \equiv k_0^{\rm ref} - k_R^{\rm ref}$ denotes the wave number of the Langmuir wave at the reference density. $f_{\rm win}$ is a window function; its role is to prevent any unphysical destabilizations of absolute instabilities at the edge of the density profile. $\omega_{\rm pe} = \omega_{\rm pe}^{\rm ref}\sqrt{n_0/n_0^{\rm ref}}$ is the local electron plasma frequency. We added a noise source term in the equation for E_L in order to properly describe the thermal equilibrium. The values of the quantities ϵ_L and ϵ_S are $\epsilon_L = \epsilon_S = 1$; taking $\epsilon_L = 0$ makes it possible to artificially suppress the coupling of the LW to the IAW; similarly, taking $\epsilon_S = 0$ makes it possible to suppress the transverse waves self-focusing.

We first show the results of 2D simulations in the case of a monospeckle laser beam generated by the flattop model [10]. The simulations parameters are the following: the laser wavelength, denoted as λ_0 and expressed in μm, is 1.06, the numerical aperture $f_\#$ of the focusing optics is $f_\# = 3$. The laser beam is focused in the center of the plasma with the intensity $I = 2 \times 10^{15}$ W/cm^2; the plasma longitudinal and transverse sizes are $L_\parallel = 127\lambda_0$ and $L_\perp = 16\lambda_0$, the electron temperature is $T_e = 1$ keV, the ion temperature is such that $ZT_e/T_i = 10$ with $Zm_e/m_i = 1/1836$. The density profile along the propagation axis is a linear ramp varying in the range $[0.08{:}0.12]n_c$; the value of the key parameter $k_L\lambda_{\rm De}$ is $k_L^{\rm ref}\lambda_{\rm De}^{\rm ref} = 0.21$ at the reference density $n_0^{\rm ref} = 0.1n_c$. The parameters lead to the following value of the Rosenbluth's amplitude gain factor $G_{\rm Ros} \sim 4.5$. The reflectivity is the ratio of the backscattered wave energy flux over the incoming wave energy flux.

It can be observed in Fig. 1 that the time averaged reflectivity $\langle R\rangle$ is higher, by the factor 5, in case (b) when BSRS is coupled to LDI and to self-focusing ($\epsilon_L = \epsilon_S = 1$) than in case (a), when BSRS is considered on its own, i.e., without any coupling to the plasma low frequency response ($\epsilon_L = 0$ and $\epsilon_S = 0$). We checked that in this case (a), the Raman reflectivity is in excellent agreement with the theoretical predictions [2] leading to

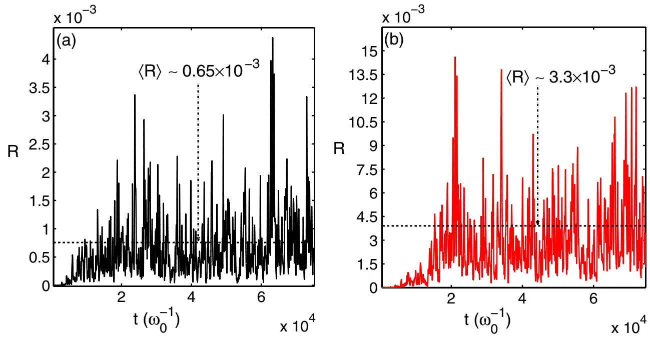

FIG. 1 (color online). 2D case. Reflectivities versus time for 2D simulations carried out in the time interval $T = 45$ ps ($7.5 \times 10^4 \omega_0^{-1}$) in the cases: (a) BSRS without LDI and without self-focusing (solid black line); the corresponding time averaged reflectivity is $\langle R \rangle = 0.65 \times 10^{-3}$ (dashed line); (b) BSRS with LDI and with autofocusing [solid red (gray) line]; the corresponding time averaged reflectivity is $\langle R \rangle = 3.3 \times 10^{-3}$ (dashed line).

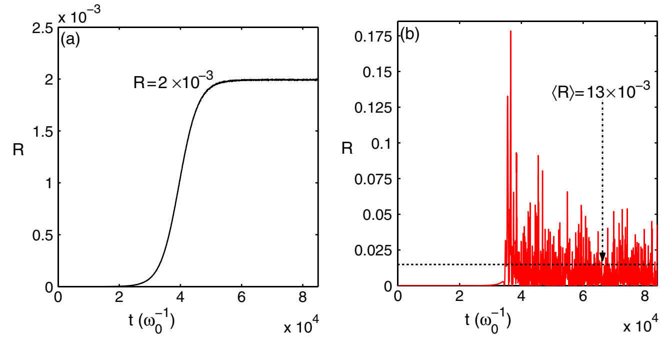

FIG. 2 (color online). 1D case. Reflectivities versus time for 1D simulations carried out in the time interval $T = 51$ ps ($8.5 \times 10^4 \omega_0^{-1}$) in the cases: (a) BSRS without LDI and without the self-focusing term (solid black line); the corresponding asymptotic reflectivity is $R = 2 \times 10^{-3}$; (b) BSRS with LDI and with the self-focusing term [solid red (gray) line]; the corresponding time averaged reflectivity is $\langle R \rangle = 13 \times 10^{-3}$ (dashed line).

the expression $\langle R \rangle = R_{\rm th} \exp(2G_{\rm Ros})$, $R_{\rm th}$ denoting the thermal level computed in Ref. [2]. We then found that the time averaged Raman reflectivity does not change significantly as compared with the full physics case ($\epsilon_L = \epsilon_S = 1$) if self-focusing is ignored ($\epsilon_S = 0$), as long as LDI is retained ($\epsilon_L = 1$). Finally, we also checked that self-focusing alone ($\epsilon_S = 1$ and $\epsilon_L = 0$) does not lead to any significant increase of the Raman reflectivity when compared with case (a). Thus, we have established the main result of our Letter: LDI is able to increase the Raman reflectivity in an inhomogeneous plasma as compared to the prediction $\langle R \rangle = R_{\rm th} \exp 2G_{\rm Ros}$ based on the Rosenbluth gain factor value. As said previously, we interpret this surprising result as being due to the fact that the LDI cascade gives rise to IAW modulations able to restore the absolute nature of the BSRS reflectivity (the maximum IAW amplitudes are observed to be in the order of $n_s/n_0 \sim 10^{-1}$).

In order to check this interpretation, we carried out extensive 1D simulations, solving the same system (1), with exactly the same parameters as in the 2D simulations. The corresponding 1D results are displayed in Fig. 2. It can be seen that the effect observed in 2D takes place similarly in 1D, namely, the averaged Raman reflectivity is increased by the factor 6.5 in case (b), when BSRS is coupled to LDI, as compared with case (a), when the coupling of the Langmuir wave to the IAW is suppressed.

Figures 3(a) and 3(b), respectively, display the snapshots of the Langmuir and ion acoustic waves at the instant $t = 21$ ps ($3.5 \times 10^4 \omega_0^{-1}$) corresponding to the reflectivity peak in Fig. 2(b). On these figures, the following features can be observed in the spatial domain $[550{:}700]c/\omega_0$ where the Langmuir wave was initially maximum: (i) the Langmuir wave is characterized by large amplitudes modulations; (ii) the IAW reaches amplitudes n_s/n_0 in the order of $|n_s/n_0| \approx 5 \times 10^{-2}$. These two features, together with their spatial correlation, are characteristic of LDI development. When looking at the same snapshots at later times, it can be seen that the spatial domain in which the two previous characteristic features are observed extends in space until it fills the whole plasma domain: then, the Langmuir and the ion acoustic waves both appear to be spatially incoherent in the entire plasma. Both waves are characterized by spatially uncorrelated high peaks, and the incoherence is manifest in the waves spectra. Figure 4 shows the snapshot (a) and the corresponding spatial spectrum (b) of the IAW at the time $t = 45$ ps ($7.5 \times 10^4 \omega_0^{-1}$); the IAW amplitude is in the order of $|n_s/n_0| \approx 10^{-1}$, which is similar to the value found in the 2D simulations. An asymmetry can also be observed in the IAW snapshot displayed in Fig. 4(a), namely, the well amplitudes are larger than the peak heights. Such an asymmetry is typical of the existence of so-called cavitons. The latter are predicted to result from the nonlinear evolution of the LDI cascade in the regime of Langmuir wave driven collapse [7,9]. The characteristic caviton size $l_{\rm cs}$ can be observed to be of the order of $1/k_s$, k_s denoting the wave number of the fundamental IAW generated by LDI. For our simulations

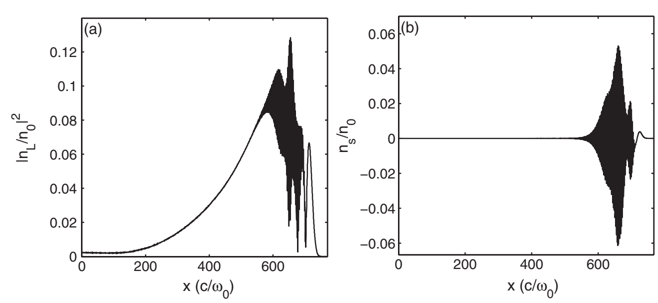

FIG. 3. 1D simulation of BSRS with LDI. Snapshots of the Langmuir wave (a) and the sound wave (b) at the time $t = 21$ ps ($3.5 \times 10^4 \omega_0^{-1}$). The plasma size is $L = 127\lambda_0(800c/\omega_0)$.

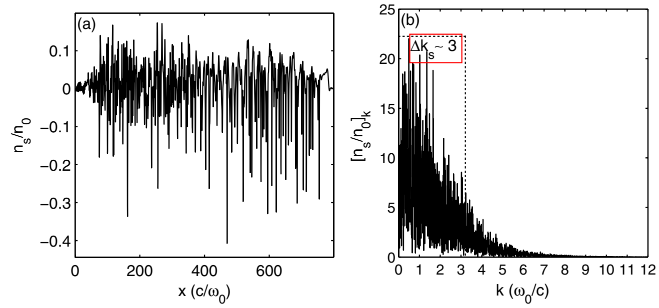

FIG. 4 (color online). 1D simulation of BSRS with LDI. Snapshot (a) and spectrum (b) of the sound wave at the time $t = 45$ ps ($7.5 \times 10^4 \omega_0^{-1}$). The plasma size is $L = 127\lambda_0 (800c/\omega_0)$.

parameters, k_s is given by $k_s = 3.1c/\omega_0$, and it can be seen on Fig. 4 that the characteristic width $\Delta k_s = 1/l_{\rm cs}$ of the IAW spectrum is indeed of the order of $k_s = 3.1c/\omega_0$. The destabilization of the absolute instabilities by incoherence in the case of an inhomogeneous plasma has been predicted in the past within the framework of the random phase approximation (RPA) theory [4]. The RPA theory is valid if the LW correlation length $l_{c_{\rm LW}}$ is shorter than the coherent gain length for BSRS, denoted as l_0. This condition can be written $\Delta k_{\rm LW} > 1/l_0$, where $\Delta k_{\rm LW}$ is the LW spectral width induced by the incoherent IAW generated by LDI, with $\Delta k_{\rm LW} \approx \frac{1}{4}(\frac{\omega_{\rm pe}}{v_{g,L}})^2 \langle |\frac{n_s}{n_0}|^2 \rangle l_{\rm cs}$. For our simulations parameters, taking $|n_s/n_0| \approx 0.1$, one obtains $\Delta k_{\rm LW}\lambda_0 \approx 0.6$ and $\lambda_0/l_0 \approx 0.4$, so that the RPA validity condition is satisfied. Within the RPA theory, the conditions for the absolute instabilities destabilization are the following inequalities: (i) $(c_s - v_{g,R})(c_s - v_{g,L}) < 0$ and $v_{g,R}v_{g,L} < 0$, which are trivially satisfied in the case of BSRS (here $v_{g,L}$ denotes the group velocity of the BSRS-generated Langmuir wave), (ii) $G_{\rm Ros} > \pi/2$, which is one of the conditions defining our study domain, and (iii) $\Delta k_{\rm LW} > 1/l_0\sqrt{\pi/G_{\rm Ros}}$, which is slightly more severe than the RPA validity conditions $\Delta k_{\rm LW} > 1/l_0$ in the subdomain $\pi/2 \leq G_{\rm Ros} \leq \pi$, and which is automatically fulfilled in the complementary subdomain $G_{\rm Ros} > \pi$. All these conditions are satisfied in the case of our simulations, so that our simulations results can be described within the RPA theory. In order to test our interpretation of the IAWs generated by LDI being able to restore the absolute nature of the BSRS instability, we carried out a 1D simulation over a very long period of time ($300\,000\omega_0^{-1}$): during the first $150\,000\omega_0^{-1}$, the time average intensities of the fluctuations source terms were kept at their thermal level, whereas for the second part of the simulation they were set at zero. We did not observe any modification of the time average reflectivity past the time $t = 150\,000\omega_0^{-1}$. This result demonstrates unambiguously that once the LDI cascade is established, BSRS is in the regime of absolute instability, so that the average reflectivity is only determined by the LDI nonlinear saturation mechanism. Finally, we observed that for long enough plasmas, the reflectivity of an inhomogeneous plasma, although larger than the usual prediction [2] $\langle R \rangle = R_{\rm th} \exp(2G_{\rm Ros})$, remains smaller than the reflectivity of a homogenous plasma of same length and characterized by its mean density.

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