Influence of Pairing on the Nuclear Matrix Elements of the Neutrinoless $\beta\beta$ Decays

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We study in this Letter the neutrinoless double beta decay nuclear matrix elements (NME's) in the framework of the interacting shell model. We analyze them in terms of the total angular momentum of the decaying neutron pair and as a function of the seniority truncations in the nuclear wave functions. This point of view turns out to be very adequate to gauge the accuracy of the NME's predicted by different nuclear models. In addition, it gives back the protagonist role in this process to the pairing interaction, the one which is responsible for the very existence of double beta decay emitters. We show that low seniority approximations, comparable to those implicit in the quasiparticle RPA in a spherical basis, tend to overestimate the NME's in several decays.

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The discovery of the massive character of the neutrinos in the recent measurement at Super-Kamiokande [1], SNO [2], and KamLAND [3], has opened a new era in the neutrino physics. However, these experiments are sensitive only to the mass differences between the three neutrino species; their absolute mass scale and hierarchy are still unknown. In addition, we do not know either if the neutrinos are Dirac or Majorana particles. The double beta decay is the rarest nuclear weak process. It takes place between two even-even isobars, when the decay to the intermediate nucleus is energetically forbidden due to the pairing interaction, which shifts the even-even and the odd-odd mass parabolas in a given isobaric chain. The two-neutrino decay is just a second order process in the weak interaction. It conserves the lepton number and has already been observed in several nuclei. A second mode, the neutrinoless decay $0\nu\beta\beta$, can only take place if the neutrino is a Majorana particle and demands an extension of the standard model of electroweak interactions because it violates the lepton number conservation. Therefore, the observation of the double beta decay without emission of neutrinos will sign the Majorana character of the neutrino, and moreover it will establish the absolute mass scale of the neutrinos, hence deciding their mass hierarchy.

The expression for the neutrinoless double beta decay half-life, in the $0^+ \to 0^+$ case, can be brought to the following form [4,5]:

$$[T_{1/2}^{(0\nu)}(0^+ -> 0^+)]^{-1} = G_{0\nu}\left[M^{(0\nu)}\left(\frac{\langle m_\nu\rangle}{m_e}\right)\right]^2 \qquad (1)$$

$$M^{(0\nu)} = M_{\rm GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)} - M_T^{(0\nu)} \qquad (2)$$

where $\langle m_\nu\rangle$ is the effective neutrino mass (a linear combination of the neutrino mass eigenvalues whose coefficients are the corresponding elements of the neutrino mixing matrix), and $G_{0\nu}$ is the kinematic phase space factor [6].

The important point at this stage is that, once the neutrinoless double beta decay be detected, to transform the measured half-life in an accurate value of the effective neutrino mass would require a precise computation of the nuclear matrix elements (NME's) of the decay operators. This, in turn, demands a detailed description of the structure of the nuclei involved in the process. A critical analysis of the available predictions for the NME's of the potential $0\nu\beta\beta$ emitters (only about one dozen) was made recently by Bahcall *et al.* [7]. Their conclusion was rather pessimistic, owing to the large dispersion of the calculated values. In a subsequent paper, Rodin *et al.* [8] have shown that many of the quasiparticle RPA (QRPA) calculations taken into account in Bahcall's survey were obsolete, and that, when these are not considered, the spread of the calculated values is much smaller. The aim of this work is to go one step further and to propose a much narrower band of values for the NME's, based in the predictions of large scale applications of the Interacting Shell Model (ISM) and in the analysis of the QRPA results in terms of the pairing content of their solutions.

The matrix elements $M_{{\rm GT},F,T}^{(0\nu)}$ can be calculated in the closure approximation, that is good to better than 90% due to the large average energy of the virtual neutrino (~100 MeV) [9]. For the Gamow-Teller channel, it reads,

$$M_{\rm GT}^{(0\nu)} = \langle 0_f^+|h(|\vec{r}_1 - \vec{r}_2|)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(t_1^- t_2^-)|0_i^+\rangle, \qquad (3)$$

and similar expressions hold for the other matrix elements. $h(|\vec{r}_1 - \vec{r}_2|)$ is the neutrino potential ($\sim 1/r$) obtained from the neutrino propagator. Higher order contributions (HOC) to the nuclear current produce the tensor term and add extra contributions to the Gamow-Teller expression of Eq. (3) [10].

Generically, the two body decay operators can be written in the Fock space representation as

$$\hat{M}^{(0\nu)} = \sum_J \bigg(\sum_{i,j,k,l} M^J_{i,j,k,l} [(a_i^\dagger a_j^\dagger)^J (a_k a_l)^J]^0 \bigg), \qquad (4)$$

where the indices i, j, k, l run over the single particle orbits of the spherical nuclear mean field. Applying the techniques of Ref. [11], we can factorize the operators as follows:

$$\hat{M}^{(0\nu)} = \sum_{J^\pi} \hat{P}^\dagger_{J^\pi} \hat{P}_{J^\pi}. \qquad (5)$$

The operators $\hat{P}_{J^\pi}$ annihilate pairs of neutrons coupled to J^π in the father nucleus, and the operators $\hat{P}^\dagger_{J^\pi}$ substitute them by pairs of protons coupled to the same J^π. The overlap of the resulting state with the ground state of the granddaughter nucleus gives the J^π-contribution to the NME. The—*a priori* complicated—internal structure of these exchanged pairs is dictated by the double beta decay operators.

The ISM calculations reported in this Letter are carried out in the spirit of the previous shell model works [12–15]. For the $A = 76$ and $A = 82$ cases, we make full calculations in the valence space ($1p_{3/2}$, $0f_{5/2}$, $1p_{1/2}$, $0g_{9/2}$) using a newly built effective interaction that, starting with a G matrix [16], has its matrix elements fitted to a large set of experimental data. For the $A = 124$, $A = 128$, $A = 130$, and $A = 136$ emitters, we make full calculations in the valence space ($0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, $0h_{11/2}$) with another interaction obtained in a similar manner. These model spaces and interactions will be discussed in detail elsewhere [17]. The dimensions of the shell model bases reach in some cases $O(10^{10})$. The present calculations adopt the closure approximation and model the short range and finite size corrections as in [13]. We use $r_0 = 1.2$ fm to make the matrix elements dimensionless and $g_A = 1.25$. The choice of $g_A = 1.25$ instead of the quenched value $g_A = 1.0$ needed for the pure Gamow-Teller processes in nuclei is consistent with the use of the closure approximation, in which the multipole decomposition of the decay plays no role at all. In a calculation without closure, the use of the quenched g_A can be justified only in the $J = 1^+$ channel, which is not the dominant one in the 0ν decay. Hence, its effects in the NME's are bound to be small and, depending on the phase of its contribution relative to those of the other multipolarities, this will either increase or decrease the NME's. Furthermore, even for this particular channel, the reasons to choose a quenched g_A are not compelling because the $J = 1^+$ operator of the 0ν decay may not resemble the pure Gamow-Teller operator of the 2ν decay. Higher order contributions to the nuclear current (HOC) [10] are explicitly included for the first time in the ISM context, leading to reductions of the NME's in the 15% range. Our final predictions for $M^{(0\nu)}$ are gathered in Table I.

Except for doubly magic ^{48}Ca, whose NME is severely quenched, our predictions cluster around a value $M^{(0\nu)} \approx$ 2. The upper bounds on the neutrino mass for a half-life of 10^{25} y, that incorporate the phase space factors, show a mild preference for the potential emitters with $A = 82$, 124, 130, and 136. The matrix elements are dominated by the Gamow-Teller contribution. The influences of the restrictions in the valence space and of the choice of the effective interaction in the ISM NME's have been studied in [18]. In all, these should result in a 20% uncertainty of our predictions. Treating the short range correlations with a prescription softer than Jastrow might produce an increase of NME's that we have not evaluated yet in the ISM context, but we do not expect it to go beyond 10–15%.

TABLE I. ISM predictions for the 0ν double beta decay matrix elements, with and without higher order contributions to the nuclear current (HOC). The effective neutrino mass corresponds to $T_{(1/2)} = 10^{25}$ y.

	$M^{(0\nu)}$ (no HOC)	$M^{(0\nu)}$	$\langle m_\nu \rangle$
^{48}Ca → ^{48}Ti	0.76	0.59	1.07
^{76}Ge → ^{76}Se	2.58	2.22	0.91
^{82}Se → ^{82}Kr	2.49	2.11	0.46
^{124}Sn → ^{124}Te	2.38	2.02	0.48
^{128}Te → ^{128}Xe	2.67	2.26	1.68
^{130}Te → ^{130}Xe	2.41	2.04	0.37
^{136}Xe → ^{136}Ba	2.00	1.70	0.47

In order to explore the structure of the $0\nu\beta\beta$ two body transition operators, we have plotted in Fig. 1 the contributions to the 0ν GT matrix element as a function of the J^π of the decaying pair.

The results are very suggestive because the dominant contribution corresponds to the decay of $J = 0$ pairs, whereas the contributions of the pairs with $J > 0$ are either negligible or have opposite sign to the leading one. This behavior is common to all the cases that we have studied, as can be seen in Table II. Notice that the cancellations are substantial. These features are also present in the QRPA

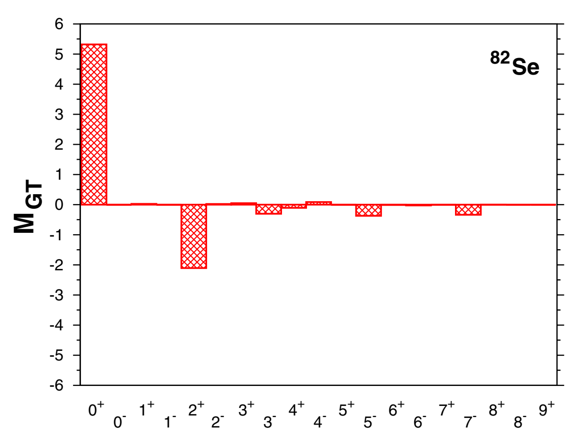

FIG. 1 (color online). Contributions to the Gamow-Teller matrix element of the ^{82}Se → ^{82}Kr decay as a function of the J^π of the transformed pair (no HOC).

TABLE II. $J = 0$ *vs* $J > 0$ pair contributions to the Gamow-Teller matrix element (no HOC).

	$M^{(0\nu)}_{\rm GT}$	$M^{(0\nu)}_{\rm GT}(J=0)$	$M^{(0\nu)}_{\rm GT}(J>0)$
^{48}Ca $\to$ ^{48}Ti	0.67	3.16	−2.49
^{76}Ge $\to$ ^{76}Se	2.35	5.59	−3.24
^{82}Se $\to$ ^{82}Kr	2.25	5.32	−3.07
^{130}Te $\to$ ^{130}Xe	2.12	6.58	−4.46
^{136}Xe $\to$ ^{136}Ba	1.77	5.72	−3.95

calculations, in whose context they had been discussed in Refs. [8,19].

To better grasp this mechanism, we have expressed the matrix elements in a basis of generalized seniority s (s counts the number of unpaired nucleons in the nucleus); $|0_i^+\rangle = \sum_s \alpha_s |s\rangle_i$; $|0_f^+\rangle = \sum_s \beta_s |s\rangle_f$. The $J = 0$ terms provide essentially all the contribution to $M^{(0\nu)}$ that is diagonal in s. The canceling parts, $J > 0$, produce almost exclusively cross terms with $\Delta s = +4$. The matrix elements $_f\langle s|\hat{M}^{(0\nu)}|s\rangle_i$ are roughly proportional to $(s_{\rm max} - s)$, averaged in parent and granddaughter, while the cross terms $_f\langle s+4|\hat{M}^{(0\nu)}|s\rangle_i$ are roughly constant—in both cases scaled by the largest oscillator quantum number in the corresponding valence space. The diagonal two body matrix elements of the operator $\hat{M}^{(0\nu)}(J=0)$ are similar to those of the isovector pairing of the realistic nuclear effective interactions; that is why it acts as a "pair counter." At present, we cannot offer a similarly simple explanation for the behavior of the $J > 0$ terms. Obviously, when the initial and final states have seniority zero, the $J = 0$ contribution is maximized, and the canceling terms are null; hence, $M^{(0\nu)}$ becomes maximal.

These results highlight the role of the seniority structure of the nuclear wave functions in the buildup of the 0ν NME's, and we shall examine this issue for the competing theoretical approaches. In the first place, we have plotted the results of the ISM calculations of the NME's as a function of the seniority in Fig. 2. The values with maximum seniority provide the exact ISM results in the corresponding valence spaces. Two aspects are worth underlining: (a) the strong reduction of the NME as the maximum allowed seniority increases (up to a factor five); and (b) the fact that, at $s \leq 4$, the NME's of the $A = 76$, 82, 128, and 130 decays miss convergence by factors 2–3. On the contrary, in the $A = 48$, $A = 124$, and 136 cases, the convergence at $s \leq 4$ is much better. The reason why these decays behave differently is very illuminating; ^{124}Sn has only neutrons in the valence space; hence, its wave function is dominated by low seniority components, and its NME at $s \leq 4$ is quite close to the exact result; in the $A = 136$ decay, the $s \leq 4$ calculation for ^{136}Xe is exact; therefore, at $s \leq 4$, the NME is also close to the exact one; finally, in the $A = 48$ decay, the $s > 4$ components are negligible both in doubly magic ^{48}Ca and in ^{48}Ti.

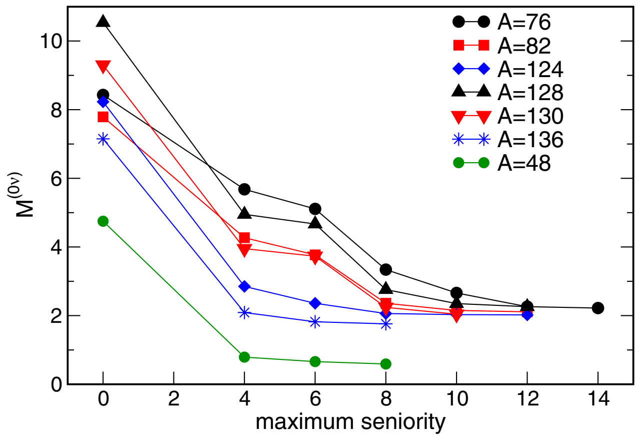

FIG. 2 (color online). The neutrinoless double beta decay NME's, defined in Eq. (2), as a function of the maximum seniority of the wave functions.

We can now proceed to compare in detail the "state of the art" ISM and QRPA [20,21] NME's in Fig. 3. The QRPA results for ^{124}Sn are not yet available. The range of QRPA values shown in the figure is that given by the authors, and derives from the different choices of g_{pp} and g_A, as well as from their use or not of a renormalized version of the QRPA. The larger values correspond to $g_A = 1.25$ and should therefore be preferred in the comparison with our predictions. Both the QRPA and the ISM calculations include the higher order corrections from Ref. [10]. For a proper comparison, the TU07 NME's have been increased by 10% due to their different choice of r_0. In all the calculations, the short range correlations are modeled by the same Jastrow factor.

Several interesting conclusions stem from this figure. First, the fact that the different QRPA calculations are now compatible. In addition, they produce NME's that are strikingly close to the ISM ones calculated at the truncation

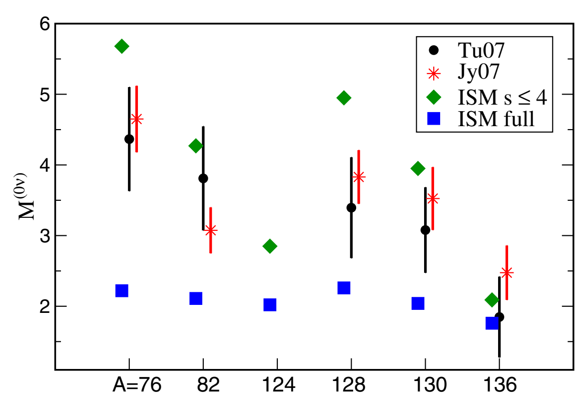

FIG. 3 (color online). The neutrinoless double beta decay NME's; comparison of ISM and QRPA calculations. Tu07; QRPA results from Ref. [20]. Jy07; QRPA results from Ref. [21]. ISM $s \leq 4$ and ISM; present work. The ISM results have uncertainties in the 20% range (see text).

level $s \leq 4$. In the $A = 136$ decay, in which the $s \leq 4$ truncation is a good approximation to the full result, the QRPA values and the ISM ones do agree (this seems to be also the case for the $A = 124$ decay [22]). This suggests that, somehow, $s \leq 4$ is the implicit truncation level of the QRPA. In the QRPA calculations in a spherical basis that we are discussing, the ground states of parent and granddaughter, calculated in the BCS approximation, have generalized seniority zero. The RPA ground state correlations of multipole character (quadrupole, octupole, etc.), bring components with $s \geq 4$ into these wave functions. But, for the RPA approximation to remain valid, their amplitudes should decrease with s. Indeed, in our ISM $s \leq 4$ results, the percentage of $s = 0$ components is always larger than 70%, a figure compatible with a QRPA description. However, in the full calculation for the $A = 76$, $A = 82$, $A = 128$, and $A = 130$ decays, this percentage can be as low as 25% (actually, in ^{76}Se, the $s = 4$ components almost double the percentage of the $s = 0$ ones). In these cases, the QRPA is bound to overestimate the amount of $s = 0$ components and, consequently, the value of the NME's. In a sense, the QRPA can be said to be a "low seniority approximation," roughly equivalent to the $s \leq 4$ ISM truncations, that overestimate the NME's when the nuclei that participate in the decay are strongly correlated by the multipole part of the effective nuclear interaction. The extent of the overestimation depends on the degree of validity of the low seniority approximation in each decaying pair.

The values of $M^{(0\nu)}$ predicted by the present ISM calculation for the $A = 76$, $A = 82$, $A = 128$, and $A = 130$ decays are smaller than the QRPA (central) ones by factors 1.5–2. Therefore, for a given value of the effective neutrino mass, the predicted ISM half-lives of the $0\nu\beta\beta$ decays are 2–4 times longer than the QRPA ones. Equivalently, for a given lower bound on the half-life, the ISM NME's produce upper bounds on the effective neutrino mass that are larger than those of the QRPA by factors 1.5–2. For instance, a bound on $T_{(1/2)}(^{76}\mathrm{Ge} \rightarrow {}^{76}\mathrm{Se})$ of 10^{25} y results in an effective neutrino mass of 910 meV with the ISM NME, and 430 meV with the QRPA one. The same bound for the half-life of the $^{130}\mathrm{Te} \rightarrow {}^{130}\mathrm{Xe}$ decay would lead to bounds on the neutrino mass of 370 and 245 meV, respectively.

In summary, we have analyzed the $0\nu\beta\beta$ NME's in terms of the J^{π} of the decaying neutron pair. We have found that in the seniority zero limit, the decays are strongly favored. When the nonzero seniority components of the wave functions, originated by the multipole terms of the nuclear effective interaction, are properly taken into account, the matrix elements are drastically reduced. In particular, when the multipole correlations are large, the low seniority truncations, $s \leq 4$, similar to those implicitly present in the spherical QRPA approaches based in a BCS treatment of the pairing interaction, are shown to overestimate the NME's. Hence, we surmise that, when the QRPA and ISM results do not agree, the true NME's should be much closer to the ISM predictions than to the QRPA ones.

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