Bimodality: A Sign of Critical Behavior in Nuclear Reactions

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The recently discovered coexistence of multifragmentation and residue production for the same total transverse energy of light charged particles, which has been dubbed bimodality like it has been introduced in the framework of equilibrium thermodynamics, can be well reproduced in numerical simulations of heavy ion reactions. A detailed analysis shows that fluctuations (introduced by elementary nucleon-nucleon collisions) determine which of the exit states is realized. Thus, we can identify bifurcation in heavy ion reactions as a critical phenomenon. Also the scaling of the coexistence region with beam energy is well reproduced in these results from the quantum molecular dynamics simulation program.

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Recently, the INDRA Collaboration has discovered [1] that, in collisions of heavy ions—Xe + Sn and Au + Au between 60 and 100 *A*MeV incident energy—in a small interval of the total transverse energy of light charged particles ($Z \leq 2$), $E_{\perp 12}$, a quantity which is usually considered as a good measure for the centrality of the reaction, two distinct reaction scenarios exist. In this $E_{\perp 12}$ interval, in the forward direction, i.e., quasiprojectile, either a heavy residue is formed which emits light charged particles only, or the system fragments into several intermediate mass fragments. This phenomenon has been named "bimodality." In addition, as shown in [1], the mean $E_{\perp 12}$ value of this transition interval scales with the projectile energy in the center of mass of the system for Au + Au reactions, between 60 and 150 *A*MeV.

This observation has created a lot of attention because a couple of years before the theory predicted [2,3] that in finite size systems, whose infinite counterparts show a first order transition, the system can—for a given temperature—be in either of the two phases if this temperature is close to that of the phase transition. Assuming that $E_{\perp 12}$ is a measure for the excitation energy, and acts as the control parameter of the system, it is tempting to identify the residue with the liquid phase of nuclear matter, and the creation of several medium or small size fragments with the gas phase. The experimental observation would then just be a realization of the theoretical prediction.

If this were the case, the long-standing problem of identifying the reaction mechanism which leads to multifragmentation would be solved. This problem arrived because many observables could be equally well described in thermodynamical or statistical theories [4,5] as in dynamical models [6,7], although the underlying reaction mechanism was quite different. The statistical models assume that at freeze-out, when the system is well below normal nuclear matter density, the fragment distribution is determined by phase space.

In dynamical models, on the contrary, fragments are surviving initial state correlations which have not been destroyed during the reaction, and equilibrium is not established during the reaction. A detailed discussion of how the reaction proceeds in these models can be found in [8].

To quantify the bimodality, one may define as in Ref. [1]

$$a_2 = (Z_{\max} - Z_{\max-1})/(Z_{\max} + Z_{\max-1}), \qquad (1)$$

where $Z_{\max}$ is the charge of the largest fragment, while $Z_{\max-1}$ is the charge of the second largest fragment, both observed in the same event in the forward hemisphere—at polar angles $\theta_{\mathrm{cm}} < 90°$—in the center of mass of the system. If the system shows bimodality, we will observe a sudden transition from small to large a_2 values. In this narrow transition region, we expect two types of events: One with a large a_2 (one big projectile residue with some very light fragments), the other with a small a_2 (two or more similarly sized fragments). Events with intermediate values of a_2 should be rare.

In order to verify whether bimodality is a "smoking gun" signal for a first order phase transition in a finite system, we have performed numerical simulations with one of the dynamical models which has frequently been used to interpret the multifragmentation observables, the quantum molecular dynamics (QMD) approach [6,8]. This approach simulates the entire heavy ion reaction, from the initial separation of projectile and target up to the final state, composed of fragments and single nucleons. Here, nucleons interact by mutual density dependent two body interactions and by collisions. The two body interaction is a parametrization of the Brückner *G* matrix supplemented by an effective Coulomb interaction. For this work, we have used a soft equation of state. The initial positions and momenta of the nucleons are randomly chosen, and respect the measured rms radius of the nuclei. Collisions take place if two nucleons come closer than $r = \sqrt{\sigma/\pi}$, where σ is the energy dependent cross section for the corresponding channel (pp or pn). The scattering angle is chosen randomly, respecting the experimentally measured $d\sigma/d\Omega$. Collisions are Pauli blocked. For details we refer to Refs. [6,7]. For the later discussion it is of importance

that, even for a given impact parameter, two simulations are not identical, because the initial positions and momenta of the nucleons as well as the scattering angles are randomly chosen.

Figure 1 shows the INDRA experimental results (left-hand panels) in comparison with those of QMD calculations (right-hand panels). The calculations have been acceptance corrected. Qualitatively, we see the same behavior also in the unfiltered simulations. In the top row, we present the most probable value of a_2 in the quasiprojectile as a function of $E_{\perp 12}/(E_{\mathrm{c.m.}}/A)$ in the quasitarget for different beam energies, where $E_{\mathrm{c.m.}}/A$ is the center of mass energy per nucleon of the colliding system. Accordingly with [1], the two observables have

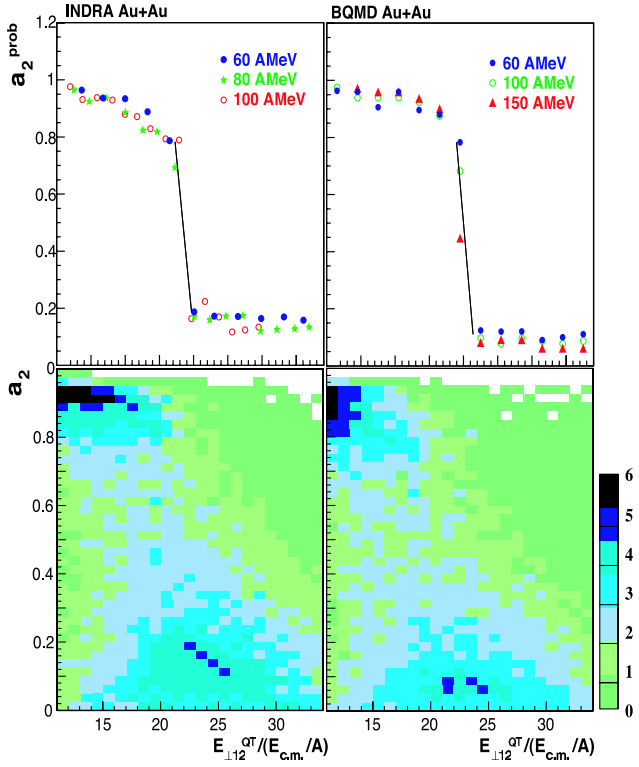

FIG. 1 (color online). Top: Most probable value of a_2 in the quasiprojectile angular range $\theta_{\mathrm{c.m.}} < 90°$ (a_2^{QP}) as a function of $E_{\perp 12}^{\mathrm{QT}}$, the total transverse energy $E_{\perp 12}$ in the quasitarget angular range $\theta_{\mathrm{c.m.}} \geq 90°$, scaled by $E_{\mathrm{c.m.}}$, the energy per nucleon of the system in the center of mass. We display INDRA experimental results (left-hand panels) extracted from [1] and QMD simulations (right-hand panels) for the Au + Au collisions at three different bombarding energies. Bottom: Differential reaction cross section (linear color scale, arbitrary unit) of a_2^{QP} as a function of $E_{\perp 12}^{\mathrm{QT}}$, in the transition region, for Au + Au at 100 *A*MeV bombarding energy. We show the INDRA experimental data and the filtered QMD simulations (left-hand and right-hand panels, respectively). As in [1], for calculating a_2, in both experimental and QMD results, it is required that at least 80% of the total charge of the projectile is detected by the INDRA setup in the forward hemisphere.

been determined in distinct angular ranges in order to minimize possible correlations between the total transverse kinetic energy of light particles and the size of the two biggest fragments inside the same spectator (quasiprojectile or quasitarget). We observe that in the experiment as in the calculation, the sudden transition between large and small a_2 values scales with $E_{\perp 12}^{\mathrm{QT}}/(E_{\mathrm{c.m.}}/A)$. Even the numerical value of this transition agrees between experiment and theory. In order to see whether this phenomenon survives at higher incident energies, the simulations have been extended up to 150 *A*MeV bombarding energy. We observe that it is the case.

The bottom row shows the transition interval in detail. Here, we display the differential reaction cross section of a_2^{QP} as a function of $E_{\perp 12}^{\mathrm{QT}}$ for Au + Au at 100 *A*MeV. From the experimental data, we observe that there is no smooth transition between the two event classes. In the simulations as well as in the experiment we see two maxima for a_2^{QP}, separated by a minimum of the distribution. QMD simulations reproduce the experimental findings qualitatively and quantitatively.

In QMD simulations, the system does not even come to a local thermal equilibrium. It is therefore necessary to explore the origin of the observed dependence of a_2 as a function of $E_{\perp 12}$. The first step toward this goal is to identify when, in the course of the reaction, the fragment pattern is determined. This is all but trivial. Fragments can easily be identified at the end of the heavy ion reaction, when they are clearly separated in coordinate space by a minimum spanning tree procedure in coordinate space. At earlier times, however, they overlap in coordinate space and, consequently, another method has to be employed. It has been proposed by Dorso and Randrup [9], and later verified in QMD simulations [10], that an early identification of fragments is possible if one uses in addition the momentum space information: If one identifies at each time step during the simulation the most bound configuration, one can establish that the fragment pattern changes only little during the time, and that the early identified fragments are the prefragments of the finally observed fragments. The most bound configuration in a simulation with N fragments is that in which

$$E_{\mathrm{bind}} = \sum_{i=1}^{N} E_i$$

is minimal. E_i is the binding energy of the fragment i which contains $m(i)$ nucleons and is given by

$$E_i = \frac{1}{2m} \sum_{k=1}^{m(i)} (p_k - \langle p_i \rangle)^2 + \sum_{k<l}^{m(i)} V_{kl},$$

where $\langle p_i \rangle$ is the average momentum of the nucleons entrained in the fragment i. Please note that E_{bind} does not contain the interaction among fragments. Therefore, its numerical value can vary, although the total energy is

conserved in the simulations. The most bound configuration has to be determined by a simulated annealing procedure [10]. With this procedure, the fragment multiplicity of a given event can already be determined when the system has passed the highest density and is starting to expand. Later, the prefragments may still emit some nucleons, but the nucleons which are entrained at the end in the fragment are part of this prefragment. These methods allow us to trace back at which time point in the reaction it is determined whether the event has a large or a small final a_2 value. Figure 2 shows QMD results for Au + Au at 80, 100, and 150 AMeV bombarding energy. We see that, whatever the incident energy, the two event classes are already formed shortly after the system has passed the highest compression stage. There we observe the highest rate of hard nucleon-nucleon collisions. These collisions transport nucleons in unoccupied regions of momentum space leaving behind holes in momentum space which create in time (due to the different trajectories) holes in coordinate space. These holes weaken the binding of projectile and target matter, leading to a fragment formation still at rather high density. Within statistical models [4,11], droplets are created with a spherical shape and have a normal density. This is possible only at a density of the system of $\rho \leq \rho_0/3$. The observation of an early creation of cluster partitions, above the critical density, has been found too in [12] with lattice-gas calculations where droplets are also defined according to energy considerations. The same conclusion has been obtained in [13] from classical molecular dynamics calculations where particles interact through Lennard-Jones plus Coulomb potentials.

Consequently, bimodality in QMD has nothing to do with the final state interaction, or with how the neck between projectile and target residue finally breaks. Whether we find a multifragmentation or a heavy residue event is determined when projectile and target nucleons still overlap almost completely in coordinate space [8]. One may conjecture that, due to the random character of the scattering angle, events with the same $E_{\perp 12}$ decelerate differently, and, therefore, a different behavior of the average momenta may be at the origin of the different a_2 values. For this purpose, we study with Au + Au at 150 AMeV incident energy, at 60 fm/c, when a_2 is decided, the dependence with the final a_2 of the average momentum of all target nucleons which are at the end entrained in $A > 4$ fragments. In Fig. 3, we display their longitudinal and transverse momentum as a function of a_2. Clearly, both average momentum are almost independent of the final a_2. The fluctuations of the momenta around the mean values are by far larger than the difference between the mean values for different a_2 values. This excludes mean deceleration of the simulation events as reason for the different reaction scenarios.

Obviously fluctuations around the mean values are at the origin of the different event classes. This phenomenon is known from nonlinear theory [14] and is called "bifurcation." We see here, in a system with a very limited particle number, a nonlinear behavior. Can we understand where it comes from? In order to answer this question, we go back

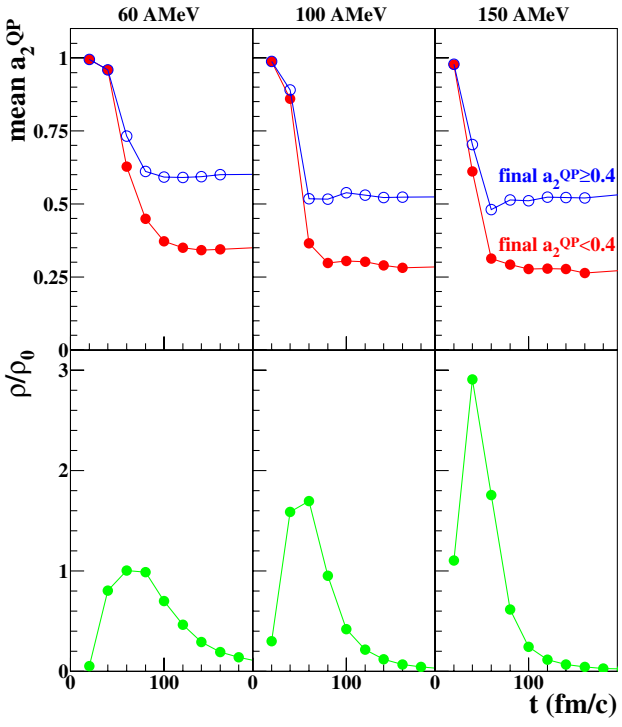

FIG. 2 (color online). Results of QMD simulations of Au + Au at 80, 100, and 150 AMeV incident energies (left-hand, middle, and right-hand panels, respectively). Top: a_2^{QP} as a function of time for the two different event classes: final (at 200 fm/c) $a_2^{QP} < 0.4$ corresponding to multifragmentation events and final $a_2^{QP} \geq 0.4$ corresponding to a projectile residue. Bottom: Central density of the system as a function of time.

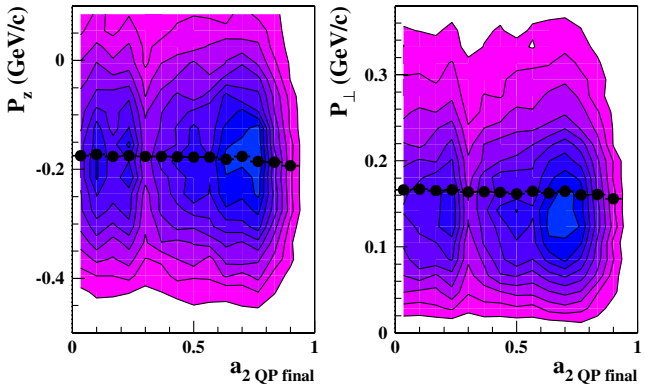

FIG. 3 (color online). QMD simulations of Au + Au at 150 AMeV incident energy, at 60 fm/c: differential cross section (colored contour levels, linear scaled) of the longitudinal (left) and transverse momentum (right) of all nucleons which are finally entrained in a fragment of size $A > 4$ as a function of final (at 200 fm/c) a_2^{QP}. The symbols represent the mean values of momentum.

to the search of the most bound configuration. For $E_{\perp 12}$ values below the transition (and hence large a_2 values), the most bound configuration is a large residue. Above the transition (where a_2 is small), several small fragments give a more bound configuration. The sum of the internal kinetic energies of the clusters

$$E_{\mathrm{mult}} = \frac{1}{2m} \sum_{i=1}^{N} \sum_{k=1}^{m(i)} (p_k - \langle p_i \rangle)^2$$

is there smaller than

$$E_{\mathrm{res}} = \frac{1}{2m} \sum_{k=1}^{m(i)+m(2)+\cdots+m(N)} (p_k - \langle p \rangle)^2,$$

the internal kinetic energy for a residue configuration, and compensates the increase of the attractive potential energy

$$V_{\mathrm{res}} - V_{\mathrm{mult}} = \frac{1}{2}\left(\sum_{k,l=1}^{m(1)+m(2)+\cdots+m(N)} V_{kl} - \sum_{i=1}^{N} \sum_{k,l=1}^{m(i)} V_{kl} \right).$$

In the transition region, we see that $E_{\mathrm{mult}} + V_{\mathrm{mult}} \approx E_{\mathrm{res}} + V_{\mathrm{res}}$. In some events, both configurations differ by 100 keV only. Therefore it may happen that the scattering angle of one single nucleon-nucleon collision, which is, see above, randomly chosen, determines the type of the most bound configuration.

It is interesting to see the differences and similarities of the origin of bifurcation in a statistical model as compared to the analysis of QMD events. In both cases, the energy is the essential quantity. In the statistical model, there is, for a given number of nucleons in a given volume, a small range of total energies for which the number of microstates with one residue is of the same order of magnitude as the number of microstates with many fragments. (In order to count the microstates, it is assumed that the fragments are in one if their eigenstates, sometimes parametrized by a level density formula.) In this energy range, bimodality appears as a global property of the systems which is dependent on the total energy of all nucleons present in the reaction. In QMD events, the essential quantity is the total binding energy of the nucleons bound in medium size or large clusters. As explained above, in the transition region, this energy is almost identical for a multifragment and for a residue configuration. Therefore, both configurations appear, and we see bimodality. The fragments are not in the ground state, their nucleons are not isotropic neither in coordinate space nor in momentum space. Thus, bimodality is a local quantity in QMD simulations, depending only on the total binding energy of a subset of the nucleons. Therefore, in QMD, bimodality makes no reference to a statistical or thermal equilibrium, neither of the system nor of the population of the excited states of the fragments.

In summary, we have shown that the experimentally observed bimodality, the sudden transition between a residue and a multifragment exit states, and the existence of a small interval in $E_{\perp 12}$ in which both channels are coexistent, is in quantitative agreement with the result of QMD simulations. Even the scaling of this transition region with the center of mass energy of the system is well reproduced. From a detailed investigation of the reaction mechanism in QMD, we have seen that bimodality has properties observed in nonlinear systems: The system shows bifurcation as a function of the control parameter $E_{\perp 12}$. Fluctuations around the mean value in the longitudinal and transverse momentum decide which exit channel the simulation will take.

Being reproduced in statistical as well as in dynamical models, bimodality reflects the same ambiguity already observed for other observables [15].

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