Observation of Photon-Assisted Tunneling in Optical Lattices

C. Sias, H. Lignier, Y. P. Singh, A. Zenesini, D. Ciampini,* O. Morsch, and E. Arimondo*

CNR-INFM, Dipartimento di Fisica "E. Fermi," Università di Pisa, Largo Pontecorvo 3, 56127 Pisa, Italy

We have observed tunneling suppression and photon-assisted tunneling of Bose-Einstein condensates in an optical lattice subjected to a constant force plus a sinusoidal shaking. For a sufficiently large constant force, the ground energy levels of the lattice are shifted out of resonance and tunneling is suppressed; when the shaking is switched on, the levels are coupled by low-frequency photons and tunneling resumes. Our results agree well with theoretical predictions and demonstrate the usefulness of optical lattices for studying solid-state phenomena.

PACS numbers: 03.75.Lm, 03.65.Xp

A number of experiments in recent years have shown that Bose-Einstein condensates (BECs) [1] loaded into optical lattices are well suited to simulating solid-state systems [2,3]. Optical lattices are created by crossing two or more laser beams, and the resulting periodic potential landscapes (arising from the ac Stark shift exerted on the condensate atoms) are intrinsically defect-free, their lattice wells have controllable depths, and it is possible to move or accelerate the entire structure. This flexibility has made it possible to study dynamical effects such as Bloch oscillations [4] and resonant tunneling [5] as well as ground-state quantum properties such as the Mott-insulator transition [6]. More recently, the coherent suppression of interwell tunneling by strong driving of the lattice has been demonstrated [7]. In this Letter, we explore an effect [8] that is analogous to photon-assisted tunneling in solids and arises from the interplay between static acceleration and strong driving of the lattice. We observe two regimes, a linear and a nonlinear one, with different dependencies of the observed tunneling on the theoretically predicted behavior.

Photon-assisted tunneling occurs when adjacent potential wells whose ground states are tuned out of resonance by a static potential are coupled by photons (see Fig. 1). The static force leads to a suppression of resonant tunneling between the ground states. This suppression and the related Wannier-Stark localization of the wave function have been intensively discussed in the theoretical literature [9,10]. In this work, we report a direct measurement of this suppression based on the spatial tunneling of the condensate atoms. When photons of an appropriate frequency are present whose energy bridges the gap created by the static potential, tunneling is (partly) restored. In solid-state systems, the photons are typically in the microwave frequency range and the static potential is provided by an electric bias field applied to the structure. So far, photon-assisted tunneling has been observed in superconducting diodes [11], semiconductor superlattices [12–14], and quantum dots [15,16].

Our system consists of a BEC inside a one-dimensional optical lattice. The static potential is provided by a constant acceleration of the lattice, resulting in a constant force F in the lattice rest frame and hence in a potential difference $\Delta E = F d_L$ between adjacent wells a distance d_L apart. The role of the photons is played by a periodic shaking of the lattice at frequency ω that leads to the creation of sidebands around the carrier frequency of the laser beam. In the limit of sufficiently deep lattice wells and neglecting higher-lying energy levels, our system can be described by the Hamiltonian [8]

$$\hat{H}_0 = -J \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + \hat{c}_j^\dagger \hat{c}_i) + \frac{U}{2} \sum_j \hat{n}_j(\hat{n}_j - 1)$$
$$+ \Delta E \sum_j j \hat{n}_j + K \cos(\omega t) \sum_j j \hat{n}_j, \quad (1)$$

where $\hat{c}_i^{(\dagger)}$ are the boson creation and annihilation opera-
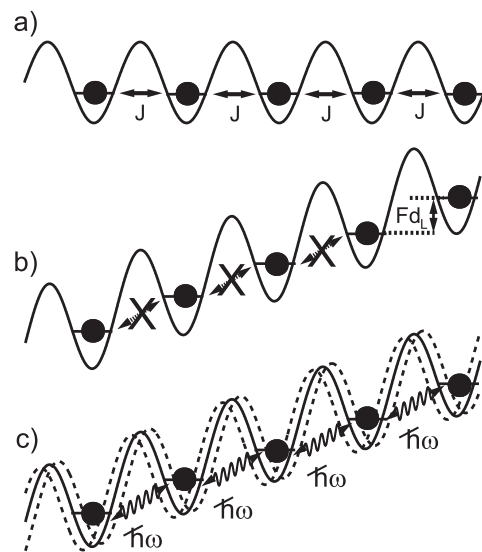

FIG. 1. Photon-assisted tunneling. (a) In an optical lattice at rest, the ground-state levels are resonantly coupled, leading to a tunneling energy J. (b) When a linear potential is applied, e.g., by accelerating the lattice, the levels are shifted out of resonance and tunneling is suppressed. (c) If the lattice is now periodically shaken at an appropriate frequency, the levels can again be coupled through photons of energy $\hbar \omega$ and tunneling is partially restored.

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tors on site i, $\hat{n}_i = \hat{c}_i^{\dagger}\hat{c}_i$ are the number operators, and K and ω are the strength and angular frequency of the shaking, respectively. The first line of this equation is the Bose-Hubbard model [17] with the tunneling matrix element J and the on-site interaction term U (in a BEC, the on-site interaction is due to atom-atom collisions and hence proportional to the s-wave scattering length and the density of the BEC). In the second line, the first term describes the constant potential, whereas the second term represents the sinusoidal shaking of the lattice. While for a sufficiently strong linear potential interwell tunneling is suppressed, leading to Wannier-Stark localization, recent theoretical work [8] predicts that the shaking term can partially restore it, leading to an effective tunneling rate

$$|J_{\text{eff}}(K_0)/J| = |\mathcal{J}_n(K_0)| \qquad (2)$$

when the resonance condition

$$n\hbar\omega = Fd_L \qquad (3)$$

is satisfied, where n is an integer denoting the order of the photon-assisted resonance, $\mathcal{J}_n$ is the nth order ordinary Bessel function, and $K_0 = K/\hbar\omega$ is the dimensionless parameter characterizing the shaking amplitude.

In our experiment we produced BECs of ^{87}Rb containing around 5×10^4 atoms in a crossed optical dipole trap. The two dipole traps were created by Gaussian laser beams at 1030 nm wavelength and a maximum power of around 1 W per beam focused to waists of 50 μm, and the frequencies of the resulting trapping potentials could be controlled independently. Subsequently, the BECs held in the dipole trap were loaded into an optical lattice created by two counterpropagating Gaussian laser beams ($\lambda = 852$ nm) with 120 μm waists by ramping up the power of the lattice beams in about 50 ms. The resulting periodic potential $V(x) = V_0 \sin^2(\pi x/d_L)$ had a lattice spacing $d_L = \lambda/2 = 426$ nm and its depth V_0 was measured in units of the recoil energy $E_{\text{rec}} = \hbar^2\pi^2/(2md_L^2)$, where m is the mass of the Rb atoms. By introducing a frequency difference $\Delta\nu$ between the two lattice beams (using acousto-optic modulators, which also control the power of the beams), the optical lattice could be moved at a velocity $v = d_L\Delta\nu$ or accelerated with an acceleration $a = d_L\frac{d\Delta\nu}{dt}$. In order to periodically shake the lattice, $\Delta\nu$ was sinusoidally varied with frequency ω and amplitude $\Delta\nu_{\text{max}}$ leading to a time-varying force (in the rest frame of the lattice)

$$F(t) = m\omega d_L \Delta\nu_{\text{max}} \cos(\omega t) = F_{\text{max}} \cos(\omega t). \qquad (4)$$

The dimensionless shaking parameter K_0 is then given by

$$K_0 = K/\hbar\omega = md_L^2\Delta\nu_{\text{max}}/\hbar = \pi^2\Delta\nu_{\text{max}}/2\omega_{\text{rec}}. \qquad (5)$$

Our method for measuring the effective tunneling parameter $|J_{\text{eff}}/J|$ (where the modulus indicates that this measurement is not sensitive to the sign of J_{eff}) is based on the free expansion of the BEC [7,18] confined only

radially but free to move along the direction of the lattice (the dipole trap frequency in that direction being on the order of a few Hz and hence negligible for our purposes). After condensation was reached in the crossed dipole trap and the optical lattice had been ramped up, the trapping beam perpendicular to the lattice direction was suddenly switched off. Subsequently, the *in situ* width of the BEC in the lattice direction was measured by flashing on a resonant beam and imaging the shadow cast by the BEC on a CCD camera. In the experiments with an accelerated and/or shaken lattice, $|J_{\text{eff}}/J|$ was determined by measuring the expansion for the same lattice depth both in the driven case and without the driving.

In a preliminary experiment, we studied the tunneling suppression caused by shifting adjacent ground states out of resonance through a constant force F acting on the BEC inside a lattice that was subjected to an acceleration a. Typical accelerations in our experiment were between 0 and 4 ms^{-2}, meaning that for an expansion time of 400 ms the lattice (and therefore the BEC) would have been displaced by up to 32 cm, 2 orders of magnitude more than the field of view of our imaging system. Also, a displacement of more than 100 μm along the lattice direction would have led to a restoring force of the longitudinal harmonic trap created by the radial dipole trap beam corresponding to an acceleration $a_{\text{restore}} > 0.1$ ms^{-2}. Therefore, in order to be able to achieve a high resolution in our measurements of the expansion rate (and hence J) of the condensate, implying a long expansion time, while keeping the displacement of the lattice below ≈ 100 μm, we used a rectangular acceleration profile that alternated between $+a$ and $-a$ and therefore "rocked" the lattice back and forth. In this way, the modulus of the resulting force and hence the energy shift between adjacent wells was constant, while the mean position of the lattice (and hence the BEC) remained close to the center of the dipole trap. In order to separate the frequency regimes of the rocking and the shaking motion, we chose a "rocking frequency" of 30 Hz, which was much smaller than the resonant frequencies for photon-assisted tunneling (typically around 150–400 Hz).

Figure 2(a) shows the results of our measurements of $|J_{\text{eff}}|$ in an accelerated (rocked) lattice. As expected, when the energy difference Fd_L between adjacent levels is increased, resonant tunneling is reduced and, for $Fd_L \approx J$, completely suppressed (as recently also observed for single-atom tunneling in a double-well structure [19]). In this limit, the energy levels in the individual wells can be viewed as Wannier-Stark levels. Our data are fitted very well by a Lorentzian, but to our knowledge there is no analytical prediction for such a dependence in the theoretical literature. In [10], an expression for the wave function in a tilted lattice as a function of the applied force is given, but an analytical calculation of $J_{\text{eff}}(F)$ has yet to be done.

Tunneling between the on-site levels shifted out of resonance by the static acceleration can be partially re-
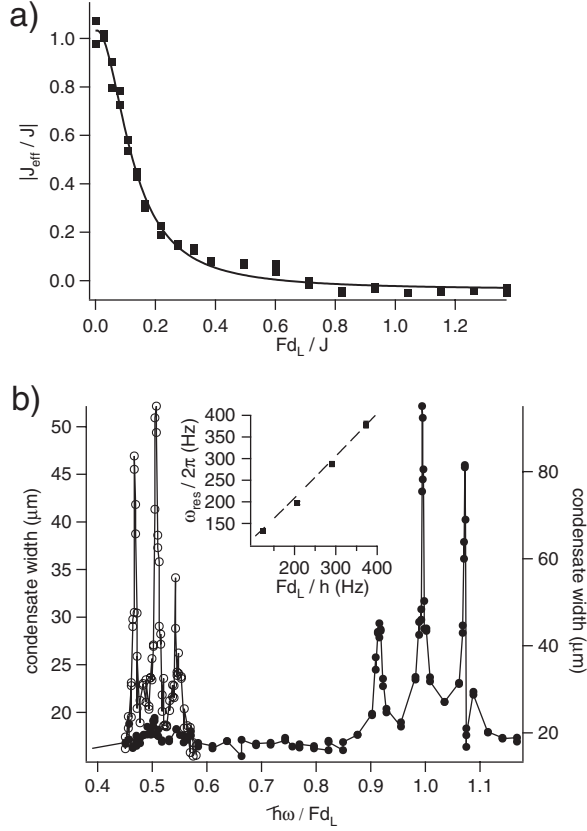
FIG. 2. (a) Suppression of tunneling by a linear potential. Shown here is the normalized effective tunneling parameter $|J_{\mathrm{eff}}/J|$ as a function of the linear potential Fd_L in units of the tunneling energy. When $Fd_L/J \approx 1$, the ground-state levels are shifted out of resonance and tunneling is suppressed almost completely. The solid line is a Lorentzian fit with a half-maximum half-width of 0.13. (b) Photon-assisted tunneling resonances in a shaken lattice. For a fixed linear potential $Fd_L/h = 380$ Hz, the condensate width is plotted as a function of the normalized shaking frequency $\hbar\omega/Fd_L$. The fixed shaking parameter was $K_0 = 1.8$ for the one-photon resonance (solid circles) and $K_0 = 3.1$ for the two-photon resonance (open circles), corresponding to the first maximum of the $\mathcal{J}_1$ and $\mathcal{J}_2$ Bessel functions, respectively. For a), $V_0/E_{\mathrm{rec}} = 5$ and $J/h = 270$ Hz, while for b) $V_0/E_{\mathrm{rec}} = 8$ and $J/h = 120$ Hz.

stored by sinusoidally shaking the lattice at a frequency ω satisfying the resonance condition of Eq. (3). The shaking of the lattice effectively creates low-frequency "photons" that bridge the energy gap between adjacent wells with n such photons [see Fig. 1(c)]. Figure 2(b) shows the condensate width after 400 ms of free expansion inside a rocked lattice with $Fd_L/J \approx 3$ as a function of the shaking frequency ω. One clearly sees two photon-assisted tunneling peaks at $\hbar\omega = Fd_L$ (for $K_0 = 1.8$, where $\mathcal{J}_1(K_0)$ has its first maximum) and at $2\hbar\omega = Fd_L$ [for $K_0 = 3.1$, the first maximum of $\mathcal{J}_2(K_0)$]. The peaks are extremely narrow, with a width of ≈ 3 Hz (when we repeated the experiment with smaller values of Fd_L, this width increased slightly). The two side peaks of each peak are evidence of

additional photon-assisted tunneling events due to the rocking motion of the lattice (for the two-photon resonance at $2\hbar\omega = Fd_L$, the side peaks are at half the distance to the main peak compared to the one-photon case, as expected).

Finally, we studied the dependence of the effective tunneling rate on the shaking parameter K_0. Figure 3 summarizes our results for the one-photon and two-photon resonances. Theory predicts [8] that the effective tunneling rates $|J_{\mathrm{eff}}/J|$ for resonances of nth order should vary as an nth order ordinary Bessel function [see Eq. (2)]. While qualitative agreement between experiment and theory is good, with the positions of the maxima and minima of the one- and two-photon resonances as a function of K_0 coinciding perfectly with theoretical predictions, the absolute values of $|J_{\mathrm{eff}}/J|$ lie consistently below the theoretical curves by a factor of about 1.3.

Interestingly, quantitative agreement between experiment and theory is better if we use the squares of the Bessel functions rather than their moduli. A dependence of the photon-assisted tunneling rate on the square of the
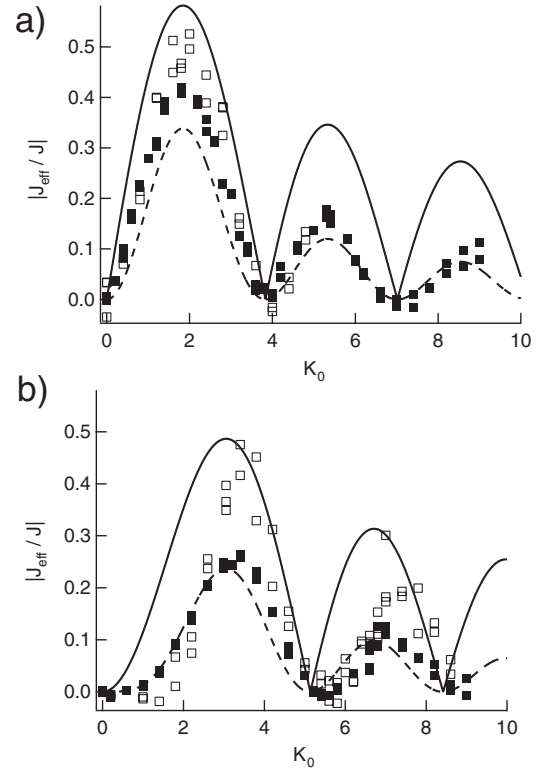

FIG. 3. Photon-assisted tunneling as a function of the shaking parameter K_0. Shown here are the one-photon resonance at $\omega/2\pi = 380$ Hz (a) and the two-photon resonance at $\omega/2\pi = 190$ Hz (b). In both graphs, the full and open squares are the measurements for $N \approx 5 \times 10^4$ and $N \approx 0.5 \times 10^4$, respectively. The solid lines are the moduli of the $\mathcal{J}_1(K_0)$ and $\mathcal{J}_2(K_0)$ Bessel functions, respectively, whereas the dashed lines are the squares of these functions. The lattice depth $V_0/E_{\mathrm{rec}} = 5$ and the constant force $Fd_L/h = 380$ Hz in this experiment, with a free expansion time $t = 100$ ms.

Bessel function is expected, e.g., for experiments on Josephson junctions irradiated by microwaves and, more generally, if the tunneling in a multiwell structure is sequential rather than coherent. The difference between coherent and sequential tunneling has been extensively studied in the theoretical literature [20]. Sequential tunneling would require a dephasing mechanism between two successive tunneling events. In our experiments, such a mechanism could be the dynamical instability inside the optical lattice [21–23]: owing to the constant applied acceleration a, the BEC moves through the Brillouin zone and hence through the dynamically unstable region within the Brillouin zone between about $0.5p_{rec}$ (where $p_{rec} = \pi\hbar/d_L$ is the recoil momentum) and the zone edge. For the values of the intrinsic nonlinearity of the BEC in our experiments (determined by the number of atoms and the trap frequencies), we found in previous experiments [21] (and have verified for this work) that the BEC loses its phase coherence after a few milliseconds inside the unstable region, which is comparable to the Bloch periods in the present experiment. Since the effective tunneling frequencies J_{eff} in our experiment are less than 100 Hz, the corresponding dephasing rate is almost an order of magnitude larger and hence it is likely that dephasing of neighboring wells occurs between two tunneling events.

In future experiments one might use, e.g., Feshbach resonances in order to tune the nonlinearity and hence move from the strongly nonlinear regime to the linear regime in order to test our hypothesis. As a preliminary test, we have repeated the measurement of the one-photon-assisted tunneling rate as a function of K_0 with the smallest atom number that allowed us to measure the free expansion ($N \approx 0.5 \times 10^4$), resulting in a condensate density that was about a factor of 3 smaller. Again, we obtained qualitatively similar results to the measurements with $N \approx 5 \times 10^4$, but this time the absolute values of $|J_{eff}/J|$ agreed better with the linear Bessel function prediction (see Fig. 3).

A further indication that dephasing might be responsible for the observed deviation from the linear Bessel functions comes from our measurements of the dynamical suppression of tunneling in a shaken lattice without linear acceleration [7], in which our measured values for J_{eff}/J agreed perfectly with the linear Bessel function prediction. In that system, the BEC does not cross the unstable region of the Brillouin zone, and we have experimentally verified that during the shaking the BEC retains its phase coherence.

If the two regimes of photon-assisted tunneling observed in our experiment do, indeed, correspond to coherent and sequential tunneling, our system is ideally suited to studying the crossover between these two extremes in a well-controlled way. However, we also have to consider the possibility that other effects play a role. In particular, it is conceivable that a self-trapping mechanism is present that depends on the relative magnitude of the nonlinearity U

and the effective tunneling parameter J_{eff}. Self-trapping in static optical lattices has already been observed [18] and could, if present also in our strongly driven system, lead to a suppression of the tunneling rate compatible with our observations, also for the case of a static force as in Fig. 2(a). On the other hand, our system is only in a self-trapping regime (as calculated from $U/h \approx 10$–30 Hz and J_{eff}) in a small region around the zeroes of the Bessel functions, whereas we observe the largest deviation from the linear prediction close to the local maxima.

In summary, we have demonstrated photon-assisted tunneling of BECs in a linearly accelerated and sinusoidally shaken optical lattice. Our results agree quantitatively with recent theoretical predictions and show the need for a theoretical investigation into the difference between the (roughly) linear and the nonlinear regimes for which we observe different dependencies of the effective tunneling rate on the predicted linear Bessel function scaling.

*Also at CNISM Unità di Pisa.
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