Validity and Breakdown of Onsager Symmetry in Mesoscopic Conductors Interacting with Environments

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We investigate magnetic-field asymmetries in the linear transport of a mesoscopic conductor interacting with its environment. Interestingly, we find that the interaction between the two systems causes an asymmetry only when the environment is out of equilibrium. We elucidate our general result with the help of a quantum dot capacitively coupled to a quantum Hall conductor and discuss the asymmetry dependence on the environment bias and induced dephasing.

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More than two decades ago it became clear [1] that the Onsager-Casimir symmetry relations [2,3] are crucial to understanding the transport properties of mesoscopic conductors. These symmetries are fundamentally a consequence of the microreversibility of the scattering matrix that describes the conductance of a phase-coherent conductor, dictating that the two-terminal linear conductance G must be symmetric under reversal of the external magnetic field B. An interesting consequence is that the phase of the Aharonov-Bohm conductance oscillations of a solid-state interferometer with a quantum dot embedded in one of its arms can take the values 0 or π only, thus leading to phase rigidity [4,5].

The scattering approach to mesoscopic transport assumes that the mesoscopic conductor preserves the electron quantum phase while inelastic processes giving rise to irreversibility take place only in the reservoirs that feed and draw the current. Therefore, close to equilibrium G is a function of the transmission T evaluated at the Fermi energy E_F common to all terminals and the Onsager symmetry implies $T(B) = T(-B)$. However, the system inevitably interacts with the environment which may give rise to irreversible processes. Theoretically, the Onsager symmetry has been proven to be valid for an isolated conductor. Therefore, it is an interesting question whether a conductor interacting with its environment still fulfills the symmetry. In this Letter, we predict that this interaction leads, in fact, to magnetic-field *asymmetries* (or, briefly, magnetoasymmetries) when the environment is driven out of equilibrium. In addition, our theory confirms why magnetic-field symmetries are preserved in previous experiments on two-terminal conductors even if they cannot avoid interactions with its environment.

Recent works [6,7] have shown that magnetoasymmetries arise in *nonlinear* mesoscopic transport, a fact which has been observed experimentally [8–13]. In contrast, here we address the magnetoasymmetry of the *linear* mesoscopic conductance when the environment is out of equilibrium.

Consider the model system sketched in Fig. 1. System C is a mesoscopic conductor coupled to reservoirs L and R. The environment, denoted as system D, is modeled as a second conductor in close proximity with system C. There exists a Coulomb interaction coupling conductor and environment electrons, but no particle exchange is permitted between the two subsystems. Experimentally, the environment can be a quantum point contact (QPC), a quantum Hall bar or any other system whose electron states depend on the electronic trajectory across the conductor. The environment can be driven out of equilibrium by applying a bias between reservoirs X and Y. As a consequence, we must consider scattering of two particles described by the following (asymptotic) states: $|1\rangle = |L\rangle_C \otimes |X\rangle_D$, $|2\rangle = |L\rangle_C \otimes |Y\rangle_D$, $|3\rangle = |R\rangle_C \otimes |X\rangle_D$, and $|4\rangle = |R\rangle_C \otimes |Y\rangle_D$. $|\cdots\rangle_C$ and $|\cdots\rangle_D$ represent the electron states at systems C and D, respectively.

Suppose that an electron from lead L is injected into the conductor. Before scattering, the initial density matrix $\rho_{\rm in}$ is given by

$$\rho_{\rm in} = (|L\rangle\langle L|)_C \otimes (n_X|X\rangle\langle X| + n_Y|Y\rangle\langle Y|)_D, \quad (1)$$

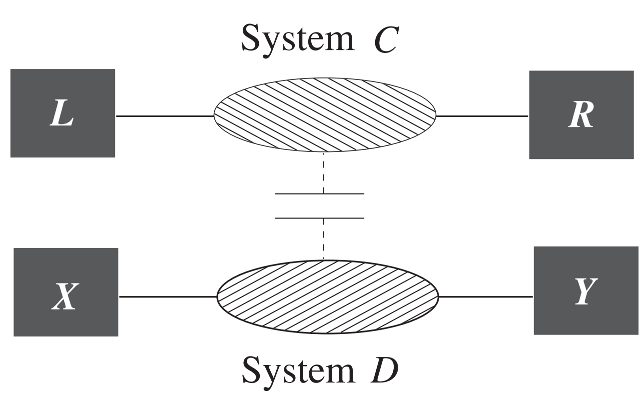

FIG. 1. Schematic representation of the system under consideration. System C is a conductor capacitively coupled to an environment (system D).

where n_X and n_Y are the transport electrons in system D that originate from leads X and Y, respectively. The normalization condition, $\mathrm{Tr}\rho_{\mathrm{in}} = 1$, gives the constraint $n_X + n_Y = 1$. At equilibrium, one has $n_X = n_Y = 1/2$ whereas out of equilibrium we rewrite ρ_{in} as

$$\rho_{\mathrm{in}} = \tfrac{1}{2}(1 + \delta\rho)|1\rangle\langle 1| + \tfrac{1}{2}(1 - \delta\rho)|2\rangle\langle 2|, \qquad (2)$$

where $\delta\rho = n_X - n_Y$ represents a nonequilibrium parameter. At small bias, $\delta\rho$ is proportional to the bias voltage while very far from equilibrium $\delta\rho = 1$. The limit of $\delta\rho = 1$ is discussed in Ref. [14]. Upon scattering, the output density matrix is [15] $\rho_{\mathrm{out}} = \hat{S}\rho_{\mathrm{in}}\hat{S}^\dagger$, where $\hat{S}$ is the two-particle scattering matrix with elements $S_{ij} = \langle i|\hat{S}|j\rangle$ $(i, j = 1, \ldots, 4)$. Note that $\hat{S}$ describes scattering between two electrons in different conductors [16]. We emphasize that $\hat{S}$ cannot, in general, be written as the product of two single-particle scattering matrices and that this nonseparability is precisely due to the interaction between the two systems [17].

The transition operator, $\hat{T}$, gives the transmission probability for an electron in system C, $T = \mathrm{Tr}(\rho_{\mathrm{out}}\hat{T})$. It reads $\hat{T} = (|R\rangle\langle R|)_C \otimes I_D = |3\rangle\langle 3| + |4\rangle\langle 4|$, where I_D is the unit operator in system D. Then we find

$$T = 1 - n_X(|S_{11}|^2 + |S_{21}|^2) - n_Y(|S_{12}|^2 + |S_{22}|^2), \qquad (3)$$

where unitarity of $\hat{S}$ has been used.

We introduce the magnetoasymmetry factor $\alpha \equiv T(B) - T(-B)$. Microreversibility implies that $S_{ij}(B) = S_{ji}(-B)$. Hence, we infer from Eq. (3) that

$$\alpha = \delta\rho(|S_{12}|^2 - |S_{21}|^2). \qquad (4)$$

Quite generally, one has $|S_{12}|^2 \neq |S_{21}|^2$ and then the transmission through the conductor is clearly not symmetric under reversal of the magnetic field. Notably, the asymmetry is proportional to $\delta\rho$; thus Eq. (4) predicts that at small bias the magnetoasymmetry in the conductor grows linearly with the voltage applied in system D. Below we perform numerical simulations in a realistic system that confirms this prediction.

Our argument can be easily extended to the case where the nonequilibrium situation of system D includes more than two leads. We note that Eq. (4) is valid when electrons are interacting within each subsystem and also when the electron at system C interacts with more than one electron at D, but the problem then becomes involved because one should resort to a multiple-particle scattering matrix. Interestingly, the magnetoasymmetry vanishes when the environment is in equilibrium ($\delta\rho = 0$), and it does not depend on a specific model for the environment. This explains why all linear-transport experiments satisfy the Onsager symmetry even though the conductors cannot be isolated from uncontrolled interactions with their environments.

It is clear from Eq. (4) that the magnetoasymmetry is nonzero only to the extent that $\hat{S}$ is nonseparable; i.e., $|S_{12}|^2 = |S_{21}|^2$ if $\hat{S}$ is given by the product $\hat{S} = \hat{S}_C \otimes \hat{S}_D$, where $\hat{S}_C$ and $\hat{S}_D$ are the single-particle scattering matrices of systems C and D, respectively. Unitarity of $\hat{S}_D$ is necessary in deriving this relation. In Ref. [18], $\hat{S}$ is expressed in terms of the scattering matrix of the uncoupled systems to leading order in the interaction coupling strength and the correction term of the transmission probability of the first system is found to depend on the injectivity of lead X of the second system. But the injectivity alone is *not* invariant under field reversal [6]. Therefore, T need not be an even function of B due to interaction between the two systems.

We focus on the zero temperature case for simplicity and assume that electrochemical potentials of leads X and Y are $E_F + eV_D/2$ and $E_F - eV_D/2$ with V_D the bias of system D. In a two-dimensional conductor, $n_X \propto E_F + eV_D/2$ and $n_Y \propto E_F - eV_D/2$. Using the constraint $n_X + n_Y = 1$, we obtain the nonequilibrium parameter $\delta\rho = n_X - n_Y = eV_D/2E_F$.

Let us now illustrate the general result given by Eq. (4) with an instructive example as depicted in Fig. 2. We consider resonant tunneling through a quantum dot which is capacitively coupled to the top edge of a quantum Hall conductor, which works as the controllable environment. We assume that the filling factor is $\nu = 1$ and thus current is carried by two edge states along opposite sides of the sample.

In addition, we consider a QPC constriction that partitions the current injected from lead X with probability T_d. For the moment, consider the case $T_d = 1$. Qualitatively, it is clear that the current traversing the dot, even close to equilibrium, is not an even function of B. This follows from the fact that the potential of the edge state U_e is in equilibrium with the electrochemical potential of the injecting lead. Then, $U_e = V_X$ for $B > 0$ and $U_e = V_Y$ for $B < 0$. Since the conductance through the dot depends on the local potential at the dot U_d, and U_d, in turn, depends on U_e via the capacitive coupling, we must have, quite generally, $G(B) \neq G(-B)$ [19].

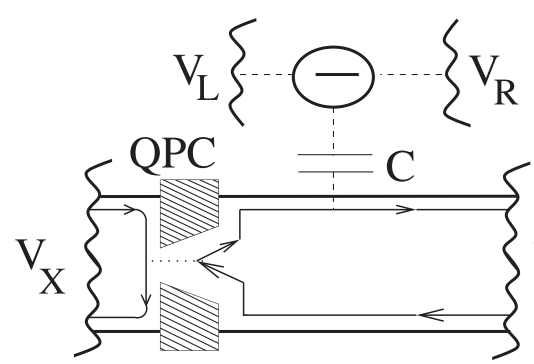

FIG. 2. Sketch of a quantum dot capacitively coupled with the top edge of a quantum Hall bar for $B > 0$. The edge state is transmitted at the QPC with probability T_d. For $B < 0$ the arrow directions are reversed.

We investigate the case in which U_d reacts continuously to a change in U_e and treat interactions in a mean-field way. We describe the scattering through the dot with a Breit-Wigner resonance with level position ε_0 and broadening $\Gamma = \Gamma_L + \Gamma_R$. Γ_L (Γ_R) denotes the resonance broadening contribution from the left (right) lead. To be definite, we take ε_0 and Γ invariant under B reversal, which is true at equilibrium. To separate equilibrium and nonequilibrium contributions, we consider the potential $U_d = U_{\rm eq} + \Delta U$. Then, away from equilibrium the screening potential ΔU follows from the charge neutrality condition which establishes that the net charge $\delta q = q_{\rm neq} - q_{\rm eq}$ must be equal to the polarization charge permitted by electrostatics:

$$\delta q(V_L, V_R, U_d) = C(U_{\rm eq} + \Delta U - U_e). \quad (5)$$

The charge injected from the left and right electrodes, $q_{\rm neq} = \int_{-\infty}^{E_F+eV_L} D_L(E - eU_{\rm eq} - e\Delta U)dE + \int_{-\infty}^{E_F+eV_R} D_R(E - eU_{\rm eq} - e\Delta U)dE$, is given by the injectivities $D_\alpha(E) = (e^2/2\pi)\Gamma_\alpha/|\Delta(E)|^2$ [20], where $\alpha = (L, R)$ and $\Delta(E) = E - \varepsilon_0 + i\Gamma/2$. The equilibrium charge, $q_{\rm eq} = \int_{-\infty}^{E_F} D_d(E - eU_{\rm eq})dE$, depends on the total density of states, $D_d = D_L + D_R$. In Eq. (5), C is the geometrical capacitance between the edge state and the dot. If the density of states of the edge state D_e is much larger than C/e^2, we simply have $U_e = V_X$ for $B > 0$. In the general case, we find

$$U_e = \frac{CU_d + e^2 D_e V_X}{C + e^2 D_e} \quad (6)$$

for $B > 0$. In the last equation, V_X should be replaced with V_Y for $B < 0$. Upon inserting Eq. (6) in Eq. (5), we obtain $\delta q = C_\mu(U_{\rm eq} + \Delta U - V_X)$, where $C_\mu^{-1} = C^{-1} + (e^2 D_e)^{-1}$ is the *electrochemical* capacitance.

Equation (5) is to be solved self-consistently. Once we numerically find ΔU, we can assess the current,

$$I = \frac{2e}{h}\int_{E_F+eV_R}^{E_F+eV_L} \frac{\Gamma_L\Gamma_R}{|\Delta(E - eU_{\rm eq} - e\Delta U)|^2} dE, \quad (7)$$

and the linear conductance $G = dI/dV|_{V=0}$ with $V = V_L - V_R$. The upper inset of Fig. 3 shows results for G as a function of the equilibrium level position $eU_{\rm eq}$ for a voltage bias $-V_X = V_Y = V_H/2 = \Gamma/2e$ applied in the Hall bar. We observe that G differs for opposite B orientations. The reason for the asymmetry is uniquely due to the asymmetry of the potential ΔU (see the lower inset of Fig. 3) arising from the asymmetry of the Hall bar injectivity. Thus, we present in Fig. 3 calculations of the dimensionless magnetoasymmetry factor $\tilde\alpha = [G(B) - G(-B)]/[G(B) + G(-B)]$ as a function of V_H for $eU_{\rm eq} = \Gamma$ and various C_μ. These results are central to our discussion. The magnetoasymmetry is larger for larger C_μ since the interaction coupling to the edge state is stronger. Of course, in the limit $C_\mu \to 0$ (which amounts to $C \to 0$), the magnetoasymmetry vanishes, fulfilling the Onsager symmetry relation. It also vanishes in the limit of an equilibrium

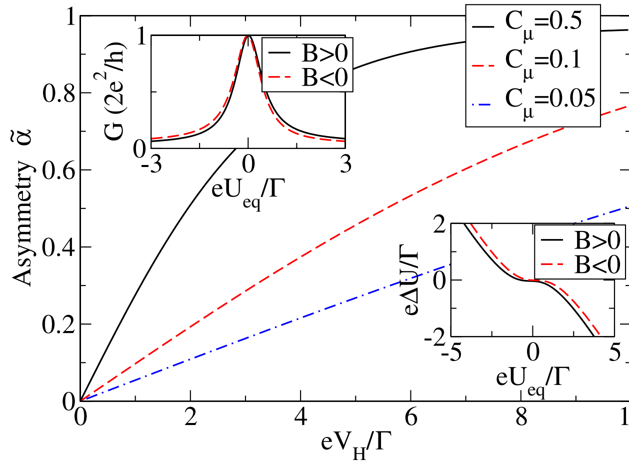

FIG. 3 (color online). Magnetic-field asymmetry of the linear conductance through the dot shown in Fig. 2 for $E_F = \varepsilon_0 = 0$, $\Gamma_L = \Gamma_R = \Gamma/2 = 0.1$, and $eU_{\rm eq} = \Gamma$ as a function of the voltage bias in the Hall bar V_H. Upper inset: Linear conductance at different polarizations of the magnetic field. Lower inset: Self-consistently calculated screening potential. In the insets we take $C_\mu = 0.1$ and $eV_H = \Gamma$.

environment ($V_H \to 0$). Moreover, $\tilde\alpha$ is linear with V_H for small V_H, in excellent agreement with Eq. (4) [21]. Saturation in $\tilde\alpha$ takes place for large V_H and C_μ, for which the precise form of the local density of states starts to play a role.

Let us now consider the case where the dot is coupled to a partitioned edge state. There is a probability T_d ($R_d = 1 - T_d$) that the edge state is transmitted (reflected) from lead X to Y (X) through the QPC. For nonzero C, we find

$$U_e(B > 0) = \frac{e^2 D_e(T_d V_X + R_d V_Y) + CU_d}{C + e^2 D_e}. \quad (8)$$

For $B < 0$ one replaces V_X with V_Y in Eq. (6). As a result, U_e is B asymmetric for nonzero T_d (for $T_d = 0$ the symmetry is restored) and depends in a generic way on T_d only for $B > 0$.

More interestingly, this geometry gives rise to *magneto-asymmetry of dephasing*. Dephasing in the dot is caused by current partition in the Hall bar [22–28]. It is intimately related to the *possibility* of extracting charge state information of the dot from the relative phase shift between the transmitted and the reflected beam at the QPC. For $B > 0$, Coulomb interaction induces dephasing because only the transmitted electron undergoes a phase shift. For $B < 0$, however, dephasing is not induced because the dot interacts with the Hall bar before the electrons arrive at the QPC. Therefore, it is clear that the magnetoasymmetry of linear transport is caused not only by the asymmetry of the local potential of Eq. (8) but also by the asymmetry of dephasing.

Dephasing induces an additional broadening for $B > 0$: $\Gamma \to \Gamma + \Gamma_\phi$ with $\Gamma_\phi = \eta T_d R_d V_H$, where η is a constant which depends on the details of the interaction between the

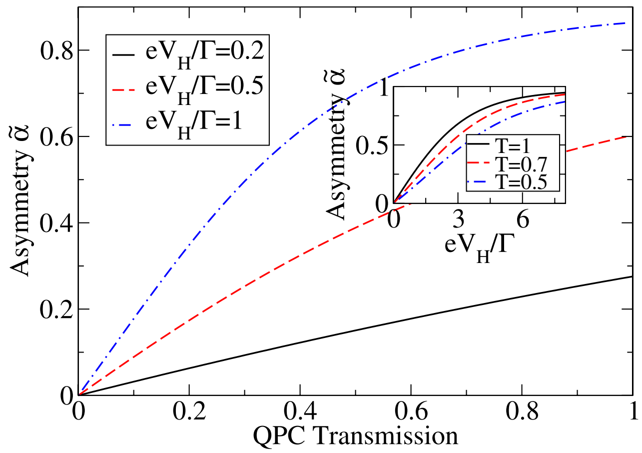

FIG. 4 (color online). Magnetic-field asymmetry as a function of the QPC transmission for various biases applied to the Hall bar V_H. Parameters are $E_F = \varepsilon_0 = 0$, $\Gamma_L = \Gamma_R = \Gamma/2 = 0.1$, $C_\mu = 0.5$, and $eU_{\rm eq} = \Gamma$. Inset: Asymmetry versus V_H for different QPC transmissions.

edge state and the dot [24–28] ($\eta = 0$ for $B < 0$). Since our goal is to offer a simple picture of the effect, we adopt the phenomenological voltage probe model [29]. It assumes a fictitious voltage probe attached to the dot with coupling Γ_ϕ. The condition $I_\phi = 0$ determines the potential at the probe, V_ϕ. Every carrier which enters the probe, is reemitted into the dot with a completely unrelated phase, thereby giving rise to dephasing. Thus, we add a term $\int_{-\infty}^{E_F+eV_\phi} D_\phi(E - eU_{\rm eq} - e\Delta U)dE$ on the left-hand side of Eq. (5) and a current contribution $(2e/h) \times \int_{E_F+eV_\phi}^{E_F+eV_L} \Gamma_L\Gamma_\phi/|\Delta(E - eU_{\rm eq} - e\Delta U)|^2 dE$ to Eq. (7). Figure 4 shows the effect of current partitioning in $\tilde\alpha$. The amount of dephasing is tuned with T_d. (To emphasize the role of T_d in the asymmetry we use a small $\eta = 10^{-3}$.) When $T_d = 0$, $U_e = V_Y$ independently of the B direction. As a result, $\tilde\alpha = 0$. When T_d increases, $\tilde\alpha$ enhances monotonically. For higher V_H, $\tilde\alpha$ becomes larger in agreement with Fig. 3. Finally, in the inset we plot $\tilde\alpha$ as a function of V_H for decreasing values of T_d, which also demonstrates that $\tilde\alpha$ vanishes for $V_H \to 0$.

In conclusion, the statement that the two-terminal linear conductance must be symmetric under reversal of the magnetic field is widely accepted and has been exhaustively confirmed. However, conductors inevitably interact with the external environment. We have shown that a magnetic-field asymmetry appears even in the linear response when the environment is out of equilibrium. This situation can be realized with another conductor in close proximity applying an electric bias across it. Importantly, we predict that the asymmetry depends on the two-particle scattering matrix. We have examined a quantum dot coupled to a quantum Hall bar and found that the asymmetry grows with the Hall bar bias and the interaction coupling strength. It also leads to an asymmetry of dephasing when the Hall current is partitioned.

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