Plateaus in the Dispersion of Two-Dimensional Magnetoplasmons in GaAs Quantum Wells: Theoretical Evidence of an Electron Reservoir

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(Received 23 August 2007; published 23 January 2008)

The filling-factor-dependent plateau-type dispersion of the long-wavelength magnetoplasmon in high-mobility two-dimensional electron system observed by Holland *et al.* [Phys. Rev. Lett. **93**, 186804 (2004)] can be explained by the well-established semiclassical dispersion, by adopting the electron reservoir hypothesis previously proposed in order to explain the integer quantum Hall effects.

DOI: 10.1103/PhysRevLett.100.036802 PACS numbers: 73.21.Fg

Recent discovery of zero-resistance states in an extremely high-mobility two-dimensional electron system (2DES) realized in GaAs quantum wells subjected to a crossed radiation field and quantizing magnetic fields [1,2] has attracted much interest [3–10]. Considering that the electromagnetic interaction should play a central role in the phenomenon and that the most important charge excitations in the electrons are plasmons, it would be natural to expect that the magnetoplasmon in such a 2DES might show certain signatures of the fundamental mechanism responsible for the phenomenon.

Indeed, Holland *et al.* [11] carried out the measurement of the long-wavelength magnetoplasmon in a high-mobility 2DES realized in a GaAs quantum well. Using the coupling between the plasmon with THz radiation, they explored a wide range of filling factors to obtain an explicit filling-factor dependence of the dispersion. They found a striking result. The observed dispersion seemed to deviate violently from the well-established semiclassical dispersion

$$\omega_{\rm mp}^2 = \omega_c^2 + \frac{2\pi e^2 N_{\rm 2DES}}{\varepsilon m} q, \tag{1}$$

where ε is the dielectric constant of GaAs semiconductor into which the 2DES is embedded, m is the electron effective mass, $-e$ is the electron charge, $N_{\rm 2DES}$ is the electron number density, q is the wave-number vector, and $\omega_c = eB/mc$ is the cyclotron frequency.

Using the resistively detected magnetoplasmon frequency $\omega_{\rm mp}^{\rm EXP}$, they defined the renormalized magnetoplasmon frequency,

$$\Omega_{\rm mp}^{\rm EXP} = \frac{\{\omega_{\rm mp}^{\rm EXP}\}^2 - \omega_c^2}{\omega_c}. \tag{2}$$

They plotted this $\Omega_{\rm mp}^{\rm EXP}$ versus the filling factor defined as

$$\nu = \frac{hcN_{\rm sample}}{eB}, \tag{3}$$

where $N_{\rm sample}$ is the electron number density of the samples, whose explicit values were given for three samples in Ref. [11]. By substituting Eqs. (1) and (3) into Eq. (2), and assuming $N_{\rm 2DES} = N_{\rm sample}$, one would straightforwardly find

$$\Omega_{\rm mp}^{\rm EXP} = \frac{2\pi ecN_{\rm sample}}{\varepsilon B} q = \frac{2\pi e^2}{\varepsilon h}\nu q. \tag{4}$$

For a fixed value of the wave-number vector q, this Eq. (4) would show that $\Omega_{\rm mp}^{\rm EXP}$ is simply proportional to the filling factor. After the measurement, however, they found an astonishing deviation from such a simple linear relation. In Ref. [11], the authors wrote: “We find a quantized dispersion with plateaus forming around even filling factors. It reveals a previously unknown relation between the magnetoplasmon and the integer quantum Hall effect (QHE) which is intriguing for investigating the nature of both.”

In this Letter we show that the filling-factor-dependent plateau-type dispersion for the long-wavelength magnetoplasmon observed by Holland *et al.* [11] can be reconstructed theoretically on the basis of the dispersion formula (1), provided that we adopt the electron reservoir hypothesis (ERH) [12] that was introduced more than two decades ago by Baraff and Tsui [13] and by Toyoda, Gudmundsson, and Takahashi [14,15] in order to explain the integer quantum Hall effect then discovered by von Klitzing [16].

If we follow carefully the quantum statistical mechanical derivation of the dispersion relation (1), then we find the quantity $N_{\rm 2DES}$ in the formula is actually the grand canonical ensemble expectation value for the electron number density in the system and should be written as [17–20]

$$N_{\rm 2DES} = \frac{eB}{hc}\sum_\alpha \sum_{n=0}^{\infty} \frac{1}{1+\exp(\beta E_{n\alpha})}, \tag{5}$$

where $\beta = 1/k_B T$ and $E_{n\alpha}$ is the energy eigenvalue of the nth Landau orbital with spin α, given by

$$E_{n\alpha} = \hbar\omega_c\left(n + \frac{1}{2}\right) + \frac{g^*}{2}\mu_B B\sigma(\alpha) - \mu. \tag{6}$$

Here we have defined the Bohr magneton $\mu_B = e\hbar/2m_0c$, the effective g-factor g^*, and the magnetic field B. The function σ is defined such that $\sigma(\uparrow) = 1$ and $\sigma(\downarrow) = -1$. Note that the electron rest mass m_0 appears in the Bohr magneton.

To proceed further, we are obliged to make a crucial judgment about whether the system has fixed number of electrons or fixed value of the chemical potential. In other words, we must choose whether to adopt the ERH or not. If we adopt ERH, the renormalized magnetoplasmon frequency defined by Eq. (2) should be replaced by

$$\Omega^T_{\mathrm{mp}} = \frac{\omega^2_{\mathrm{mp}} - \omega^2_c}{\omega_c} = \frac{2\pi ecN_{\mathrm{2DES}}}{\varepsilon B}q, \tag{7}$$

where ω_{mp} is given by Eq. (1). The only difference between Eqs. (4) and (7) is the electron number density. In Eq. (4), N_{sample} is a given quantity. On the other hand, in Eq. (7), N_{2DES} is a function of the temperature T, magnetic field strength B, and chemical potential μ, as given by Eq. (5). Substituting Eq. (5) into Eq. (7), we find

$$\Omega^T_{\mathrm{mp}} = \frac{2\pi q}{\varepsilon}\frac{e^2}{h}\sum_{\alpha}\sum_{n=0}^{\infty}\frac{1}{1+\exp\beta E_{n\alpha}} \equiv \Omega^T_{\mathrm{mp}}(T, B, \mu). \tag{8}$$

Apart from the factor $2\pi q/\varepsilon$, the resemblance to the quantum Hall conductivity formula derived in Refs. [14,15] is unmistakable.

In order to plot the dispersion given by Eq. (8), we need experimental values for g^* and m that appeared in the energy spectrum $E_{n\alpha}$. These parameters can be determined independently of the measurement of the magnetoplasmon dispersion. Although the values of m for the samples used in the measurement of the magnetoplasmon dispersion by Holland *et al.* are given in Ref. [11], the values of g^* for the same samples are not available at present. Therefore, we consider the case $g^* = 2$ and also the region $0 < g^* < 6$. According to Ref. [11], the temperature of the measurement is $T = 1.8$ K.

Because the variables μ, T, and B are thermodynamically independent, we may consider a situation that T and B are very small. Then, by calculating the grand canonical ensemble expectation value for the electron number density, we can readily find the relation, $\mu = \hbar\omega_c n_F$, where n_F is the number of fully occupied Landau levels. Here we have assumed two conditions such that $n_F \gg 1$ and that the only partially filled Landau level is the $(n_F + 1)$th level. These two conditions define the precise meaning of the assumption that T and B are very small [21].

Now that we have all the values of the relevant variables and parameters at hand, we may plot the dispersion curve given by Eq. (8) as a function of the filling factor that is

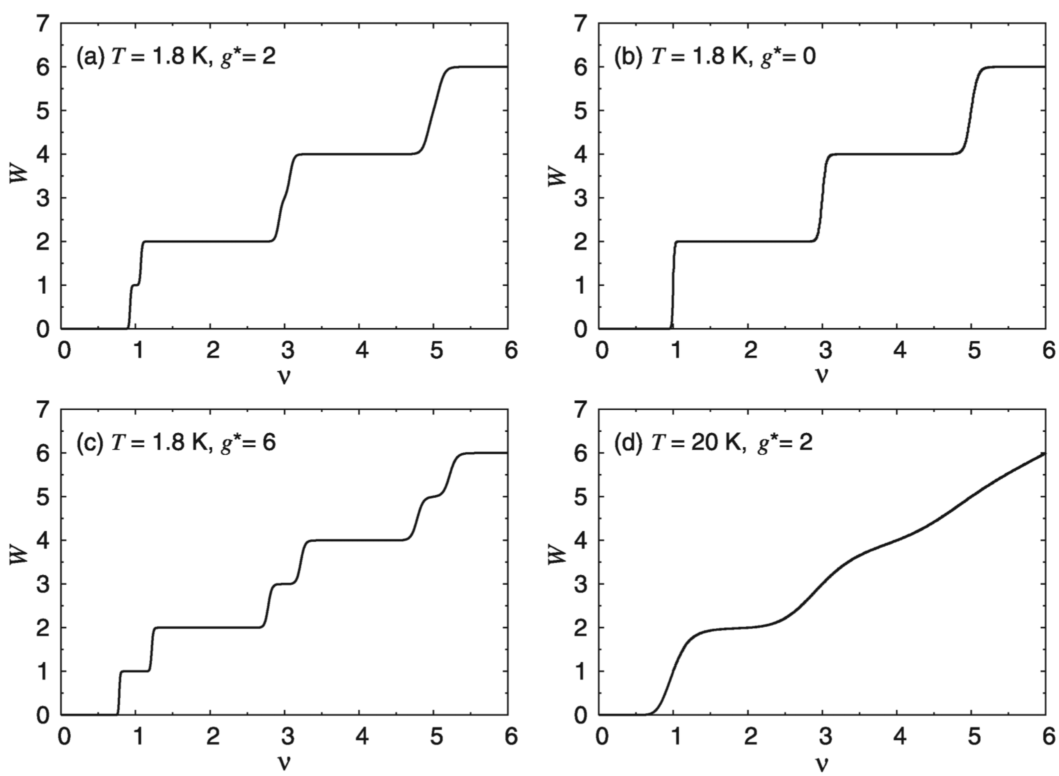

FIG. 1. Magnetoplasmon dispersion as a function of the filling factor. From Eq. (9), the dimensionless quantity $W = (\varepsilon h/2\pi qe^2)\Omega^T_{\mathrm{mp}}$ is plotted as a function of ν: (a) $T = 1.8$ K, $g^* = 2$, (b) $T = 1.8$ K, $g^* = 0$, (c) $T = 1.8$ K, $g^* = 6$, and (d) $T = 20$ K, $g^* = 2$.

defined as a function of B by the formula (3), where the value of N_{sample} is given in Ref. [11], i.e., $\nu \propto 1/B$.

Holland *et al.* [11] used three different samples in their measurement of the magnetoplasmon dispersion. Among the three samples, the data taken from sample M1218 show the most distinct filling-factor-dependent plateaus in the magnetoplasmon dispersion. Therefore, we use the parameters for sample M1218 to plot the theoretical dispersion given by Eq. (8). In order to see clearly the functional behavior of the dispersion, let us first define $D \equiv 2\pi\hbar^2 N_{\text{sample}}/m$ and $\Delta \equiv g^* m/4m_0$. Then, we define dimensionless parameter $a \equiv \beta D$. For the chemical potential we use $\mu = \hbar\omega_c n_F$. Using these parameters, the ν-dependent part of the dispersion can be written as

$$\Omega_{\text{mp}}^T = \frac{2\pi q}{\varepsilon}\frac{e^2}{h}\sum_{n=0}^{\infty}\{F_n^+(\nu) + F_n^-(\nu)\} \equiv \frac{2\pi q}{\varepsilon}\frac{e^2}{h}W(\nu) \tag{9}$$

with

$$F_n^{(\pm)}(\nu) = \frac{1}{1 + \exp[a\{\frac{1}{\nu}(n + \frac{1}{2} \pm \Delta) - \frac{1}{2}\}]}. \tag{10}$$

For the effective mass for sample M1218, we use $m = 0.0726m_0$ [11]. This gives $\Delta = 0.0182g^*$. The value of the parameter a is determined by the temperature such that $a = 424/T$ for sample M1218. In Fig. 1(a), we plot W versus ν for $T = 1.8$ K and $g^* = 2$. The curve shows plateaus around the even integer ν: $\nu = 0$, $\nu = 2$, $\nu = 4$, and $\nu = 6$. This result perfectly confirms the experimental result by Holland *et al.* The plateau at $\nu = 0$ is expected, because $\nu = 0$ corresponds to an extremely strong magnetic field, which certainly forces the electrons to oscillate with the given cyclotron frequency. In Fig. 1(b), we plot W versus ν for $T = 1.8$ K and $g^* = 0$. In Fig. 1(c), we plot W versus ν for $T = 1.8$ K and $g^* = 6$ [22]. For this enhanced g-factor, plateaus at odd integer ν also emerge. In Fig. 1(d), we plot W versus ν for $T = 20$ K and $g^* = 2$. At this higher temperature, the plateaus disappear. This temperature dependence is very similar to the integer quantum Hall effect [15]. To illustrate the temperature dependence of W more precisely, in Fig. 2(a) we plot W for $10 < a < 300$, which corresponds to $1.41 < T < 42.4$ K. As the temperature increases, the dispersion becomes its classical limit except near the origin ($\nu \approx 0$), where the magnetic field is so strong, the Landau quantization is still effective. In Fig. 2(b), we plot W for $0 < g^* < 6$. This 3D plot shows that around $g^* \approx 3$, plateaus at odd integer ν emerge. It should be noted that Ω_{mp}^T given in Eq. (9) is an even function of g^*. Therefore, the dispersion curves are independent of the sign of the effective g-factor.

In conclusion, we believe that the excellent agreement of our theoretically derived dispersion curve with the experimental one obtained by Holland *et al.* [11] clearly supports the correctness of the ERH. As to a possible microscopic mechanism that realizes the electron reservoir, the model proposed by Baraff and Tsui [13] seems to be most likely. In their model, the donor impurities in the barrier that confines the 2DES may act as a reservoir, but there has been no direct experimental verification of the model. Although the microscopic mechanism that realizes the electron reservoir is still being sought after, there have been a number of both experimental and theoretical investigations that seem to support strongly the ERH [23–30]. The next task should be to reveal the microscopic mechanism that realizes such an electron reservoir in GaAs semiconductors. We would leave this challenging task to experimental physicists.

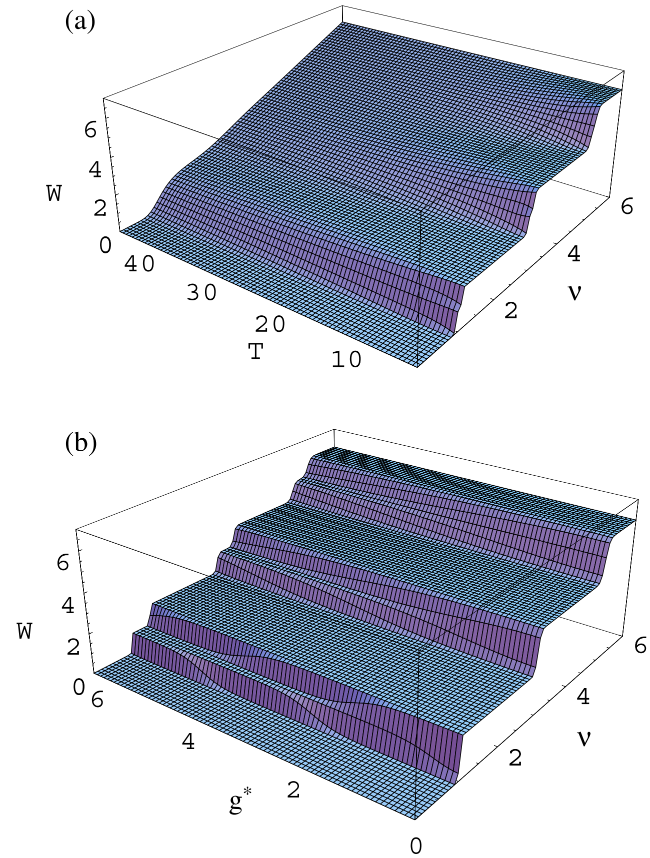

FIG. 2 (color online). Magnetoplasmon dispersion as a function of the filling factor, temperature, and effective g-factor. From Eq. (9), the dimensionless quantity $W = (\varepsilon h/2\pi q e^2)\Omega_{\text{mp}}^T$ is plotted (a) as a function of ν and a for $g^* = 2$, for the temperature range $1.41 < T < 42.4$ K, and (b) as a function of ν and g^* at $T = 1.8$ K.

We thank Professor Toshiyuki Toyoda for valuable comments.

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