Screened Radiative Corrections from Hyperfine-Split Dielectronic Resonances in Lithiumlike Scandium

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Term energies for dielectronic-recombination Rydberg resonances below 0.07 eV are determined for Sc^{18+} with absolute accuracies below 0.0002 eV by electron collision spectroscopy in an ion storage ring, using the twin-electron-beam technique and a cryogenic photocathode. The lithiumlike $2s_{1/2}$-$2p_{3/2}$ transition energy for $Z = 21$ is determined to 4.6 ppm, less than 1% of the few-body effects on radiative corrections. Features from the hyperfine structure of the $2s$ state could be resolved in the dielectronic-recombination spectrum.

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Atomic energy levels, in particular for heavier elements, feature important radiative corrections arising from the production of virtual particles and their dynamics in strong Coulomb fields [1]. For experimentally analyzing these quantum-electrodynamical (QED) effects, especially their convergence for multiple virtual-particle loops, lithiumlike ions play a crucial role as the transition energies between the fine structure levels $2s$ and $2p$ in their valence shell have particularly large relative QED corrections and are in the reach of precision measurements for many elements up to the highest nuclear charge Z [2–10]. At high Z especially, these level energies presently offer the most precise access to single-electron radiative corrections at the forefront of experimental [9] and theoretical research [11,12], representing the corrections due to two loops of virtual particles. Nevertheless, few-body effects severely interfere in extracting the higher-order single-electron effects from such measurements. Thus, the few-body corrections [13–20] to the single-electron radiative shift (often denoted as screening corrections to QED) must be accurately determined and their uncertainty, together with that of the nuclear radius, limits the precision on the two-loop contribution, as shown in the recent study on the $2s$-$2p_{1/2}$ transition in U^{89+} [9] or the earlier one on $2s$-$2p_{3/2}$ in Bi^{80+} [5]. For intermediate Z, the higher-order single-electron effects as well as the nuclear size affect the transition energy much less, while the few-body corrections retain their relative magnitude and can in this Z range be studied by dedicated precision measurements.

In the present Letter, electron collision spectroscopy employing new techniques is shown to reach a <5 ppm energy uncertainty relative to the $2s$-$2p$ transition for $Z = 21$. This yields the radiative correction at the 0.1% level and its screening part within 1%, unaffected by higher-order single-electron QED and significantly more accurate than the present scatter of $\sim 3\%$ in the screening calculations used to obtain the two-loop correction in the recent high-Z experiment [9,11]. The relative precision on the $2s$-$2p$ transition energy surpasses that of all data at intermediate and high Z [2–10] by more than a factor of 3.

The method [21,22] is based on a nearly complete cancellation between the valence excitation energy, which includes the radiative correction, and the binding energy of an outer electron attached to the valence-excited ion. In this case, a narrow dielectronic resonance occurs in the near-threshold ionization continuum of the ground-state lithiumlike ion whose interrogation with slow electrons of variable energy yields a sharp peak at the resonance energy by dielectronic recombination (DR) [23]. Since near-threshold electron collision spectroscopy can reach high energy accuracy and the radiative corrections in the Rydberg binding energy can be essentially neglected, this offers a precision approach focused on the radiative energy shift of the valence excitation.

Near cancellations leading to DR resonances at very low collision energy for lithiumlike systems occur for a few Z values only [22]. The ion Sc^{18+} ($Z = 21$) is particularly attractive in this respect as isolated members of the $2p_{3/2}10d$ Rydberg fine structure resonance array occur at collision energies of 0.02–0.08 eV only. It is studied here with a new twin-electron-beam technique [24] at an ion storage ring, performing high-resolution merged-beams collision spectroscopy on a continuously cooled and energy-stabilized stored ion beam. Moreover, a new cryogenic GaAs photocathode source is applied, delivering an electron beam with thermal energies of only 1 meV in its co-moving reference frame. With these additions, resonance energies can be measured with errors below 0.0002 eV, a factor of 10 smaller than in our previous storage-ring study [22] of this system. The low-energy spread for the first time allows hyperfine components of a DR resonance to be resolved in an electron collision

experiment, going beyond the isotope-dependent shifts within complex hyperfine arrays seen before [25] and revealing the hyperfine doublet [26] of the $2s_{1/2}$ ground state due to the $I = 7/2$ nuclear spin of ^{45}Sc. The analysis of the more complex hyperfine structure for the Rydberg DR resonances allows us to determine the resonance term energies from the hyperfine-split experimental peaks. The $2s$-$2p_{3/2}$ transition energy is found from these terms using relativistic many-body perturbation theory (RMBPT) improving on the previous approach [22] and yielding the Rydberg binding energies to $\sim$0.0001 eV.

The experiment at the storage ring TSR of the Max-Planck-Institut für Kernphysik (MPIK), Heidelberg, Germany used a beam of 177-MeV $^{45}Sc^{18+}$ ions from the MPIK tandem accelerator, circulating in the ring at injection energy with $\sim$1 μA typical current. Phase-space cooling is applied [22] with the collinear electron beam of the TSR electron cooler (laboratory energy 2.15 keV, electron density $n_e = 1.2 \times 10^7$ cm^{-3}) yielding a stored ion beam with a relative momentum spread of $<10^{-4}$ and $\sim$1 mm diameter. Unlike previously [22], the electron cooler is operated at a fixed highly stabilized voltage which precisely defines the ion beam velocity and quickly damps any small velocity deviations with a time constant of order 10 ms. The electron target [24] of the TSR recently installed in a separate ring section is used for the collision spectroscopy, applying an electron density almost 2 orders of magnitude lower than that of the cooler. Recombination rates between Sc^{18+} ions and the co-moving electrons are measured by counting the Sc^{17+} ions from the interaction region on a detector behind the first bend in the ion orbit downstream of the electron target. Their collision energies E in the center-of-mass (c.m.) frame are varied by tuning the acceleration voltage U of the electron target close to its value for matched velocities, thus scanning DR resonances at small E as well as the cusp in the radiative recombination (RR) rate at $E = 0$ [23]. The c.m. energy corresponding to the average longitudinal velocity difference of both beams is denoted by E_d and derived from U as described below. Through the small velocity spread in the interacting beams (mainly the electron beam) the energies E are distributed within only $\sim$2 meV from E_d. This twin-electron-beam technique avoids uncontrolled ion velocity variations causing a severe limitation [22] for collision spectroscopy with a single-electron beam in an ion storage ring. For the quasistationary conditions during twin-beam operation at $E_d > 0.01$ eV, the relative velocity variations of the ion beam were found by separately monitoring its Schottky noise spectrum to be $<2 \times 10^{-5}$ and thus irrelevant for the experimental error.

The electron target used a magnetically guided, dc electron beam of $\sim$200 μA, emitted in space-charge limited mode by a 3-mm diam. p-GaAs(Cs,O) photocathode cooled to $\sim$110 K to reduce the thermal electron energies at emission [27,28]. The beam was magnetically expanded (magnetic field ratio 28) to reach $\sim$16 mm diam. and thermal electron energies transverse to the magnetic guiding field (0.058 T) of only $\sim$1 meV, accelerated to its final laboratory energy of $\sim$2.2 keV ($n_e \sim 2 \times 10^5$ cm^{-3}), and overlapped with the Sc^{18+} beam over 1.5 m. The electron acceleration voltages of the target and the cooler were locked to each other by a direct electrical connection, using only a small bias supply at the photocathode to control their difference. The adjustment of the electron target ensures an angular alignment of the interacting beams within $\pm$0.2 mrad and their centering relative to each other within $\sim$2 mm.

The recombination rate shows the RR peak at an acceleration voltage U_{RR} close to 2155 V, while the investigated DR resonances occur at nearly symmetric positions above and below U_{RR} in a range of $\pm$25 V around this center. To find E_d, a space-charge correction U_{sc} is subtracted from both U and U_{RR}. It had a typical size of 0.3 V and was determined from the electron current together with the electron current profile analyzed separately. Since the electron density varied only little over the scan range, essentially the same space-charge correction is subtracted from both voltages, leaving for the difference $\Delta U = U - U_{RR}$ only a small relative correction of $+U_{sc}/2U_{RR} \sim 7 \times 10^{-5}$ on the U-scale.

The measured rate spectrum was converted [22] to a rate coefficient as a function of the c.m. energy E_d using the measured ion and electron-beam currents, the electron target acceleration voltage, and the exact kinematical relations for the space-charge corrected electron energy. To lowest order in $\Delta U/U_{RR}$, the detuning energy in the c.m. frame amounts to $E_d \simeq e(\Delta U)^2/4U_{RR}$. The resonances in the c.m. energy range of 0.02–0.08 eV are shown in Fig. 1. Through the kinematical transformation, the accuracy in E_d is boosted relative to that in ΔU by the factor of $\sqrt{eU_{RR}/E_d} \sim$ 160–330 for $E_d = 0.08$–0.02 eV. The sharpest structure of $\sim$0.002 eV width at $E_d \sim 0.025$ eV thus corresponds to an interval of $\sim$0.6 V in ΔU. The voltage control applied on top of the highly stabilized electron cooler power supply is estimated to be precise within $\pm$0.02 V. The position U_{RR} of the RR cusp used to define ΔU was determined within $\pm$0.02 V by requiring resonances for $\Delta U > 0$ to appear at identical positions as those for $\Delta U < 0$ after the conversion to the E_d scale. The space-charge correction introduces negligible errors in the range considered, the full space-charge effect leading to only a relative correction of $+2U_{sc}/U_{RR} \sim 1.4 \times 10^{-4}$ on the E_d scale in the approximation for $|\Delta U/U_{RR}| \ll 1$. The total systematic uncertainty of E_d, including the sources from the previous discussion [22] as far as still applicable, is estimated to $\pm$0.00017 eV as a mean value over the scan range.

The low-energy DR of Sc^{18+} up to $\sim$0.08 eV is governed by the three narrow resonance terms $(2p_{3/2}10d_{5/2})_{J=4}$, $(2p_{3/2}10d_{3/2})_{J=2}$, and $(2p_{3/2}10d_{3/2})_{J=3}$ predicted at

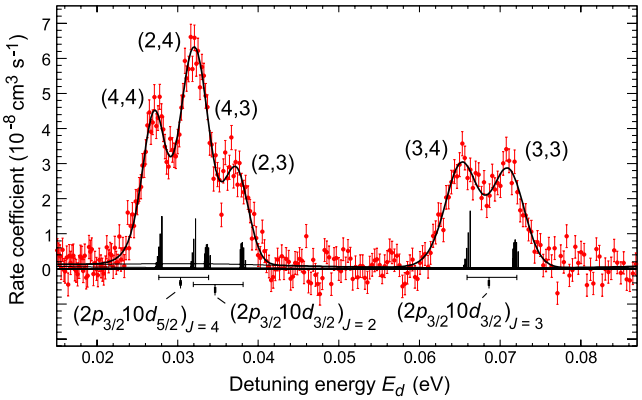

FIG. 1 (color online). Electron collision spectrum of the lowest Sc^{18+} DR resonances assigned to three doubly excited Rydberg terms of different J, whose hyperfine-unsplit energies are indicated in the bottom together with the energy shifts caused by the splitting between two F-levels of the $1s^2 2s_{1/2}$ ground state. Labels (J, F) are applied near the resulting hyperfine-split contributions in the experimental spectrum. The further splitting of each (J, F) contribution through the HFS in $1s^2 2p_{3/2}$ and the relative intensities of the HFS subcomponents are marked by stick diagrams. The fitted rate coefficient and broad minor features included in the fit are indicated by thick and thin curves, respectively.

0.0029, 0.0034, and 0.068 eV, respectively (Table III of Ref. [22]). Through the hyperfine structure (HFS) of the $Sc^{18+}(1s^2 2s_{1/2})$ ground state with the levels $F = 3$ and 4 separated by about 0.006 eV [26], each term (labeled by its J) is expected to produce two resonance groups (J, F) corresponding to the Sc^{18+} initial HFS levels. In contrast to the previous unresolved spectrum (Fig. 5 of [22]), we clearly identify these hyperfine groups, showing that the F states are about equally populated in the stored Sc^{18+} beam as expected at 300 K ambient temperature. In addition, the previously unresolved terms $J = 4$ and 2 near 0.03 eV can now be separately identified.

Each (J, F) resonance group is further split by small amounts through the HFS terms $F' = 2 \ldots 5$ in the $2p_{3/2}$ excited valence state. Using the RMBPT results and the approach of Ref. [29], we obtain for this state an interval factor of 0.0000779 eV [22] and a splitting constant of -0.0000284 eV due to the quadrupole moment [30]. For the outer electron (both in the continuum and in $n = 10$), the hyperfine interaction can be safely neglected. Thus, for the up to eight closely spaced components of each group, energetic positions relative to the term energy and relative intensities follow from angular momentum recoupling [31]. The required angular momentum assignments for the Ryberg electron and the partial-wave contributions in the collision are available from the RMBPT calculations of the DR rate coefficient and confirmed by their good agreement with experiment [22].

For fitting the spectrum, the rate coefficient was found by averaging $\sigma(E)\sqrt{2E/m}$ over the c.m. energy distribution $f(E, E_d; T_\perp, T_\parallel)$ of the electrons (mass m) for Maxwellian temperatures $T_\perp$, $T_\parallel$ transverse and parallel to the beam [22]. In addition to the narrow DR resonances with their calculated natural widths close to 0.0004 eV [22], the recombination cross section $\sigma(E)$ also included the theoretical RR and the DR of the broad resonance $(2p_{3/2}10p_{3/2})_{J=0}$ [22]. Zeeman splitting is expected to only increase the broadening by <0.0001 eV and thus neglected. The splitting and the relative intensities in each (J, F) group were fixed, while the resonance term energies $E_r(J)$, the Sc^{18+} ground-state HFS determining the spacing between the groups, and the intensity ratio between the two hyperfine groups were fitted. The fit also yielded $T_\parallel = 2.23(3) \times 10^{-5}$ eV and $T_\perp = 0.00111(10)$ eV from the high- and low-energy slopes of the resonances, respectively. Variations of the fixed parameters within reasonable limits had little effect on the fit results, most strongly on $T_\perp$, but remaining within the error given. $T_\perp$ causes an asymmetric broadening; however, its value is well defined from the different steepnesses of both slopes and the valleys between the resonance groups. This allows us to extract the resonance term energies as given in Table I. The hyperfine splitting in the Sc^{18+} ground state is found as 0.00620(8) eV in reasonable agreement with the predictions converging at 0.0060633(33) eV [20] and 0.006063(7) eV [32].

The term energies $E_r(J)$ represent the $2s_{1/2}$-$2p_{3/2}$ core excitation energy reduced by the binding energy $E_b(J)$ of the Rydberg electron on the $1s^2 2p_{3/2}$ core. They contain QED corrections arising from the core only, while an upper limit for the QED corrections to $E_b(J)$ is estimated to only 2×10^{-5} eV (αZ times the Breit contributions). The RMBPT calculations [22] of $E_b(J)$ were revisited with minimal changes. The errors represent neglected contributions of high partial-wave angular momenta ($l = 12$ to ∞) as well as of uncalculated higher-order correlation, conservatively estimated from the calculated all-order ladder

TABLE I. Resonance term energies and inferred $2s_{1/2}$-$2p_{3/2}$ transition energy using the calculated binding energies (in eV). From the average over the three terms, the final value of this transition energy is given as 44.30943(20) eV.

	$J = 4$	$J = 2$	$J = 3$
Resonance term energy $E_r(J)$ (experiment)	0.03036(10)	0.03465(10)	0.06861(10)
Rydberg binding energy $E_b(J)$ (RMBPT)	44.27916(11)	44.27480(11)	44.24071(9)
$2s_{1/2}$-$2p_{3/2}$ transition energy	44.30952(15)	44.30945(15)	44.30932(15)

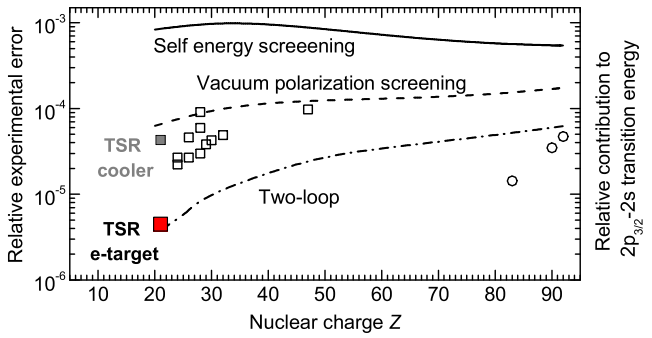

FIG. 2 (color online). Relative contributions to the $2s_{1/2}$-$2p_{3/2}$ transition energy in lithiumlike ions from the screening of the electron-nucleus QED [16,18] and from two-loop QED [11,19,34] compared to experimental errors for electron-beam ion traps [3–5] (circles), optical spectroscopy ([6] and references therein, squares), and previous [22] and present electron collision spectroscopy (filled squares) on the $2s_{1/2}$-$2p_{3/2}$ splitting.

diagrams. The sum $E_r(J) + E_b(J)$ yields three inferred values for the core excitation energy (Table I) consistent within the errors. From the average and the systematic energy uncertainty given above, the $2s_{1/2}$-$2p_{3/2}$ transition energy is obtained as 44.30943(20) eV. This 4.6 ppm result contains total radiative corrections of ~0.22 eV [22] thus determined on the 0.1% level.

Our result strongly improves the relative accuracy on two-electron (screening) effects in the QED corrections (Fig. 2) and lies well within the ~47 ppm uncertainty range of the recent prediction of 44.3091(21) eV [20]. We can also use it to reduce the limits on the QED screening correction specified in this unified full-QED many-body calculation. The difference between our experimental value and the sum of all listed contributions in Table I of Ref. [20] except the QED screening correction confine the value of the latter, given as 0.0331(20) eV [20], to 0.0335(5) eV, the error limit including the theoretical uncertainties [20]. Bearing in mind that the exact theoretical division into RMBPT and QED depends on the starting potential, we also note that the QED part of the two-photon exchange correction considered recently [19,33] is found to be 0.0011 eV for Sc^{18+} [19]. Our result for the transition energy is sensitive to this contribution within ~20%.

In summary, twin-beam electron collision spectroscopy together with non-QED atomic structure calculations experimentally infers the $2s_{1/2}$-$2p_{3/2}$ transition energy for $Z = 21$ accurately enough to find the few-body effects in its QED corrections within 1%. Its high accuracy qualifies the result as a benchmark in the ongoing precision calculations, in particular, within unified full-QED few-body theory.

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