Empirical Transverse Charge Densities in the Nucleon and the Nucleon-to-Δ Transition

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(Received 3 October 2007; published 23 January 2008)

Using only the current empirical information on the nucleon electromagnetic form factors we map out the transverse charge density in proton and neutron as viewed from a light front moving towards a transversely polarized nucleon. These charge densities are characterized by a dipole pattern, in addition to the monopole field corresponding with the unpolarized density. Furthermore, we use the latest empirical information on the $N \to \Delta$ transition form factors to map out the transition charge density which induces the $N \to \Delta$ excitation. This transition charge density in a transversely polarized N and Δ contains both monopole, dipole and quadrupole patterns, the latter corresponding with a deformation of the N and Δ charge distribution.

DOI: 10.1103/PhysRevLett.100.032004 PACS numbers: 14.20.Dh, 13.40.Gp, 14.20.Gk

Electromagnetic form factors (FFs) of the nucleon are the standard source of information on the nucleon structure and as such have been studied extensively; for recent reviews see, e.g., Refs. [1–3]. The FFs describing the transition of the nucleon to its first excited state, $\Delta(1232)$, contain complementary information, such as the sensitivity on the nucleon shape; see Ref. [4] for a recent review.

In more recent years, generalized parton distributions (GPDs) have been discussed (see, e.g., Refs. [5–8] for some reviews) as a tool to access the distribution of partons in the transverse plane [9], and first calculations of these spatial distributions have been performed within lattice QCD [10,11] and hadronic models (see, e.g., [12] for a recent evaluation). By integrating the GPDs over all parton momentum fractions, they reduce to FFs. Given the large amount of precise data on FFs it is of interest to exhibit directly the spatial information which results from these data. This has been done recently in Ref. [13] for an unpolarized nucleon. In this Letter we extend that work to the case of a transversely polarized nucleon as well as to map out the transition charge density which induces the $N \to \Delta$ excitation.

In the following we consider the electromagnetic (e.m.) $N \to N$ and $N \to \Delta$ transitions when viewed from a light front moving towards the baryon. Equivalently, this corresponds with a frame where the baryons have a large momentum-component along the z axis chosen along the direction of $P = (p + p')/2$, where p (p') are the initial (final) baryon four-momenta. We indicate the baryon light-front + component by P^+ (defining $a^\pm \equiv a^0 \pm a^3$). We can furthermore choose a symmetric frame where the virtual photon four-momentum q has $q^+ = 0$, and has a transverse component (lying in the xy plane) indicated by the transverse vector $\vec{q}_\perp$, satisfying $q^2 = -\vec{q}_\perp^{\,2} \equiv -Q^2$. In such a symmetric frame, the virtual photon only couples to forward moving partons and the + component of the electromagnetic current J^+ has the interpretation of the quark charge density operator. It is given by $J^+(0) = +2/3\bar{u}(0)\gamma^+ u(0) - 1/3\bar{d}(0)\gamma^+(0)d(0)$, considering only u and d quarks. Each term in the expression is a positive operator since $\bar{q}\gamma^+ q \propto |\gamma^+ q|^2$.

Following [9,13], one can then define quark transverse charge densities in a nucleon as

$$\rho_0^N(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{i\vec{q}_\perp\cdot\vec{b}} \frac{1}{2P^+} \times \left\langle P^+, \frac{\vec{q}_\perp}{2}, \lambda | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, \lambda \right\rangle, \quad (1)$$

where the two-dimensional vector $\vec{b}$ denotes the position (in the xy plane) from the transverse c.m. of the nucleon, and $\lambda = \pm 1/2$ denotes the nucleon (light-front) helicity.

Using the Dirac FF F_1, Eq. (1) becomes [13]:

$$\rho_0^N(b) = \int_0^\infty \frac{dQ}{2\pi} Q J_0(bQ) F_1(Q^2), \quad (2)$$

with J_n denotes the cylindrical Bessel function of order n. Note that ρ_0^N only depends on $b = |\vec{b}|$. It has the interpretation of a quark charge density in the transverse plane for an unpolarized nucleon, and is well defined for all values of b, even when b is smaller than the nucleon Compton wavelength. In contrast, the usual three-dimensional Fourier transform of the matrix elements of J^μ in the Breit frame (parameterized in terms of the Sachs FFs) becomes intrinsically ambiguous [14], due to the Lorentz contraction of the nucleon along its direction of motion. Although this does not affect the densities at larger distances (typically larger than about 0.5 fm) the value for the densities at smaller densities is merely a reflection of the prescription how to relate the experimentally measured Sachs FFs at large Q^2 with the intrinsic charge and magnetization FFs. A feature of viewing the nucleon when "riding a photon" is that one gets rid of the longitudinal direction. This allows one to project the charge density (in the case of the J^+ operator) on the transverse plane, which does not get Lorentz contracted. In this way, it was found, e.g., in Ref. [13] that the neutron charge density reveals the well-known negative contribution at large distances, around

1.5 fm, due to the pion cloud, a positive contribution at intermediate b values, and a negative core at b values smaller than about 0.3 fm. One can understand the negative value of the neutron $\rho_0(b = 0)$ from Eq. (2) and the observation that over the whole measured Q^2 range the neutron FF F_1 is negative. In contrast, the Breit-frame density, corresponding with the Fourier transform of G_E, is positive at $b = 0$ for the neutron due to the neutron FF G_E, which is positive over the whole measured Q^2 range.

It was shown in Ref. [9] that one can also define a probability distribution to find a quark with a given momentum fraction x of P^+, and at a given transverse position b in the nucleon, when considering a nucleon polarized in the xy direction. We denote this transverse polarization direction by $\vec{S}_\perp = \cos\phi_S \hat{e}_x + \sin\phi_S \hat{e}_y$. When integrating the resulting GPD, depending on x and $\vec{b}$, over x, one can define a quark charge density in the transverse plane for a transversely polarized nucleon as

$$\rho_T^N(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{i\vec{q}_\perp\cdot\vec{b}} \frac{1}{2P^+} \left\langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} \middle| J^+(0) \middle| P^+, -\frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} \right\rangle, \tag{3}$$

with $s_\perp$ the nucleon spin projection along the direction of $\vec{S}_\perp$. The transverse spin state can be expressed in terms of the light-front helicity spinor states as $|s_\perp = +\frac{1}{2}\rangle = (|\lambda = +\frac{1}{2}\rangle + e^{i\phi_S}|\lambda = -\frac{1}{2}\rangle)/\sqrt{2}$. By working out the Fourier transform in Eq. (3), one obtains

$$\rho_T^N(\vec{b}) = \rho_0^N(b) - \sin(\phi_b - \phi_S) \times \int_0^\infty \frac{dQ}{2\pi} \frac{Q^2}{2M_N} J_1(bQ) F_2(Q^2), \tag{4}$$

where the second term, which describes the deviation from the circular symmetric unpolarized charge density, depends on the orientation of $\vec{b} = b(\cos\phi_b \hat{e}_x + \sin\phi_b \hat{e}_y)$. In Eq. (4), F_2 is the Pauli FF and M_N the nucleon mass.

In the following we are using the current empirical information on the nucleon e.m. FFs to extract the transverse charge density in a transversely polarized nucleon, complementing the pictures given in [13], for the transverse charge densities in an unpolarized nucleon. For the proton e.m. FFs, we use the recent empirical parameterization of [15] and show the resulting transverse charge density for a proton polarized along the x axis (i.e., for $\phi_S = 0$) in Fig. 1. One notices from Fig. 1 that polarizing the proton along the x axis leads to an induced electric dipole moment along the negative y axis which is equal to the value of the anomalous magnetic moment, i.e., $F_2(0)$ (in units $e/2M_N$) as first noticed in [9]. One can understand this induced electric dipole field pattern based on the classic work of Ref. [16] (see also the pedagogical expla-

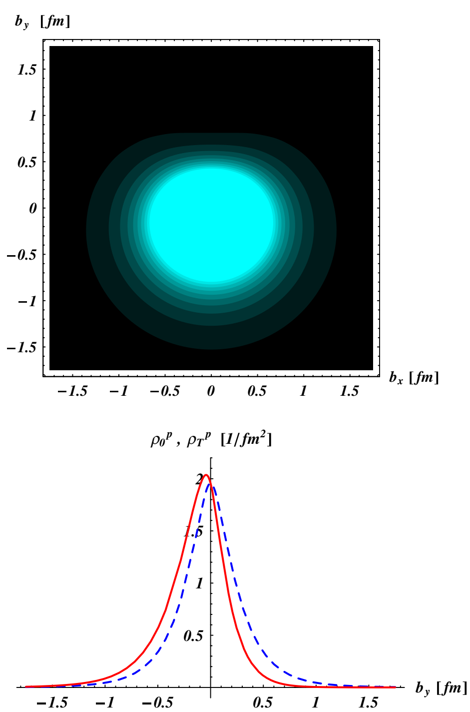

FIG. 1 (color online). Quark transverse charge densities in the *proton*. The upper panel shows the density in the transverse plane for a proton polarized along the x axis. The light (dark) regions correspond with largest (smallest) values of the density. The lower panel compares the density along the y axis for an unpolarized proton (dashed curve), and for a proton polarized along the x axis (solid curve). For the proton e.m. FFs, we use the empirical parameterization of Arrington *et al.* [15].

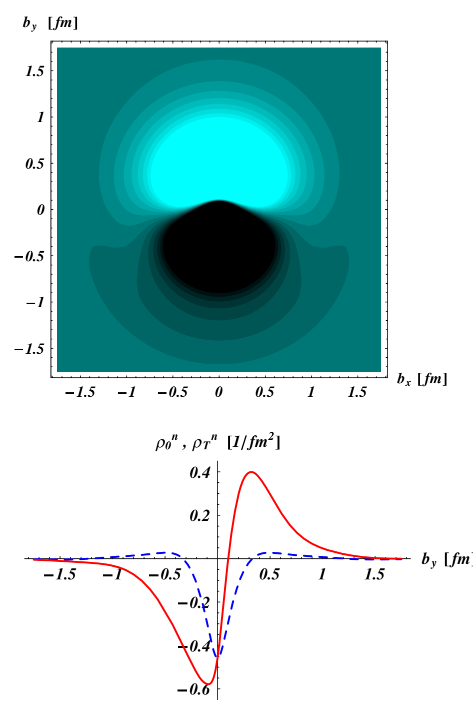

FIG. 2 (color online). Same as Fig. 1 for the quark transverse charge densities in the *neutron*. For the neutron e.m. FFs, we use the empirical parameterization of Bradford *et al.* [18].

nation in [17]). The nucleon spin along the x axis is the source of a magnetic dipole field, which we denote by $\vec{B}$. An observer moving towards the nucleon with velocity $\vec{v}$ will see an electric dipole field pattern with $\vec{E}' = -\gamma(\vec{v} \times \vec{B})$ giving rise to the observed effect.

For the neutron e.m. FF, we use the recent empirical parameterization of Ref. [18]. The corresponding transverse charge density for a neutron polarized along the x axis is shown in Fig. 2. One notices that the neutron's unpolarized charge density gets displaced significantly due to the large (negative) value of the neutron anomalous magnetic moment, $F_{2n}(0) = -1.91$, which yields an induced electric dipole moment along the positive y axis.

We next generalize the above considerations to the $N \to \Delta$ e.m. transition as it allows access to $l = 2$ angular momentum components in the nucleon and/or Δ wave functions. We will use the empirical information on the $N \to \Delta$ transition FFs to study the quark transition charge densities in the transverse plane which induce the e.m. $N \to \Delta$ excitation. It is customary to characterize the three different types of the $\gamma N \Delta$ transitions in terms of the Jones-Scadron FFs G_M^*, G_E^*, G_C^* [19], corresponding with the magnetic dipole ($M1$), electric quadrupole ($E2$) and Coulomb quadrupole ($C2$) transitions, respectively; see Ref. [4] for details and definitions.

We start by expressing the matrix elements of the $J^+(0)$ operator between N and Δ states as

$$\left\langle P^+, \frac{\vec{q}_\perp}{2}, \lambda_\Delta | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, \lambda_N \right\rangle = (2P^+) e^{i(\lambda_N - \lambda_\Delta)\phi_q} \times G^+_{\lambda_\Delta \lambda_N}(Q^2), \quad (5)$$

where λ_N (λ_Δ) denotes the nucleon (Δ) light-front helicities, and where $\vec{q}_\perp = Q(\cos\phi_q \hat{e}_x + \sin\phi_q \hat{e}_y)$. The helicity FFs $G^+_{\lambda_\Delta \lambda_N}$ depend on Q^2 only and can equivalently be expressed in terms of G_M^*, G_E^*, and G_C^*.

We define a transition charge density for the unpolarized $N \to \Delta$ transition, given by the Fourier transform

$$\rho_0^{N\Delta}(b) = \int_0^\infty \frac{dQ}{2\pi} Q J_0(bQ) G^+_{+(1/2)+(1/2)}(Q^2), \quad (6)$$

where the helicity conserving $N \to \Delta$ FF $G^+_{+(1/2)+(1/2)}$ can be expressed in terms of G_M^*, G_E^*, and G_C^* as

$$\begin{aligned} G^+_{+(1/2)+(1/2)} = {} & I \frac{(M_\Delta + M_N)}{M_N Q_+^2} \sqrt{\frac{3}{2}} \left(-\frac{Q^2}{4}\right) \\ & \times \left\{ G_M^* + G_E^* \frac{3}{Q_-^2} [(3M_\Delta + M_N) \right. \\ & \times (M_\Delta - M_N) - Q^2] \\ & \left. + 2G_C^* \left[-\frac{(M_\Delta + M_N)}{M_\Delta} + \frac{3Q^2}{Q_-^2} \right] \right\}, \end{aligned} \quad (7)$$

with $M_\Delta = 1.232$ GeV the Δ mass, and where the isospin factor $I = \sqrt{2/3}$ for the $p \to \Delta^+$ transition, which we consider in all of the following. We also introduced the shorthand notation $Q_\pm \equiv \sqrt{(M_\Delta \pm M_N)^2 + Q^2}$.

The above unpolarized transition charge density gives us one combination of the three independent $N \to \Delta$ FFs. To get information from the other combinations, we consider the transition charge densities for a transversely polarized N and Δ, both along the direction of $\vec{S}_\perp$ as

$$\rho_T^{N\Delta}(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{i\vec{q}_\perp \cdot \vec{b}} \frac{1}{2P^+} \left\langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp^\Delta = +\frac{1}{2} | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, s_\perp^N = +\frac{1}{2} \right\rangle, \quad (8)$$

where $s_\perp^N = +1/2$ ($s_\perp^\Delta = +1/2$) are the N (Δ) spin projections along the direction of $\vec{S}_\perp$, respectively. By working out the Fourier transform in Eq. (8), one obtains

$$\begin{aligned} \rho_T^{N\Delta}(\vec{b}) = \int_0^\infty \frac{dQ}{2\pi} \frac{Q}{2} \{ & J_0(bQ) G^+_{+(1/2)+(1/2)} + \sin(\phi_b - \phi_S) J_1(bQ) [\sqrt{3} G^+_{+(3/2)+(1/2)} + G^+_{+(1/2)-(1/2)}] \\ & - \cos 2(\phi_b - \phi_S) J_2(bQ) \sqrt{3} G^+_{+(3/2)-(1/2)} \}. \end{aligned} \quad (9)$$

One notices from Eq. (9) that besides half the unpolarized transition density, one obtains two more linearly independent structures. The $N \to \Delta$ FF combination with one unit of (light-front) helicity flip, which corresponds with a dipole field pattern in the charge density, can be expressed in terms of G_M^*, G_E^*, and G_C^* as

$$\sqrt{3} G^+_{+(3/2)+(1/2)} + G^+_{+(1/2)-(1/2)} = I \frac{(M_\Delta + M_N)}{M_N Q_+^2} \sqrt{\frac{3}{2}} Q \left\{ G_M^* (M_\Delta + M_N) + G_C^* \frac{Q^2}{2M_\Delta} \right\}, \quad (10)$$

whereas the $N \to \Delta$ form factor with two units of (light-front) helicity flip, corresponding with a quadrupole field pattern in the charge density, can be expressed as

$$G^+_{+(3/2)-(1/2)} = I \frac{(M_\Delta + M_N)}{M_N Q_+^2} \frac{3}{4\sqrt{2}} Q^2 \left\{ G_M^* + G_E^* \left[1 - \frac{4M_\Delta (M_\Delta - M_N)}{Q_-^2} \right] - G_C^* \frac{2Q^2}{Q_-^2} \right\}. \quad (11)$$

We show the results for the $N \to \Delta$ transition densities both for the unpolarized case and for the case of transverse polarization in Fig. 3. We use the empirical information on the $N \to \Delta$ transition FFs from [20]. One notices that the

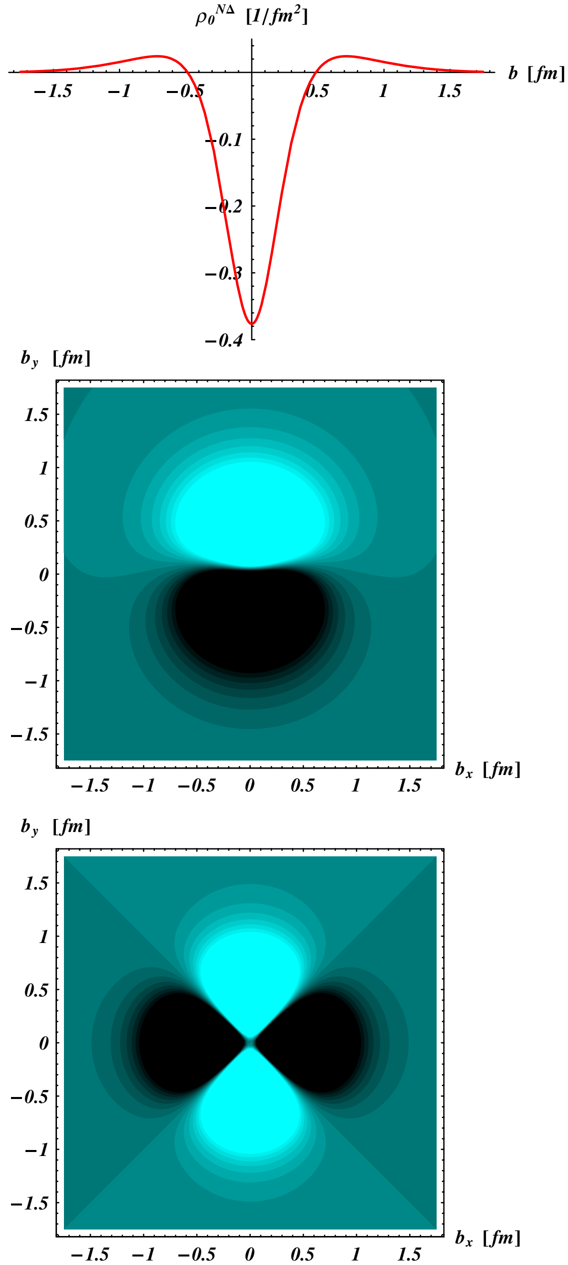

FIG. 3 (color online). Quark transverse transition charge density corresponding with the $p \to \Delta^+$ transition. Upper panel: when N and Δ are unpolarized ($\rho_0^{N\Delta}$). Middle panel: when N and Δ are polarized along the x-axis ($\rho_T^{N\Delta}$). Lower panel: quadrupole contribution to $\rho_T^{N\Delta}$. For the $N \to \Delta$ e.m. FFs, we use the empirical parameterization of MAID2007 [20].

unpolarized $N \to \Delta$ transition density displays a behavior very similar to the neutron charge density (dashed curve in Fig. 2), having a negative interior core and becoming positive for $b \geq 0.5$ fm. The density in a transversely polarized N and Δ shows both a dipole pattern, and a quadrupole pattern. The latter, shown separately in Fig. 3, allows to cleanly quantify the deformation in this transition charge distribution.

In summary, we used the recent empirical information on the nucleon and $N \to \Delta$ e.m. FFs to map out the transverse charge densities in unpolarized and transversely polarized nucleons and for the $N \to \Delta$ transition. The nucleon charge densities are characterized by a dipole pattern, in addition to the monopole field corresponding with the unpolarized density. The $N \to \Delta$ transition charge density in a transversely polarized N and Δ contains both monopole, dipole and quadrupole patterns, the latter corresponding with a deformation of the N and Δ charge distribution.

The authors thank C. Papanicolas and L. Tiator for useful discussions. The work of C. E. C. is supported by the National Science Foundation under Grant No. PHY-0555600. The work of M. V. is supported in part by DOE Grant No. DE-FG02-04ER41302 and Contract No. DE-AC05-06OR23177 under which Jefferson Science Associates operates the Jefferson Laboratory.

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