

Scalar Modifications to Gravity from Unparticle Effects May Be Testable

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Interest has focused recently on low energy implications of a nontrivial scale invariant sector of an effective field theory with an IR fixed point, manifest in terms of “unparticles” with peculiar properties. If unparticle stuff exists it could couple to the stress tensor and mediate a new “fifth” force (“ungravity”). Under the assumption of strict conformal invariance in the hidden sector down to low energies, we compute the lowest order ungravity correction to the Newtonian gravitational potential and find scale invariant power law corrections of type $(R_G/r)^{2d_U-1}$, where d_U is an anomalous unparticle dimension and R_G is a characteristic length scale where the ungravity interactions become significant. It is shown that a discrimination between extra dimension models and ungravity is possible in future improved submillimeter tests of gravity.

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Recently much interest has been generated by the possibility that a nontrivial scale invariant sector of an effective field theory [1] characterized by unparticles [2] could play a role in low energy physics. This has led to several further investigations of unparticle effects in collider physics and elsewhere [3]. In models of this type one assumes that the ultraviolet theory has hidden sector operators O_{UV} of dim d_{UV} possessing an IR fixed point. These couple via a connector sector with the standard model operators O_{SM}^n of dimension n , generating an effective interaction $O_{UV}O_{SM}^n/M_U^{d_{UV}+n-4}$. It is then assumed that the fields of the hidden sector undergo dimensional transmutation at scale Λ_U generating scale invariant unparticle fields O with dimension d_U which give the interaction

$$\left(\frac{\Lambda_U}{M_U}\right)^{d_{UV}+n-4} \frac{OO_{SM}^n}{\Lambda_U^{d_U+n-4}}. \quad (1)$$

The operator O could be a scalar, a vector, a tensor, or even a spinor. If O is a tensor of rank two it could couple to the stress tensor and its exchange between physical particles could lead to a modification of Newtonian gravity. We discuss this issue in this Letter.

We work strictly within the framework where conformal invariance holds down to low energies, and thus we forbid scalar unparticle operators of dimension $d_U < 2$ which could have couplings to the Higgs field of the type H^2O . The presence of a super-renormalizable operator destroys conformal invariance once H develops a VEV [4]. In our analysis here we consider an effective operator of the type

$$\kappa_* \frac{1}{\Lambda_U^{d_U-1}} \sqrt{g} T^{\mu\nu} O_{\mu\nu}^U, \quad (2)$$

where κ_* is defined by $\kappa_* = \Lambda_U^{-1}(\Lambda_U/M_U)^{d_{UV}}$. We assume that $O_{\mu\nu}^U$ transforms like a tensor under the general coordinate transformations, and thus the interaction of Eq. (2) gives an action which is invariant under the transformations. For convenience we assume that $O_{\mu\nu}^U$ is traceless.

The addition of Eq. (2) to the action changes the stress-energy tensor so that the new tensor is $\mathcal{T}^{\mu\nu} = T^{\mu\nu} + (\kappa_*/\Lambda_U^{d_U-1})g^{\mu\nu}T^{\sigma\rho}O_{\sigma\rho}^U$. The conservation condition in this case is $\mathcal{T}_{;\nu}^{\mu\nu} = 0$. The interaction of Eq. (2) implies that the unparticles can be exchanged between massive particles, and this exchange creates a new force, a “fifth” force, which we call “ungravity” which adds to the force of gravity. We wish to compute the correction to the Newtonian gravitational potential arising from the exchange of the unparticles to the lowest order. In this case one may neglect all the gravitational effects and replace $g_{\mu\nu}$ by $\eta_{\mu\nu}$. The quantity that enters in the computation of the unparticle exchange contribution is the unparticle propagator

$$\Delta_U^{\mu\nu\sigma\rho}(P) = \int e^{iPx} \langle 0 | T [O_U^{\mu\nu}(x) O_U^{\sigma\rho}] | 0 \rangle d^4x. \quad (3)$$

An analysis of this propagator using spectral decomposition [2,3] gives

$$\Delta_U^{\mu\nu\sigma\rho}(P) = \frac{A_{d_U}}{\sin(\pi d_U)} P^{\mu\nu\sigma\rho} (-P^2)^{d_U-2}, \quad (4)$$

where $P^{\mu\nu\sigma\rho}$ has the form $P^{\mu\nu\sigma\rho} = \frac{1}{2}(P^{\mu\sigma}P^{\nu\rho} + P^{\mu\rho}P^{\nu\sigma} - \alpha P^{\mu\nu}P^{\sigma\rho})$. Here $\alpha = 2/3$ and $P^{\mu\nu} = (-\eta^{\mu\nu} + P^\mu P^\nu/P^2)$, and A_{d_U} is given by

$$A_d = \frac{16\pi^{5/2}}{(2\pi)^{2d_U}} \frac{\Gamma(d_U + 1/2)}{\Gamma(d_U - 1)\Gamma(2d_U)}. \quad (5)$$

Further, since we are interested in computing the effects of the unparticles to the lowest order, we can replace $\mathcal{T}^{\mu\nu}$ by $T^{\mu\nu}$ and replace $\mathcal{T}_{;\nu}^{\mu\nu} = 0$ by $T_{;\nu}^{\mu\nu} = 0$. This condition implies that momentum factors when acting on the source give a vanishing contribution, and the relevant part of $P^{\mu\nu\sigma\rho}$ can be written as $P^{\mu\nu\sigma\rho} = \frac{1}{2}(\eta^{\mu\sigma}\eta^{\nu\rho} + \eta^{\mu\rho}\eta^{\nu\sigma} - \alpha\eta^{\mu\nu}\eta^{\sigma\rho})$. For the case of the graviton exchange $\alpha = 1$, and retaining α in the analysis will provide a quick check to the graviton exchange limit.

We have carried out an analysis of the unparticle exchange along with the one graviton exchange and computed the effective potential in the nonrelativistic limit. We find

$$V(r) = -m_1 m_2 G \left(\frac{1}{r} + \frac{\xi^2}{\Lambda_U^{2d_U-2}} \frac{(2-\alpha)}{(2\pi)^{2(d_U-1)}} \frac{2}{\sqrt{\pi}} \right. \\ \left. \times \frac{\Gamma(2-d_U)\Gamma(d_U+\frac{1}{2})}{\Gamma(2d_U)} f_{d_U}(r) \right), \quad (6)$$

where $\xi = \kappa_*/\kappa$ and $\kappa = M_{\text{Pl}}^{-1}$ where M_{Pl} is the Planck mass $M_{\text{Pl}} = 2.4 \times 10^{18}$ GeV, and where $f_{d_U}(r)$ is given by $f_{d_U}(r) = 4\pi \int [d^3 q / (2\pi)^3] e^{-i\mathbf{q}\cdot\mathbf{r}} / (\mathbf{q}^2)^{2-d_U}$. The first term in the parentheses in Eq. (6) is the Newtonian term, while the second term is the ungravity correction. One can easily check that the ungravity correction produces the correct Newtonian potential for the case $d_U = 1$ and $\alpha = 1$ since $f_{d_U}(r) = 1/r$ in this case. However, for d_U different from unity the ungravity effects produce an r dependence of the form $1/r^{2d_U-1}$ which can be differentiated from the effects of ordinary gravitation. An explicit evaluation gives

$$V(r) = -\frac{m_1 m_2 G}{r} \left[1 + \left(\frac{R_G}{r} \right)^{2d_U-2} \right], \\ R_G = \frac{1}{\pi \Lambda_U} \left(\frac{M_{\text{Pl}}}{M_*} \right)^{(1/d_U-1)} \\ \times \left(\frac{2(2-\alpha)}{\pi} \frac{\Gamma(d_U+\frac{1}{2})\Gamma(d_U-\frac{1}{2})}{\Gamma(2d_U)} \right)^{(1/2d_U-2)}, \quad (7)$$

where $M_* = \kappa_*^{-1}$. The choice $d_U < 1$ will produce corrections to the gravitational potential which fall off slower than $1/r$ and thus would modify the very large distance behavior of the gravitational potential, which appears undesirable. Thus a sensible constraint on d_U is $d_U > 1$ in which case the ungravity contribution to the potential falls off faster than $1/r$ and the short distance behavior will be affected. Constraints of conformal invariance in this case require [5,6] $d_U > 1 + s$, where s is the spin of the operator, and thus for a rank one tensor operator $d_U > 2$ and for a rank two $d_U > 3$. Let us now consider a spin zero unparticle operator with $d_U \geq 2$ with coupling of the type $\kappa_* \sqrt{g} T^\mu_\nu O^U / \Lambda_U^{d_U-1}$. In this case the modified Newtonian potential can be gotten from Eq. (7) by replacing $(2-\alpha)$ by 2. With this choice the corrections to the potential can begin with terms of $O(1/r^{(4+2\delta)})$, $\delta > 0$.

The short distance ungravity contribution is constrained by the recent precision submillimeter tests of the gravitational inverse square law [7,8]. The current experiment probes short distances up to around 0.05 mm, and no significant deviation from the inverse square law is seen. However, better precision experiments in the future will be able to explore the parameter space of the model more thoroughly. Returning now to Eq. (7), the quantity R_G may be constrained by experiment. The usual parametrization of the correction to Newtonian gravity in terms of a Yukawa term is not directly suitable for the present case.

Instead, we extrapolate the power law limits in Table I of Ref. [8] to obtain an upper limit on R_G as a function of d_U . The result of this exercise is shown in the left panel of Fig. 1, where the current data excludes the region above the curve. In the right panel in Fig. 1 we present an analysis of the allowed region of the $\Lambda_U - M_*$ parameter space which follows from Eq. (7) when combined with the constraint on R_G . Here the regions below the curves are excluded by the current data.

It is of interest that for $M_* \approx M_{\text{Pl}}$ the value of Λ_U required for proximity to the present bound is very low. In order to assess this possibility and explore the constraint of a conformal fixed point we examine an $SU(N)$ gauge theory with N_f massless Dirac fermions. In this case an infrared fixed point occurs at a coupling [9] $\alpha_* = -4\pi(11N - 2N_f)/[34N^2 - 10NN_f - 3(N^2 - 1)N_f/N]$. For values of N_f close to and below $11N/2$ but above $N_f^c = N(100N^2 - 66)/(25N^2 - 15)$ where the chiral symmetry breaking occurs, one is in the region of a conformal fixed point. In this region the scale Λ_U is roughly given by the scale Λ in Ref. [9]:

$$\Lambda_U \approx M_G \exp \left[-\frac{1}{b\alpha_*} \ln \left(\frac{\alpha_* - \alpha(M_G)}{\alpha(M_G)} \right) - \frac{1}{b\alpha(M_G)} \right], \quad (8)$$

where $b = (11N - 2N_f)/6\pi$. Thus for $N = 3$, the region of the conformal fixed point is $16.5 > N_f > 11.9$. To get an estimate we set $M_G = 1 \times 10^{16}$ GeV, $\alpha(M_G) \approx 0.04$, $N = 3$, and $N_f = 12$ and find an infrared fixed point at $\alpha_* = 0.75$ which gives $\Lambda_U \approx 10^{-11}$ GeV. This is an explicit demonstration that an IR fixed point can occur with Λ_U very small, which is of interest in our analysis.

The modification of gravity discussed here differs from the modification induced by extra dimensions in several aspects [10]. First, in extra dimension Arkani-Hamed–Dimopoulos–Dvali models [10] the corrections to the potential from extra dimensions falls off exponentially at large distances $r/R > 1$, where R is a compactification length scale, while at short $r/R \ll 1$, the r dependence has the form $1/r^{n+1}$ where n is an integer. This is to be contrasted with Eq. (7) where the correction from ungravity

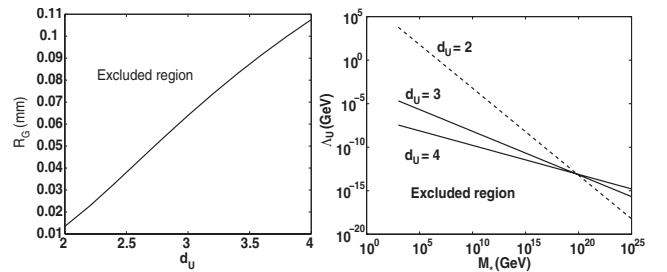


FIG. 1. Left side: allowed region (below curve) for R_G [Eq. (7)] for a region of d_U . Right side: allowed region in $M_* - \Lambda_U$ parameter space (above curves) for various values of d_U . The seeming confluence of the three lines at a single point is not exact.

ity has the r dependence of the form $1/r^{2d_U-1}$ both at short as well as at large distances, and further d_U can take on noninteger values. Further, for the case of extra dimensions the constraint that the physics of the solar system not be modified eliminates $n = 1$ [10], and one has modifications of the Newtonian potential for $n = 2$ of the form $1/r^3$. For the case of the warped extra dimension model [11–13] the correction to the gravitational potential can interpolate between $n = 1$ and $n = 2$ for the case with small warping [13], i.e., between the form $1/r^2$ and $1/r^3$. However, both for the warped and the unwarped dimension case the analytic form of the correction to the potential is significantly different from the one in ungravity. Thus it should be feasible to distinguish between extra dimension models including models with warped dimensions from the ungravity correction to the gravitational potential.

We note that purely kinematical corrections to the Newtonian potential have been computed in general relativity [14]. The sign of this correction as well as its r dependence differs from the one computed here. Further, the effective R_G in this correction is $R_{GR} = G(m_1 + m_2)/c^2$ and is of the size of the Planck length or smaller. Thus in the context of submillimeter experiments these corrections are negligible. Finally, it is interesting to note that renormalization group analyses of quantum gravity in 4 and higher dimensions [15] show that the graviton propagator near an ultraviolet fixed point scales as $\mathcal{G}(p) \sim 1/p^{2(1-\eta/2)}$ where $\eta = 4 - D$ near the fixed point with D the number of space-time dimensions. This propagator has a resemblance to the one that appears in Eq. (4). Of course the typical length scale in quantum gravity is the Planck length while the length scale in ungravity can lie in the submillimeter region and be accessible to experiment.

The interaction operator $\kappa_* \sqrt{g} T_{;\mu}^\mu O^U / \Lambda_U^{d_U-1}$ can also play a role in high energy scattering, and its domain of validity is also constrained from that consideration. Consider the process $f\bar{f} \rightarrow$ scalar unparticle (f is a fermion), which would give a Feynman amplitude $\mathcal{M} = m\bar{u}(p_1)v(p_2)/M_*\Lambda_U^{d_U-1}$, where m is the mass of the fermion and p_1, p_2 are the incident momenta. Using the notation and phase space calculation of [2], we find a cross section

$$\sigma(f\bar{f} \rightarrow \mathcal{U}) = \frac{1}{4s} \left(\frac{m}{M_*} \right)^2 \left(\frac{\sqrt{s}}{\Lambda_U} \right)^{2d_U-2} A_d. \quad (9)$$

(Restriction to the inclusive reaction enables us to probe dimensions $d_U > 2$ without encountering the pole term $\sin(\pi d_U)$ [2].) Since the annihilation to the unparticle proceeds through a single partial wave (s wave), the cross section is bounded by unitarity, $\sigma < 16\pi/s$. From Eqs. (5) and (9) this gives an upper bound on the energy for the compatibility of the unparticle effective Lagrangian with unitarity [16]:

$$\sqrt{s} < \frac{1}{R_*} \left(\sqrt{\frac{64\pi M_{pl}}{A_d m}} \right)^{1/d_U-1}, \quad (10)$$

where we have expressed the unitarity constraint in terms of the quantity $R_* \equiv (1/\Lambda_U)(M_{pl}/M_*)^{1/d_U-1}$ proportional to the quantity R_G defined in Eq. (7). The present upper bound on R_G (see Fig. 1) can be rewritten in terms of R_* : for the region of interest $2 < d_U < 3$, a convenient parametrization $R_*^{\max} \simeq [0.5 + 1.75(d_U - 2)] \times 10^{12} \text{ GeV}^{-1}$ will suffice.

If the exchange of the scalar unparticle is to be consistent with the present Newton's law experiments, yet have a chance of showing up in future experiments, R_* must lie below R_*^{\max} but above (say) $0.1R_*^{\max}$. Inserting this in (10), we obtain the following result: for the worst-case scenario, with the fermion being the top quark, unitarity is not violated up to 1.2 TeV for $2 < d_U < 2.3$. (Above this energy, rescattering corrections are significant.) For the other fermions, of course, the range of validity is larger. Even if we require compatibility with perturbative QCD for the light quarks (including the b -quark), which is a tighter constraint, it allows $2 < d_U < 2.2$ for $\sqrt{(s)_{\max}} \simeq 1.2 \text{ TeV}$. There are similar bounds if T_μ^μ is saturated with the gluon trace anomaly. To sum up, we can maintain compatibility of scale invariance with both high and low energy constraints and simultaneously not rule out seeing corrections to Newton's law in the next generation of gravitational experiments.

Corrections to Coulomb's law can also be similarly computed if one assumes couplings of a vector unparticle operator O_μ^U to the conserved em current J^μ with an interaction of type $(e_*/\Lambda_U^{d_U-1})J^\mu O_\mu^U$, where $d_U \geq 2$. An analysis similar to the above gives the following modified Coulomb potential:

$$\begin{aligned} V_C(r) &= \frac{K e_1 e_2}{r} \left[1 + \left(\frac{R_C}{r} \right)^{2d_U-2} \right], \\ R_C &= \frac{1}{\pi \Lambda_U} \left(\frac{|e_*|}{|e|} \right)^{1/d_U-1} \\ &\quad \times \left(\frac{2}{\pi} \frac{\Gamma(d_U + \frac{1}{2}) \Gamma(d_U - \frac{1}{2})}{\Gamma(2d_U)} \right)^{1/2d_U-2}. \end{aligned} \quad (11)$$

Coulomb's law is not tested beyond the Fermi scale. Setting $R_C < 10^{-13} \text{ cm}$, $d_U = 2$, and keeping $\Lambda_U = 10^{-11} \text{ GeV}$, one finds the constraint $e^*/e < 10^{-11}$. Thus a sensitive probe could unravel the effects of unparticle exchange to Coulomb's law below such scales.

In summary, we have investigated the implications of a scenario where conformal invariance of the hidden sector strictly holds down to very low energies. This requires constraining the dimensionality of the scalar unparticle operators which might couple to the Higgs field so that $d_U > 2$ in order not to spoil the conformal invariance of the hidden sector. Under the assumption that a traceless rank two unparticle operator can couple to the stress tensor, we have computed corrections to the inverse square law and find scale invariant power law corrections which can be discriminated from similar corrections from extra dimension models. We also find the corrections from the

exchange of a scalar operator (with $d_U > 2$) which couples to the trace of the stress tensor. These corrections are testable in future experiments on the submillimeter probes of gravity. We note in passing that the analysis of spin 2 operators in the context of collider phenomenology is discussed in [17]. Corrections to Coulomb's law from the exchange of vector unparticle operators were also computed.

Finally, we remark that the fractional modifications of the inverse square law was studied by Dvali [18] and was seen to lead to strong coupling effects. Dvali's discussion was premised on infrared modifications of gravity which dominate the Einsteinian term at a scale $r \gg r_c$ which leads to the strong coupling referred to above. However, in our case, the modification of gravity at large scales does not dominate the Einsteinian term. In momentum space, the conformal propagator goes like $P^{(2d-4)}$, which for $d > 1$ is suppressed relative to the Einstein case, $P^{(-2)}$, while the propagator considered in the Dvali analysis behaves as $P^{(-2\alpha)}$, ($\alpha < 1$), which indeed dominates the Einsteinian term. Thus our setup escapes the strong coupling effect encountered in [18].

We end with a note of caution, in that a fully consistent formulation of unparticles does not exist and this feature carries over also to ungravity. Nonetheless, if unparticle stuff exists, and one assumes strict conformal invariance of the hidden sector, a new gravitational size force, ungravity, could generate power law modification of gravity, and the new effects fall within the range of future submillimeter tests of gravity. Further, it is possible to distinguish between modifications of corrections due to extra dimensions and corrections from ungravity effects. It should be interesting to build explicit models of the hidden sector where strict conformal invariance is realized while also realizing couplings via a connector sector to the standard model fields of the type discussed here. The strict conformal invariance of the hidden sector required by our model is also suggestive of an AdS5 connection. However, such issues lie outside the scope of this work.

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Note added.—Recently, another work [19] in a similar spirit examined the correction to the long range forces from couplings to the baryon and lepton number currents and found that such corrections are significantly constrained by data.

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