Monte Carlo Studies of Supersymmetric Matrix Quantum Mechanics with Sixteen Supercharges at Finite Temperature

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We present the first Monte Carlo results for supersymmetric matrix quantum mechanics with 16 supercharges at finite temperature. The recently proposed *nonlattice* simulation enables us to include the effects of fermionic matrices in a transparent and reliable manner. The internal energy nicely interpolates the weak coupling behavior obtained by the high temperature expansion, and the strong coupling behavior predicted from the dual black-hole geometry. The Polyakov line asymptotes at low temperature to a characteristic behavior for a deconfined theory, suggesting the absence of a phase transition. These results provide highly nontrivial evidence for the gauge-gravity duality.

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Introduction.—In the past decade we have witnessed the increasing importance of large-N gauge theories in particle physics. For instance, the holographic principle, which was inspired originally by the Bekenstein-Hawking formula for black-hole entropy, has been given a concrete manifestation as a conjectured duality between the strongly coupled large-N gauge theory and weakly coupled supergravity. The best understood example is the so-called AdS/CFT correspondence [1], in which the gauge theory is a 4D conformal field theory (CFT) and the dual supergravity solution is given by the anti–de Sitter (AdS) space. There are numerous extensions to nonconformal cases as well. In particular, large-N gauge theories in low dimensions have been studied intensively at finite temperature, which revealed intriguing connections to black-hole thermodynamics [2–7].

Large-N gauge theories in low dimensions also play an important role in formulating superstring or M theory nonperturbatively based on the idea of matrix models, which was successful in the case of noncritical string theories. For instance, it is conjectured that critical string or M theories can be formulated in terms of matrix models, which can be formally obtained by dimensionally reducing U(N) super Yang-Mills theory in ten dimensions to $D = 0, 1, 2$ dimensions. In particular, the $D = 1$ case corresponds to the matrix theory [8], which is conjectured to describe the M theory [9] microscopically.

In order to confirm these conjectures or to make use of them, it is clearly important to study large-N gauge theories from first principles. Monte Carlo simulation is expected to be very useful for that purpose. In particular, totally reduced models [10] (the $D = 0$ case) have been studied in Refs. [11–15]. The complex Pfaffian, which appears from integration over the fermionic matrices, causes a technical obstacle in numerical simulation, which may however be overcome by a new method proposed in Ref. [15]. In fact, the phase of the Pfaffian is speculated to induce the spontaneous breaking of SO(10) symmetry down to SO(4), a scenario for the dynamical generation of four-dimensional space-time [16] suggested from the Gaussian expansion method [17].

In the $D = 1$ case, some sort of "discretization" is needed in order to put the theory on a computer. Given the well-known problems with the conventional lattice discretization, three of us have proposed a *nonlattice* simulation method [18], which is useful for studying supersymmetric quantum mechanics. The crucial point was that the gauge dynamics is almost trivial in 1D, and therefore we can choose a natural gauge slice nonperturbatively. This allows us to introduce a Fourier mode cutoff Λ without violating the gauge symmetry. In the bosonic case the new method reproduced the lattice results in the continuum limit. In the supersymmetric case with 4 supercharges, it reproduced the results of the high temperature expansion in the continuum. The same model has been studied also by the lattice approach [19].

In this Letter we apply the nonlattice simulation method to the most interesting case with 16 supercharges. While the model is formally identical to the matrix theory, here we focus on its behavior in a different parameter region, which corresponds to the 't Hooft large-N limit at finite temperature. At strong 't Hooft coupling, in particular, the 1D gauge theory has a dual description [2] in terms of a black D0-brane solution in type IIA supergravity in 10D. We are able to confirm some predictions of the gauge-gravity duality from first principles for the first time by solving the strongly coupled dynamics of the gauge theory directly. In doing this, it was crucial to reduce the computational effort by using the rational hybrid Monte Carlo (RHMC) algorithm [20].

As discussed in Ref. [18], our action is nothing but the gauge-fixed action in the continuum except for having a Fourier mode cutoff. Supersymmetry, which is mildly broken by the cutoff, is shown to be restored much faster than the continuum limit is achieved. In fact, the continuum limit is also approached faster than one would naively expect from the number of degrees of freedom. This is understandable from the fact that the modes above the cutoff are naturally suppressed by the kinetic term. A further (albeit technical) advantage of our formulation is that the Fourier acceleration, which eliminates the critical slowing down completely [21], can be implemented *without extra cost* since we are dealing with Fourier modes directly. We consider that all the theoretical and technical merits of the present approach compensate for the superficial increase in the computational effort by the factor of $O(\Lambda)$ compared to the lattice approach [19] with the same number of degrees of freedom.

Simulation techniques.—The model can be obtained formally by dimensionally reducing 10D super Yang-Mills theory to 1D. The action is given by

$$S = \frac{1}{g^2}\int_0^\beta dt\,\mathrm{tr}\Big\{\frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2 + \frac{1}{2}\psi_\alpha D_t \psi_\alpha - \frac{1}{2}\psi_\alpha(\gamma_i)_{\alpha\beta}[X_i, \psi_\beta]\Big\}, \tag{1}$$

where $D_t = \partial_t - i[A(t), \cdot]$ represents the covariant derivative with the gauge field $A(t)$ being an $N \times N$ Hermitian matrix. It can be viewed as a one-dimensional U(N) gauge theory with adjoint matters. The bosonic matrices $X_i(t)$ ($i = 1, \ldots, 9$) come from spatial components of the 10D gauge field, while the fermionic matrices $\psi_\alpha(t)$ ($\alpha = 1, \ldots, 16$) come from a Majorana-Weyl spinor in 10D. The 16×16 matrices γ_i in (1) act on spinor indices and satisfy the Euclidean Clifford algebra $\{\gamma_i, \gamma_j\} = 2\delta_{ij}$. We impose periodic and antiperiodic boundary conditions on the bosons and fermions, respectively. The extent β in the Euclidean time direction then corresponds to the inverse temperature $\beta \equiv 1/T$. The 't Hooft coupling constant is given by $\lambda \equiv g^2 N$, and the dimensionless effective coupling constant is given by $\tilde{\lambda} = \lambda/T^3$. Without loss of generality we set $\lambda = 1$; hence, low (high) T corresponds to strong (weak) coupling strength, respectively.

We fix the gauge by the static diagonal gauge $A(t) = \frac{1}{\beta}\mathrm{diag}(\alpha_1, \ldots, \alpha_N)$, where α_a are chosen to satisfy the constraint $\max_a(\alpha_a) - \min_a(\alpha_a) < 2\pi$ using the large gauge transformation. We have to add to the action a term $S_{\mathrm{FP}} = -\sum_{a<b} 2\ln|\sin\frac{\alpha_a - \alpha_b}{2}|$, which appears from the Faddeev-Popov procedure.

We expand $X_i^{ab}(t) = \sum_{n=-\Lambda}^{\Lambda} \tilde{X}_{in}^{ab} e^{i\omega n t}$ and $\psi_\alpha^{ab}(t) = \sum_{r=-\Lambda'}^{\Lambda'} \tilde{\psi}_{\alpha r}^{ab} e^{i\omega r t}$ into Fourier modes, where $\omega = \frac{2\pi}{\beta}$ and $\Lambda' \equiv \Lambda - 1/2$. The indices n and r take integer and half-integer values, respectively, corresponding to the imposed boundary conditions. Introducing a shorthand notation $(f^{(1)} \cdots f^{(p)})_n \equiv \sum_{k_1+\cdots+k_p=n} f_{k_1}^{(1)} \cdots f_{k_p}^{(p)}$, we can write the action (1) as $S = S_b + S_f$, where

$$\begin{aligned} S_b &= N\beta\Big[\frac{1}{2}\sum_{n=-\Lambda}^{\Lambda}\Big(n\omega - \frac{\alpha_a - \alpha_b}{\beta}\Big)^2 \tilde{X}_{i,-n}^{ba}\tilde{X}_{in}^{ab} - \frac{1}{4}\,\mathrm{tr}([\tilde{X}_i, \tilde{X}_j]^2)_0\Big], \\ S_f &= \frac{1}{2}N\beta\sum_{r=-\Lambda'}^{\Lambda'}\Big[i\Big(r\omega - \frac{\alpha_a - \alpha_b}{\beta}\Big)\tilde{\psi}_{\alpha,-r}^{ba}\tilde{\psi}_{\alpha,r}^{ab} - (\gamma_i)_{\alpha\beta}\,\mathrm{tr}\{\tilde{\psi}_{\alpha,-r}([\tilde{X}_i, \tilde{\psi}_\beta])_r\}\Big]. \end{aligned} \tag{2}$$

The fermionic action S_f may be written in the form $S_f = \frac{1}{2}\mathcal{M}_{A\alpha r;B\beta s}\tilde{\psi}_{\alpha r}^A\tilde{\psi}_{\beta s}^B$, where we have expanded $\tilde{\psi}_{\alpha r} = \sum_{A=1}^{N^2}\tilde{\psi}_{\alpha r}^A t^A$ in terms of U(N) generators t^A. Integrating out the fermions, we obtain the Pfaffian $\mathrm{Pf}\mathcal{M}$, which is complex in general. However, we observe that it is actually real positive with high accuracy in the temperature region studied in the present work. Hence we can replace it by $|\mathrm{Pf}\mathcal{M}| = \det(\mathcal{D}^{1/4})$, where $\mathcal{D} = \mathcal{M}^\dagger\mathcal{M}$.

The trick of the RHMC algorithm is to use the rational approximation $x^{-1/4} \simeq a_0 + \sum_{k=1}^{Q}\frac{a_k}{x+b_k}$, which has sufficiently small relative error within a certain range required by the system to be simulated. (The real positive parameters a_k and b_k can be obtained by a code [22] based on the Remez algorithm.) Then the Pfaffian is replaced by $|\mathrm{Pf}\mathcal{M}| = \int dF dF^* e^{-S_{\mathrm{PF}}}$, where $S_{\mathrm{PF}} = a_0 F^* F + \sum_{k=1}^{Q} a_k F^*(\mathcal{D} + b_k)^{-1}F$, using the auxiliary complex variables F, the so-called pseudofermions.

At this point we apply the usual prescription of the HMC algorithm to the whole system as described in Ref. [18], except that now we introduce the momentum variables conjugate to the pseudofermions F as well as the bosonic matrices $\tilde{X}_i$ and the gauge variables α_a. When we solve the auxiliary classical Hamiltonian dynamics, it is important to apply the Fourier acceleration [21] to all the variables. The main part of the computation comes from solving a linear system $(\mathcal{D} + b_k)\chi_k = F$ ($k = 1, \ldots, Q$). We solve the system for the smallest b_k using the conjugate gradient method, which reduces the problem to the iterative multiplications of $\mathcal{M}$ to a pseudofermion field, each of which requires $O(\Lambda^2 N^3)$ arithmetic operations. The solutions for larger b_k's can be obtained as by-products using the idea of the multimass Krylov solver [23]. This avoids the factor of Q increase of the computational effort.

Infrared instability.—Since the integration domain for the bosonic matrices is noncompact, the convergence of the partition function is not obvious. In particular, there exists a potential danger in the flat direction corresponding to commuting matrices. Such an issue has been addressed in the totally reduced model [11,12,16,24]. In the present $D = 1$ case, let us expand the cutoff theory (2) around

the commuting background $\tilde{X}_{i0} = \mathrm{diag}(x_{i1}, \ldots, x_{iN})$. When both T and all of $|x_{ia} - x_{ib}|$ are large, the fluctuations become very massive, and the one-loop approximation is justified. The one-loop effective action for the moduli parameters x_{ia} and α_a can be easily obtained as

$$W = \sum_{a<b} 4 \log\left(\frac{\prod_n \{(2\pi n - \alpha_{ab})^2 + (\beta x_{ab})^2\}}{\prod_r \{(2\pi r - \alpha_{ab})^2 + (\beta x_{ab})^2\}} \right), \qquad (3)$$

where $\alpha_{ab} \equiv \alpha_a - \alpha_b$ and $x_{ab} \equiv \sqrt{(x_{ia} - x_{ib})^2}$. In Eq. (3) we have omitted terms independent of x_{ab}, which actually vanish in the $\Lambda \to \infty$ limit. When $x_{ab} \ll T$ and $\alpha_{ab} \ll 2\pi$, the $n = 0$ term dominates and yields a logarithmic attractive potential $W \simeq \sum_{a<b} 4 \log\{(\alpha_{ab})^2 + (\beta x_{ab})^2\}$ among α_a and among x_{ia}. This agrees with the well-known result in the totally reduced bosonic model [12], which describes the high temperature limit of the present model. In fact one obtains $x_{ab} \sim T^{1/4}$ according to the high temperature expansion (HTE) [25]. On the other hand, when $T \ll x_{ab} \ll 2\pi\Lambda T$, the denominator and the numerator in Eq. (3) cancel each other almost completely. This implies the existence of an instability.

As T is lowered, the instability region approaches the typical value of $x_{ab} \sim T^{1/4}$ representing the high temperature behavior. However, after taking the sum over all the pairs of indices (a, b), a tiny difference between the denominator and the numerator is roughly raised to the power of N^2. This shifts the lower edge of the instability region by a factor of N. We found empirically that the instability can be avoided by taking $N \gtrsim \frac{6}{T}$, which is consistent with the above considerations.

For N satisfying this inequality, we expect the finite-N effects to be $O(\frac{1}{N^2})$ as observed in HTE [25]. The finite-Λ effects, on the other hand, are expected to become negligible for $\Lambda \gtrsim \frac{c}{T}$, where empirically we find $c \simeq 2$ for the energy and $c \simeq 4$ for the Polyakov line.

Results.—In Fig. 1 we plot the absolute value of the Polyakov line, which is the order parameter for the spontaneous breaking of the U(1) symmetry. Unlike in the bosonic case [6,26,27], where the phase transition associated with the symmetry breaking occurs around $T \simeq 1$, the Polyakov line changes smoothly [28] for the range of T investigated. At low T it can be fitted nicely to the asymptotic behavior

$$\langle |P| \rangle \equiv \left\langle \left| \frac{1}{N} \sum_{i=1}^{N} e^{i\alpha_i} \right| \right\rangle = \exp\left(-\frac{a}{T} + b \right), \qquad (4)$$

which is characteristic to a deconfined theory. This implies the absence of a phase transition in the present case, as predicted by the gauge-gravity duality [3,4,7].

In Fig. 2 we plot the internal energy defined by $E = \frac{\partial}{\partial \beta}(\beta \mathcal{F})$, where $\mathcal{F}$ is the free energy of the system. In practice, we calculate it using a formula, which follows from a simple scaling argument [19]. In our case it reads

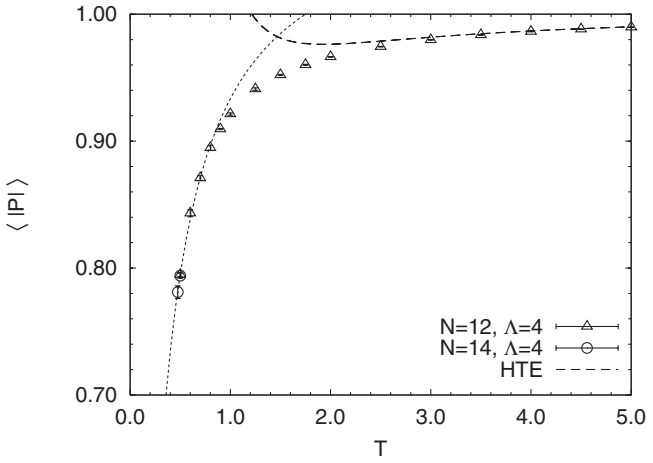

FIG. 1. The absolute value of the Polyakov line is plotted against T. The dashed line represents the result obtained by HTE up to the next leading order for $N = 12$ [25]. The dotted line represents a fit to Eq. (4) with $a = 0.15$ and $b = 0.072$.

$E = -3T[\langle S_b \rangle - \frac{9}{2}\{(2\Lambda + 1)N^2 - 1\}]$. Our results interpolate smoothly between the weak coupling behavior—calculated by the HTE up to the next leading order [25]—and the strong coupling behavior $E/N^2 = 7.4T^{2.8}$ predicted by gauge-gravity duality [2] from dual black-hole geometry [29]. The power-law behavior seems to set in at $T \simeq 0.5$, which is reasonable since the effective coupling constant is given by $\tilde{\lambda} = 1/T^3$ in our convention. More data points at lower temperature are desirable to confirm the duality unambiguously.

In Ref. [5] the energy was obtained by the Gaussian expansion method at the *leading* order, and the power-law

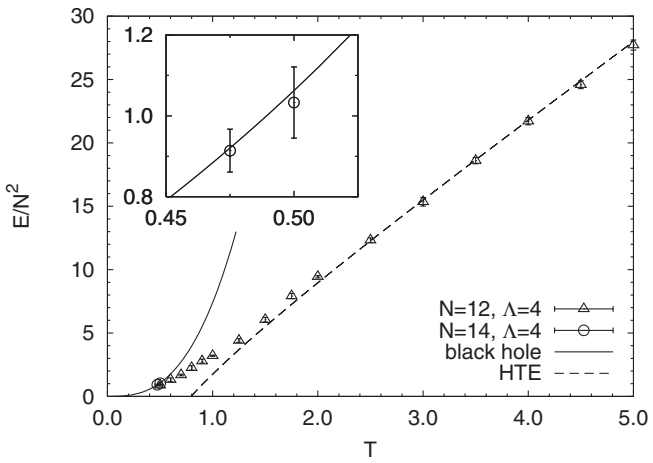

FIG. 2. The internal energy is plotted against T. The dashed line represents the result obtained by HTE up to the next leading order for $N = 12$ [25]. The solid line represents the asymptotic behavior at small T predicted by the gauge-gravity duality based on the results obtained from dual black-hole geometry. The upper left panel magnifies the region, where the power-law behavior seems to set in.

behavior $E/N^2 = 3.4T^{2.7}$ was seen within $0.25 \lesssim T \lesssim 1$. Their results are in reasonable agreement with our data at $T \sim 1$, but disagree at lower temperature. Higher order calculations in the Gaussian expansion method are worthwhile.

Summary and outlook.—We have presented the first Monte Carlo results for the maximally supersymmetric matrix quantum mechanics. New techniques enabled us to study the low temperature behavior, which was not accessible by HTE. As the temperature was lowered, we observed the infrared instability, which is eliminated, however, by increasing N. We gave a natural explanation to this phenomenon based on the one-loop effective action.

The internal energy asymptotes at low T to the power-law behavior obtained from 10D nonextremal black-hole geometry. Note that these are based on independent calculations from both sides of the gauge-gravity duality, and thus provide highly nontrivial evidence for it. They also imply that the microscopic origin of the Bekenstein-Hawking entropy for the 10D nonextremal black hole can be accounted for by the open strings attached to the D0-branes, which are described by the gauge theory.

Assuming the duality to hold in the stronger sense, one may investigate the quantum and stringy corrections to black-hole thermodynamics from the gauge theory side as finite-N and finite-$\tilde{\lambda}$ effects. In particular, it would be interesting to understand the physical meaning of the observed infrared instability from that perspective.

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