P H Y S I C A L   R E V I E W   L E T T E R S

Spatial Line Nodes and Fractional Vortex Pairs in the Fulde-Ferrell-Larkin-Ovchinnikov Vortex State of Spin-Singlet Superconductors

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A Zeeman magnetic field can induce a Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase in spin-singlet superconductors. Here we argue that there is a nontrivial solution for the FFLO vortex phase that exists near the upper critical field in which the wave function has only spatial line nodes that form intricate and unusual three-dimensional structures. These structures include a crisscrossing lattice of two sets of nonparallel line nodes. We show that these solutions arise from the decay of conventional Abrikosov vortices into pairs of fractional vortices. We propose that neutron scattering studies can observe these fractional vortex pairs through the observation of a lattice of $1/2$ flux quanta vortices. We also consider related phases in noncentrosymmetric superconductors.

A Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase predicted in Refs. [1,2] appears to have been discovered in CeCoIn$_5$ in the high magnetic field region of the superconducting phase diagram [3,4]. This discovery has generated tremendous interest both experimentally and theoretically [5]. FFLO phases have also been argued to be of importance in understanding ultracold atomic Fermi gases [6] and in the formation of color superconductivity in high density quark matter [7]. The understanding of these phases has become a relevant and topical pursuit in physics. One central issue is the role vortices play in these phases: in CeCoIn$_5$ the FFLO phase appears deep within a vortex phase [3,4]; and ultracold atomic Fermi gases can be rotated to create vortices within an FFLO phase [8].

Here we address the nature of the FFLO vortex phase. Previous studies have concluded that the superconducting gap function in this phase is, for example, $\Delta(\boldsymbol{R}) = \cos(qz)\phi_n(\boldsymbol{r})$ where the magnetic field is applied along the $\hat{z}$ direction, $\hat{z} \cdot \boldsymbol{r} = 0$, and $\phi_n(\boldsymbol{r})$ describes a vortex lattice constructed from a Landau level (LL) with index n [9–13]. This solution has intersecting spatial nodes along planes perpendicular to the z axis and along the vortex lines parallel to the z axis. We show that there is another realistic solution for the FFLO vortex phase in which there are only spatial line nodes in the gap function. We show that the existence of this solution is a consequence of the decay of conventional vortices into pairs of fractional vortices. These fractional vortices exist because of the broken translational symmetry inherent in FFLO superconductors. By suitably choosing an order parameter that correctly exhibits this broken translational symmetry, these fractional vortices naturally appear within the theory. We propose that a small angle neutron scattering (SANS) measurement of the resulting magnetic field distribution may observe a lattice of $1/2$ flux quanta near to the upper critical field. We further argue that this phase is stable within weak-coupling theories of superconductivity and consider related phases in noncentrosymmetric (NC) superconductors.

We use a phenomenological approach pioneered by Buzdin and Kachkachi to describe the FFLO phase [11–14] and extend it to include NC superconductors. We begin with the following free energy

$$F = \int d^3R\{\alpha|\Delta|^2 + \beta|\Delta|^4 + \nu|\Delta|^6 + \kappa|\boldsymbol{D}\Delta|^2$$
$$+ \delta|\boldsymbol{D}^2\Delta|^2 + \mu|\Delta|^2|\boldsymbol{D}\Delta|^2 + \eta[(\Delta^*)^2(\boldsymbol{D}\Delta)^2$$
$$+ (\Delta)^2(\boldsymbol{D}^*\Delta^*)^2] + \epsilon\boldsymbol{B} \cdot [\Delta^*(\boldsymbol{D}\Delta) + \Delta(\boldsymbol{D}\Delta)^*]\}, \quad (1)$$

where $\boldsymbol{D} = -i\boldsymbol{\nabla} - 2e\boldsymbol{A}$ and $\boldsymbol{B} = \nabla \times \boldsymbol{A}$. The coefficients that appear in this free energy are typically determined from a microscopic BCS theory [14]. The ϵ term applies only to NC superconductors. It results in the helical phase discussed previously [15]. In this phase, the gap function becomes $\Delta(\boldsymbol{R}) = \psi_1 e^{i\boldsymbol{q}\cdot\boldsymbol{R}}$. The orientation of $\boldsymbol{q}$ is determined by the free energy invariant denoted by ϵ in Eq. (1). We have chosen this invariant so that the theory applies to Li$_2$Pt$_3$B with point group O [16]. Consequently, $\boldsymbol{q}$ is parallel to $\boldsymbol{B}$. With $\epsilon = 0$, Eq. (1) has been justified previously [14].

We consider a magnetic field along the $\hat{z}$ direction and ignore screening currents in determining the high field ground state structure of the gap function (this is reasonable for strongly type II superconductors). In the normal state there will be translational invariance along the magnetic field direction. Therefore, Fourier modes along this direction will be eigenstates of the linear gap equation. Typically, the eigenstate with the lowest energy corresponds to the Fourier mode $q = 0$. However, in FFLO superconductors, the eigenstate with the lowest energy has finite q. The states $\pm q$ are degenerate and this degeneracy is broken by nonlinear terms in the free energy. Consequently, to describe the FFLO phase near the upper critical field, it suffices to keep the two modes $\pm q$. We therefore write $\Delta(\boldsymbol{R}) = \psi_1(\boldsymbol{r})e^{iqz} + \psi_2(\boldsymbol{r})e^{-iqz}$, where $\boldsymbol{r}$ is orthogonal to the magnetic field and $\boldsymbol{q}$ is parallel to the field. This yields the following free energy for the new

order parameter $\boldsymbol{\psi} = (\psi_1, \psi_2)$:

$$F = L_z \int d^2r \{\alpha_1|\psi_1|^2 + \alpha_2|\psi_2|^2 + \beta_1|\boldsymbol{\psi}|^4 + \beta_2|\psi_1|^2|\psi_2|^2 + \nu|\boldsymbol{\psi}|^6 + 6\nu|\boldsymbol{\psi}|^2|\psi_1|^2|\psi_2|^2 + \kappa_1(|\boldsymbol{D}\psi_1|^2 + |\boldsymbol{D}\psi_2|^2)$$
$$+ \kappa_2(|\boldsymbol{D}^2\psi_1|^2 + |\boldsymbol{D}^2\psi_2|^2) + \mu(|\boldsymbol{D}\psi_1|^2|\psi_1|^2 + |\boldsymbol{D}\psi_2|^2|\psi_2|^2) + \eta[(\boldsymbol{D}\psi_1)^2(\psi_1^*)^2 + (\boldsymbol{D}\psi_2)^2(\psi_2^*)^2 + \text{c.c.}]$$
$$+ 4\eta[(\boldsymbol{D}\psi_1)\psi_1^*(\boldsymbol{D}\psi_2)\psi_2^* + \text{c.c.}] + \mu[(\boldsymbol{D}\psi_1)\psi_1^*(\boldsymbol{D}\psi_2)^*\psi_2 + \text{c.c.}] + \mu[|\boldsymbol{D}\psi_1|^2|\psi_2|^2 + |\boldsymbol{D}\psi_2|^2|\psi_1|^2]\}, \quad (2)$$

where L_z is the size of the system along the z direction, $\boldsymbol{D} = (D_x, D_y)$, and c.c. means complex conjugate. The coefficients in Eq. (2) now depend upon q [17]. For FFLO superconductors $\alpha_1 = \alpha_2$. Equation (2) should be optimized with respect to q and we assume this has been done. This ensures that there is no net current flowing along the z direction [11,15].

The choice of order parameter $\boldsymbol{\psi}$ manifestly exhibits the broken translational symmetry that characterizes the FFLO state. This broken symmetry is hidden when considering Δ. By considering $\boldsymbol{\psi}$ explicitly, new and general features of the theory appear naturally. In particular, notice that Eq. (2) is independent of separate rotations of the phases of ψ_1 and ψ_2, revealing a global $U(1) \times U(1)$ gauge invariance. This follows from translational invariance of the normal state along the z direction and usual gauge invariance. In particular, consider a general term $\psi_1^n \psi_2^m (\psi_1^*)^p (\psi_2^*)^q$ appearing in the free energy; usual gauge invariance requires $n + m - p - q = 0$ and translational invariance requires $n - m - p + q = 0$. These two conditions imply that $n = p$ and $m = q$ which leads to the $U(1) \times U(1)$ invariance. A $U(1) \times U(1)$ symmetry has been examined to discuss possible topological structures in two-band superconductors [18]. Related topological structures have also been discussed in other contexts [19–21].

The vortices of a $U(1) \times U(1)$ theory can be classified [18] by two integers (n, m) which denote a $2n\pi$ phase change in $\psi_1(\boldsymbol{r})$ and a $2m\pi$ phase change in $\psi_2(\boldsymbol{r})$ as the vortex core is encircled. Of particular interest here are the $(1, 1)$, $(1, 0)$, and $(0, 1)$ vortices. The $(1, 1)$ vortex is the usual Abrikosov vortex and it contains a magnetic flux of Φ_0 (the usual flux quantum). In the FFLO phase, when $|\psi_1| = |\psi_2|$ (often called the LO phase), the corresponding $(1, 0)$ vortex contains a fractional flux $\Phi_0/2$ [18]. We are interested in the appearance of bound pairs of these vortices in the vortex lattice phase. Consequently, we consider generalized Abrikosov vortex lattice states and show the usual FFLO vortex solution is often unstable to a new lattice solution. In this new solution each of the conventional $(1, 1)$ vortices decays into a pair of $(1, 0)$ and $(0, 1)$ vortices.

We now turn to an analysis valid near the upper critical field. The vortex solutions are eigenstates of the operator $\boldsymbol{D}^2 = (-i\boldsymbol{\nabla} - 2e\boldsymbol{A})^2$ which has eigenvalues $(2n + 1)/l^2$ and $l^2 = \Phi_0/(2\pi H)$ and $n = 0, 1, 2, \ldots$ is the LL index. The usual BCS theory predicts a $n = 0$ LL solution is the most stable solution, but it has been shown that for FFLO

superconductors $n > 0$ LL solutions can also be stable [5]. It is well known that the LL exhibit a macroscopic degeneracy. Abrikosov exploited this degeneracy to construct a vortex lattice solution which we label as $\phi_n(\boldsymbol{r})$ [22]. We label the unit cell of the vortex lattice by the lattice vectors $\boldsymbol{a} = (a, 0)$ and $\boldsymbol{b} = (b \cos\alpha, b \sin\alpha)$. We take $\boldsymbol{r}$, a, and b to be in units l. Then $ab \sin\alpha = 2\pi$ gives one flux quantum per unit cell. In this basis, we set $\boldsymbol{\psi}(\boldsymbol{r}) = [\eta_1 \phi_n(\boldsymbol{r}), \eta_2 \tilde{\phi}_n(\boldsymbol{r} + \boldsymbol{\tau})]$, where $\tilde{\phi}_n(\boldsymbol{r} + \boldsymbol{\tau}) = e^{-i\tau_y x}\phi_n(\boldsymbol{r} + \boldsymbol{\tau})$. The additional phase factor that appears in $\tilde{\phi}_n$ ensures that both ψ_1 and ψ_2 lie in the same LL. It appears as a consequence of applying a translation in a uniform magnetic field. The new feature in this analysis is the appearance of the translation vector $\boldsymbol{\tau} = (\tau_x, \tau_y)$ that displaces the nodes of the two components (ψ_1, ψ_2). Previous results can be recovered with $\boldsymbol{\tau} = 0$ [11–13]. A similar solution has been used for UPt$_3$ [23]. Substituting the above solution for $\boldsymbol{\psi}(\boldsymbol{r})$ yields the free energy density (here we have considered only the $n = 0$ LL)

$$f = \tilde{\alpha}_1|\eta_1|^2 + \tilde{\alpha}_2|\eta_2|^2 + \tilde{\beta}_1 \beta_A(0)|\boldsymbol{\eta}|^4 + [(2\tilde{\beta}_1 + \tilde{\beta}_2)\beta_A(\boldsymbol{\tau})$$
$$- 2\tilde{\beta}_1 \beta_A(0)]|\eta_1|^2|\eta_2|^2 + \nu\gamma_A(0)|\boldsymbol{\eta}|^6$$
$$+ \nu[9\gamma_A(\boldsymbol{\tau}) - 3\gamma_A(0)]|\boldsymbol{\eta}|^2|\eta_1|^2|\eta_2|^2, \quad (3)$$

where the coefficients $\tilde{\alpha}_1$, $\tilde{\alpha}_2$, $\tilde{\beta}_1$, and $\tilde{\beta}_2$ do not depend upon the vortex lattice structure [24]. The vortex lattice structure appears entirely in the generalized Abrikosov coefficients $\beta_A(\boldsymbol{\tau}) = 2\pi \int_{\text{u.c.}} d^2r |\phi_0(\boldsymbol{r})|^2|\phi_0(\boldsymbol{r} + \boldsymbol{\tau})|^2$ and $\gamma_A(\boldsymbol{\tau}) = (2\pi)^2 \int_{\text{u.c.}} d^2r |\phi_0(\boldsymbol{r})|^4|\phi_0(\boldsymbol{r} + \boldsymbol{\tau})|^2$. Using the approach of Ref. [12] yields

$$\beta_A(\boldsymbol{\tau}) = \sum_{\mathbf{G}} e^{-G^2/2} e^{i\mathbf{G}\cdot\boldsymbol{\tau}} \quad (4)$$

and

$$\gamma_A(\boldsymbol{\tau}) = \sum_{\mathbf{G},\mathbf{G}'} e^{i\hat{z}\cdot(\mathbf{G}\times\mathbf{G}')/2} e^{i\mathbf{G}\cdot\boldsymbol{\tau}} e^{-(G^2+G'^2+\mathbf{G}\cdot\mathbf{G}')/2}, \quad (5)$$

where $\mathbf{G} = m\mathbf{g}_1 + n\mathbf{g}_2$ (n, m are any integer), $\mathbf{g}_1 = \sqrt{2\pi\sigma}\hat{x} - \sqrt{2\pi\rho^2/\sigma}\hat{y}$, $\mathbf{g}_2 = \sqrt{2\pi/\sigma}\hat{y}$, and $\rho + i\sigma = e^{i\alpha}b/a$. Below, the ground state lattice structures are numerically found by minimizing Eq. (3) with respect ρ, σ, and $\boldsymbol{\tau}$.

Single-q to multiple-q transition in NC superconductors.—Here, $\tilde{\alpha}_1 \neq \tilde{\alpha}_2$. When $\tilde{\alpha}_1 < 0$ and $\tilde{\alpha}_2 > 0$, $\eta_1 \neq 0$ and $\eta_2 = 0$, the stable structure is the usual hexagonal vortex lattice. If $\tilde{\beta}_2 < 2\tilde{\beta}_1[\beta_A^{(0)}(0) - \beta_A^0(\boldsymbol{\tau})]/\beta_A^0(\boldsymbol{\tau})$ then

a second transition can occur into a state in which both η_1 and η_2 are nonzero. This transition has been found within weak-coupling theories of NC superconductors [25–27]. This phase has two possible solutions. The first has $\tau = 0$ and remains a conventional hexagonal lattice. This occurs when $2\tilde{\beta}_1 + \tilde{\beta}_2 < 0$. The second solution has $\tau = (a + b)/3$ and occurs for $2\tilde{\beta}_1 + \tilde{\beta}_2 > 0$. To address which of these possibilities occur, we note that weak-coupling microscopic studies show that the phase diagram contains a line along which $\beta_2 = 0$ [26,27]. This implies that the finite $\tau = (a + b)/3$ phase is the ground state.

The spatial nodes of $\Delta(\mathbf{R}) = e^{iqz}\eta_1\phi_0(\mathbf{r}) + e^{-iqz}\eta_2\tilde{\phi}_0(\mathbf{r} + \tau)$ are given by $|\eta_1\phi_0(\mathbf{r})| = |\eta_2\phi_0(\mathbf{r} + \tau)|$ and $\cos[qz + (\theta_1 - \theta_2)/2] = 0$, where $\theta_1 = \theta_1(\mathbf{r})$ is the phase of $\phi_0(\mathbf{r})$ and $\theta_2(\mathbf{r})$ is the phase of $\tilde{\phi}_0(\mathbf{r} + \tau)$. For small η_2, these zeroes lie on small circles surrounding each of the zeroes of ψ_1. Around these circles, the phase $\theta_1(\mathbf{r}) = \phi$ since we are encircling a vortex core of ψ_1 (here, ϕ is the polar angle of the circle) and $\theta_2 \approx$ const since we are far away from the zeros of $\psi_2(\mathbf{r})$. Consequently, the nodes of $\Delta(\mathbf{R})$ are given by $qz = \phi/2 + n\pi + c$, where c is a constant and n is any integer. This describes the equation of a helix spiraling about the z direction. This is depicted in Fig. 1. As η_2 grows, the pitch of the helix grows larger. It is possible for two adjacent helices to merge for large enough η_2. This results in a crisscrossing lattice of line nodes like that discussed below in the context of the FFLO case. This analysis reveals that the $(n, m) = (1, 1)$ Abrikosov vortices have each separated into a pair of $(1, 0)$ and $(0, 1)$ vortices. The $(1, 0)$ vortices appear where $\psi_1(\mathbf{r}) = 0$ and the $(0, 1)$ vortices appear where $\psi_2(\mathbf{r}) = 0$.

Second order transition into the FFLO phase.—Here, $\alpha_1 = \alpha_2$, $\nu = 0$, and there is a second order transition from the normal state into the FFLO state. There are three possible solutions for this phase. The first has $\eta_2 = 0$ and $\eta_1 \neq 0$; this is the FF (or single-q) state with a conventional hexagonal lattice. This phase is stable when $(2\tilde{\beta}_1 + \tilde{\beta}_2)\beta_A(\tau) - 2\tilde{\beta}_1\beta_A(0) > 0$ for all τ. One of the other two solutions is stable if $(2\tilde{\beta}_1 + \tilde{\beta}_2)\beta_A(\tau) - 2\tilde{\beta}_1\beta_A(0) < 0$
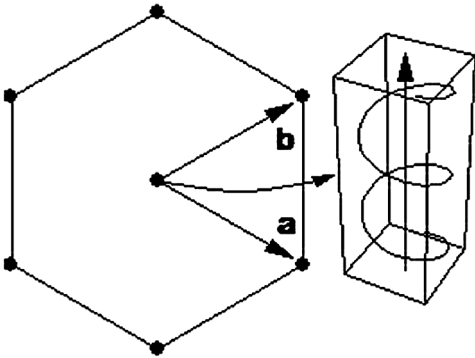

FIG. 1.   Helical spatial line nodes in the gap for the multiple-q phase of NC superconductors. The centers of the helices form a 2D hexagonal lattice perpendicular to the field.

for any τ. The second solution corresponds to $|\eta_1| = |\eta_2|$ with $\tau = 0$ and is the LO (or multiple-q) phase with a conventional hexagonal lattice. This state requires $2\tilde{\beta}_1 + \tilde{\beta}_2 < 0$ to be stable. The final state corresponds to $|\eta_1| = |\eta_2|$ with a rectangular unit cell for which $b/a = \sqrt{3}$ and $\tau = (a, b)/2$ for $\tilde{\beta}_2 = 0$. More generally, we find the same lattice but with $b/a \neq \sqrt{3}$ (this includes a possible first order FFLO transition). These solutions are stable for $2\tilde{\beta}_1 + \tilde{\beta}_2 > 0$. To understand which of these states may be stable within microscopic theories, note that the calculations of Ref. [11] imply that there is a line in the phase diagram along which $\tilde{\beta}_2 = 0$ in the weak-coupling theory of a clean s-wave superconductor with vortices. Near this line, $\tau = (a, b)/2$ gives the stable phase. Whenever the FF phase is close in energy to the LO phase (that is $|\tilde{\beta}_2| \ll \tilde{\beta}_1$), then the LO vortex phase with $\tau = (a, b)/2$ is the stable vortex phase since $\beta_A(\tau) \leq \beta_A(0)$ for any $\tau \neq 0$. It appears that this is generic for weak-coupling theories where varying gap symmetry, impurities, and vortices lead to a variety of different phase diagrams containing both the FF and LO phases [11,13,14,28].

We now focus on the LO phase with $\tau = (a, b)/2$. This phase can be understood as having conventional $(1, 1)$ Abrikosov vortices that have each separated into a pair of $(1, 0)$ and $(0, 1)$ vortices. As discussed previously the $(1, 0)$ and $(0, 1)$ vortices in this LO phase can be interpreted as containing flux $\Phi_0/2$. To understand if this may manifest itself experimentally, we have performed an Abrikosov analysis [22] on Eq. (2) to determine the field distribution $h_s(\mathbf{r})\hat{z}$ due to screening currents to lowest order in the gap function. This results in $h_s(\mathbf{r}) \propto |\psi_1(\mathbf{r})|^2 + |\psi_2(\mathbf{r})|^2$. Consequently, for $\tilde{\beta}_2 = 0$, $h_s(\mathbf{r})$ has a hexagonal symmetry even though the nodes of $\psi_1(\mathbf{r})$ and $\psi_2(\mathbf{r})$ separately form a rectangular lattice with $b/a = \sqrt{3}$. A measurement of the hexagonal unit cell lattice vector will yield a flux per unit cell that is $\Phi_0/2$ (this generalizes to nonhexagonal unit cell geometries). This can be seen through SANS measurements by observing the Bragg peaks of the vortex lattice with neutrons that have momenta perpendicular to the applied field. We emphasize that our solution is valid at H_{c2} and at lower fields it is possible that the $\Phi_0/2$ vortices are more tightly bound (e.g., $\tau \neq 0$ but $|\tau| < |a + b|/2$).

Here we give the positions of the line nodes for $\tau = (a, b)/2$ and $b/a > 3$. In the x, y plane the point zeros lie along the lines $y_1 = -3b/4$, $y_2 = -b/4$, $y_3 = b/4$, and $y_4 = 3b/4$ (the unit cell has doubled along the y direction). The x, z coordinates (measured in units a, π/q, respectively) for these four lines are given by $z_1 = n + 1/2 - x_1/2$, $z_2 = n + 1/2 + x_2/2$, $z_3 = n - x_3/2$, and $z_4 = n + x_4/2$. This results in a lattice of crisscrossing nodal lines as viewed from a direction normal to the y axis (Fig. 2).

For the FFLO phase in CeCoIn$_5$, the $\tau = (a, b)/2$ solutions may help to understand some experiments [5]. In particular, measurements in the FFLO vortex phase find that the thermal conductivity parallel to the applied field is
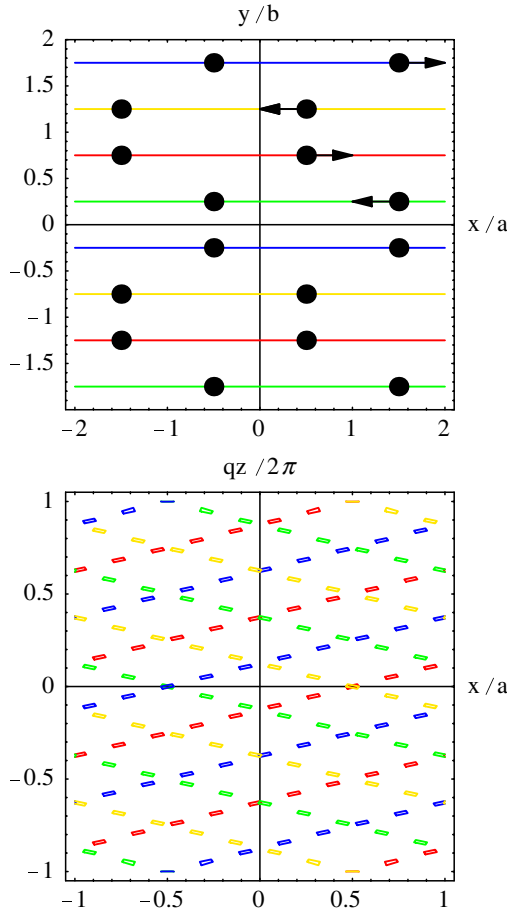
FIG. 2 (color online).   Crisscrossing lattice of nodal lines in the FFLO vortex phase with $\tau = (a, b)/2$. The ● in the top figure shows the nodes perpendicular to the applied magnetic field for $z = 0$. As z is changed slightly, these nodes move as illustrated by the arrows in the upper right of this figure. The lower figure shows a cross section as seen from the y direction. The different colored lines correspond to the nodal lines with different y positions.

greater than that perpendicular to the applied field [29]. This is not expected for a gap function with spatial plane nodes perpendicular to the field (which occurs if $\tau = 0$). However, it can be qualitatively understood if $\tau = (a, b)/2$. Note that magnetic order in the FFLO nodal planes has been proposed [30] and this may account for the thermal conductivity results when $\tau = 0$.

In conclusion, we have argued that the vortex lattice phases in FFLO and NC superconductors contain gap functions with spatial line nodes that form a variety of three-dimensional spatial configurations. These configurations include a lattice of helices in NC superconductors and a crisscrossing lattice of nodal lines in FFLO superconductors. These structures stem from the breakup of conventional vortices into pairs of fractional vortices. SANS studies of the magnetic field distribution can provide evidence for these structures.

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