Effects of Interactions in Transport through Aharonov-Bohm-Casher Interferometers

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We study the conductance through a ring described by the Hubbard model (such as an array of quantum dots), threaded by a magnetic flux and subject to Rashba spin-orbit coupling (SOC). We develop a formalism that is able to describe the interference effects as well as the Kondo effect when the number of electrons in the ring is odd. In the Kondo regime, the SOC reduces the conductance from the unitary limit, and, in combination with the magnetic flux, the device acts as a spin polarizer.

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Advances in semiconductor technology have provided useful tools to test fundamental concepts of quantum physics, such as the superposition principle and the existence of topological phases [1]. Beautiful demonstrations of these are studies of the Aharonov-Bohm (AB) effect [2] in mesoscopic rings, particularly with embedded quantum dots (QDs) [3,4]. The effect of interactions in these systems is still a matter of debate. Despite the enormous effort to describe transport through interacting regions [5], at present we do not have a unified procedure to extend the results of the single particle case to many-body cases. In the case of transport through interacting rings, even knowing the exact eigenstates of the ring, there is no simple procedure to calculate the conductance G. When the coupling V of the ring to the conducting leads is small, Jagla and Balseiro (JB) used a perturbative expression in V for G that is exact for any V in the noninteracting limit [6]. Similar equations were used recently, assuming that a Zeeman term destroys the Kondo effect in the system [7,8]. Another expression in order V^2 was proposed last year [9]. Unfortunately, these expressions are not valid in the Kondo regime, in which the number of electrons in the ring is odd, because the resulting Kondo physics cannot be described by perturbation theory in V. The ideal conductance in the Kondo regime was recovered by mapping the model into an impurity Anderson model, but in this formulation interference effects were lost [8].

Recently, the Aharonov-Casher (AC) effect [10], the charge-spin dual of the AB effect, has been demonstrated experimentally in semiconductor mesoscopic rings [11,12]. The AC phases are originated due to the Rashba spin-orbit coupling (SOC) in the ring, resulting from electronic motion in the presence of an electric field normal to the plane of the ring. The interference between electrons of given spin traveling clockwise and anticlockwise produces a strong modulation of the electronic current through the device. Recent theoretical research [13] has successfully explained the modulation of the conductance in terms of noninteracting electrons. However, the single-electron picture turns out to be inadequate to describe electronic transport in the strongly interacting case, particularly in the Kondo regime, as we will show.

In this Letter, we describe a systematic procedure to calculate the equilibrium conductance G through a ring of an interacting system weakly coupled to conducting leads that takes into account both the effects of interference and correlations in the presence of a magnetic flux and SOC. Using a non-Abelian gauge transformation (NAGT), we show that for on-site interactions, the SOC can be absorbed in opposite AC phases for spin up and down in an adequately chosen quantization axis. For a Hubbard model (that describes a ring of an even number of QDs) in the absence of SOC, G vanishes when the magnetic flux amounts to half a flux quantum. For other fluxes in the Kondo regime, G reaches the unitary limit (ideal conductance [4]). When the SOC is turned on, the ideal conductance is destroyed and G shows a strong spin dependence in this regime.

Our first task is to derive the appropriate extension to the Hubbard model to include the SOC in an adequate representation that simplifies our subsequent calculations. To illustrate the procedure, it is easier to begin with noninteracting electrons in the continuum. The correct Hamiltonian for this case was derived by Meijer *et al.* [14]. The SOC is $H_{\mathrm{SOC}} = \alpha' \vec{\sigma} \cdot \vec{E} \times (\vec{p} - e\vec{A})$, where α' is the Rashba constant and $\vec{E}$ is the electric field, which in our case is in the z direction, perpendicular to the plane of the ring. Including SOC and the orbital effects of the magnetic field, but neglecting the Zeeman term (usually several orders of magnitude smaller than the Kondo energy scale in QDs [4]), the Hamiltonian can be written in the form [13(b)]

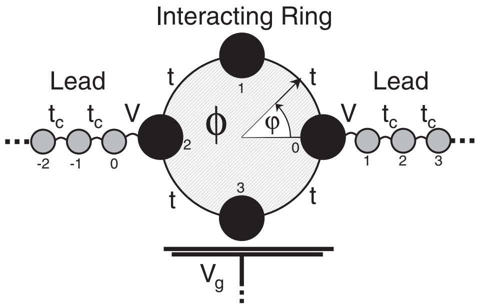

FIG. 1. Scheme of the system.

$$H_{\rm NI} = \hbar\Omega\left[-i\frac{\partial}{\partial\varphi} - \frac{\phi}{\phi_0} + \frac{\omega_{so}}{2\Omega}\sigma_r(\varphi)\right]^2, \qquad (1)$$

where $\Omega = \hbar/(2m^* r^2)$, m^* is the effective electron mass, r is the radius of the ring, $\omega_{\rm so} = \alpha/\hbar r$, $\alpha = \hbar\alpha' E_z$, $\phi = B\pi r^2$ is the magnetic flux, $\phi_0 = h/e$ is the flux quantum and $\sigma_r(\varphi) = \sigma_x \cos\varphi + \sigma_y \sin\varphi$ is the Pauli matrix in the radial direction, and φ is the azimuthal angle (see Fig. 1). Although the Schrödinger equation $H_{\rm NI}\chi(\varphi) = E\chi(\varphi)$ (where χ is a spinor) has been solved [13], we are interested in a simplification of this equation that can be extended to the interacting case. This can be achieved by a NAGT $\chi(\varphi) = \hat{U}(\varphi)\chi'(\varphi)$, where the operator $\hat{U}(\varphi)$ satisfies the differential equation

$$i\frac{\partial}{\partial\varphi}\hat{U}(\varphi) = \left[-\frac{\phi}{\phi_0} + \frac{\omega_{\rm so}}{2\Omega}\sigma_r(\varphi)\right]\hat{U}(\varphi). \qquad (2)$$

It can easily be checked that in the transformed Hamiltonian, $H'_{\rm NI} = \hat{U}^\dagger H_{\rm NI}\hat{U} = -\hbar\Omega\partial^2/\varphi^2$, the magnetic flux and the SOC disappeared, and enter now in the boundary condition, since $\chi(2\pi) = \chi(0)$ implies $\chi'(2\pi) = \hat{U}^\dagger(2\pi)\chi'(0)$. The solution of Eq. (2) with $\hat{U}(0) = 1$ is

$$\hat{U}(\varphi) = \exp\left[-i\sigma_z\frac{\varphi}{2}\right]\exp\left[i\vec{\sigma}\cdot\vec{n}_\theta\frac{\varphi'}{2}\right]\exp\left[i\frac{\phi}{\phi_0}\varphi\right], \qquad (3)$$

where $\vec{n}_\theta = (-\sin\theta, 0, \cos\theta)$, $\theta = \arctan(\omega_{\rm so}/\Omega)$, and $\varphi' = \varphi\sqrt{1 + (\omega_{\rm so}/\Omega)^2}$.

To construct the tight-binding version of $H'_{\rm NI}$, let us assume that we have N sites, lattice parameter a (with $Na = 2\pi r$) and site 0 at angle $\varphi = 0$. For simplicity we consider only hopping between nearest neighbors (NN). Then, we can take a constant hopping t between all NN, except between sites $N-1$ and 0, in which the boundary condition should be included. The matrix $\hat{U}(2\pi)$ is easily diagonalized in the quantization axis $\vec{n}_\theta$, and its eigenvalues are $\exp[i(\Phi_{\rm AB} + \sigma\Phi_{\rm AC})]$, where $\sigma = 1\,(-1)$ for spin up (down) in this direction, $\Phi_{\rm AB} = 2\pi\phi/\phi_0$, and $\Phi_{\rm AC} = \pi\{[1 + (\omega_{\rm so}/\Omega)^2]^{1/2} - 1\}$. Therefore, destroying a particle with spin σ at site $N-1$ and creating it at site 0 should be accompanied by the corresponding exponential factors. On-site interactions are not affected by the NAGT. With a convenient choice of phases, the transformed Hubbard model in the ring becomes

$$H'_r = \sum_{i=0,\sigma}^{N-1} t[e^{i(\Phi_{\rm AB}+\sigma\Phi_{\rm AC})/N} d^\dagger_{i+1\sigma}d_{i\sigma} + \text{H.c.}] + U d^\dagger_{i\uparrow}d_{i\uparrow}d^\dagger_{i\downarrow}d_{i\downarrow}. \qquad (4)$$

From the curvature of the dispersion relation at small wave vector $t = \hbar^2/(2m^* a^2)$, and then $\omega_{\rm so}/\Omega = \alpha N/(2\pi t a)$. Thus, the AC phase can be written as

$$\frac{\Phi_{\rm AC}}{N} = \sqrt{\left(\frac{\pi}{N}\right)^2 + \left(\frac{\alpha}{2ta}\right)^2} - \frac{\pi}{N}. \qquad (5)$$

Therefore, for large α or N, the properties of the system are periodic with α as observed experimentally [11,12].

The fact that the SOC can be gauged away in one dimension has been noted previously [15], but the explicit form of the transformation has not been derived. This transformation has important consequences. In the thermodynamic limit the boundary conditions are irrelevant and therefore the thermodynamic properties of the system should be identical to those of the Hubbard model without SOC. This is not obvious in alternative treatments [16]. In particular, it seems that the opening of a spin gap in the system requires long-range interactions.

To study the conductance, we must consider the Hamiltonian of the complete system $H = H_l + H'_r + H_V$, where with the appropriate quantization axis [17] and choice of phases $H_l = t_c(\sum_{i=0,\sigma}^{-\infty} c^\dagger_{i-1,\sigma}c_{i\sigma} + \sum_{i=1,\sigma}^{\infty} c^\dagger_{i+1,\sigma}c_{i\sigma} + \text{H.c.})$ describe the noninteracting leads, and $H_V = V(\sum_\sigma c^\dagger_{0\sigma}d_{N/2,\sigma} + c^\dagger_{1\sigma}d_{0\sigma} + \text{H.c.})$ is the coupling between the ring and leads. As an example of a system of few QDs, we consider the particular case $N = 4$, illustrated in Fig. 1. We assume that the leads are described by a constant density of states $\rho_0 = 1/W$, and we take for the bandwidth of the leads $W = 60t$ (W is usually much larger than t in QD arrays). The Fermi level is set at $\epsilon_F = 0$. To control the charge in the ring, we add to H'_r a term $-V_g\sum_{i\sigma}d^\dagger_{i\sigma}d_{i\sigma}$ that represents the effect of a gate voltage. Our approximations to calculate G amount to a truncation of the Hilbert space of H'_r and a slave-boson mean-field approximation for the resulting generalized Anderson model (GAM). H'_r can be diagonalized exactly (numerically for not too large N). We retain only two neighboring charge configurations with n and $n-1$ particles, and we have chosen $n = 4$. Furthermore, we retain only the lowest lying singlet state for 4 particles ($|\psi^4_0\rangle$ with energy E^4_0) and all doublets for 3 particles. This procedure is valid for small enough V [18]. Calculating the matrix elements of H_V in the truncated Hilbert space leads to a GAM

$$H_{\rm GAM} = H_l + \sum_{j,\sigma} E^3_{j\sigma}|\psi^3_{j\sigma}\rangle\langle\psi^3_{j\sigma}| + E^4_0|\psi^4_0\rangle\langle\psi^4_0| + V\sum_{j,\sigma,\eta=0,1}(\alpha^\eta_{j\sigma}|\psi^4_0\rangle\langle\psi^3_{j\sigma}|c_{\eta\sigma} + \text{H.c.}), \qquad (6)$$

where $|\psi^3_{j\sigma}\rangle$ and $E^3_{j\sigma}$ denote the jth eigenvector and eigenvalue of H'_r in the configuration with 3 particles with spin projection σ, in ascending order of energy and

$$\alpha^1_{j\sigma} = \langle\psi^4_0|d^\dagger_{0\sigma}|\psi^3_{j\bar{\sigma}}\rangle, \qquad \alpha^0_{j\sigma} = \langle\psi^4_0|d^\dagger_{N/2,\sigma}|\psi^3_{j\bar{\sigma}}\rangle. \qquad (7)$$

$H_{\rm GAM}$ can be expressed exactly in terms of a slave-boson representation similar to that proposed in Ref. [19]: $|\Psi^3_{j\sigma}\rangle\langle\Psi^3_{j\sigma}| \to f^\dagger_{j\sigma}f_{j\sigma}$, $|\Psi^4_0\rangle\langle\Psi^4_0| \to b^\dagger b$,

$|\Psi^3_{j\sigma}\rangle\langle\Psi^4_0| \to f^\dagger_{j\sigma} b$, and $|\Psi^4_0\rangle\langle\Psi^3_{j\sigma}| \to b^\dagger f_{j\sigma}$, where the operators $b^\dagger$ and $f^\dagger_{j\sigma}$ create a boson and a fermion, respectively, and are subject to the constraint $\sum_{j\sigma} f^\dagger_{j\sigma} f_{j\sigma} + b^\dagger b = 1$, which is incorporated in the Hamiltonian with a Lagrange multiplier λ. We perform a saddle-point approximation in the bosonic degrees of freedom, which reproduces the Kondo physics at low energies and temperatures [19]. The problem becomes equivalent to an effective noninteracting fermionic Hamiltonian, with parameters b_0, λ (where $b_0 = \langle b^\dagger\rangle = \langle b\rangle$), which are determined by minimization of the free energy. Thus, we can use the two-terminal Landauer formula to calculate the conductance, giving at zero temperature [5]

$$G = \sum_\sigma G_\sigma, \tag{8}$$

$$\frac{G_\sigma}{G_0} = 2(\pi\rho_0 V^2 b_0^2)^2 \left| \sum_{i,j} \bar{\alpha}^0_{i\sigma} \alpha^1_{j\sigma} g^\sigma_{i,j}(\epsilon_F) \right|^2, \tag{9}$$

where G_0 is $2e^2/h$ and

$$g^\sigma_{i,j} = g^{0\sigma}_{i,j} + \frac{b_0^2 V^2 g^{0\sigma}_{ii} g^{0\sigma}_{jj}}{A^\sigma_{11}A^\sigma_{00} - A^\sigma_{10}A^\sigma_{01}} \sum_{\eta,\eta'} \alpha^\eta_{i\sigma} \bar{\alpha}^{\eta'}_{j\sigma} A^\sigma_{\eta\eta'},$$

where $g^{0\sigma}_{ij} \equiv g^{0\sigma}_{ij}(\omega) = \delta_{ij}(\omega - E^3_{j\sigma} - \lambda)^{-1}$ is the propagator of the jth state of 3 particles with spin projection σ in the isolated ring, and the functions $A^\sigma_{\eta\eta'}$ are $A^\sigma_{\eta\eta} = 1 - b_0^2 V^2 g^{(0)}_\eta \sum_m |\alpha^\eta_{m\sigma}|^2/(\omega - E^3_m - \lambda)$ for $\eta = \eta'$ and $A^\sigma_{\eta\eta'} = b_0^2 V^2 g^{(0)}_\eta \sum_m \alpha^\eta_{m\sigma} \bar{\alpha}^{\eta'}_{m\sigma}/(\omega - E^3_m - \lambda)$ for $\eta \neq \eta'$, where $g^{(0)}_\eta(\omega)$ is the Green's function at site η of the corresponding isolated lead ($V = 0$).

We have calculated G as a function of $\Phi_{\rm AB}$ for $\alpha = 0$ and $V_g = 0.8t$, which corresponds to the nonmagnetic regime $E^{(4)}_0 < E^{(3)}_{0\sigma}$. In this regime, correlations play a minor role, and one expects that the JB formula [6], which is exact in the noninteracting case, gives accurate values for G. Our results (not shown) are very similar, with low values of $G(\Phi_{\rm AB})$ and $G(\pi) = 0$ due to destructive interference. In fact, for small V it can be demonstrated that both approaches are equivalent in this regime. We have also checked that in the noninteracting case (tight-binding model), the results of the JB formula coincide qualitatively to those obtained by Shen *et al.* and Molnár *et al.* [13] for free electrons.

The difference $E^{(4)}_0 - E^{(3)}_{0\sigma}$ can be reduced and turned negative applying a negative gate voltage. The most important results of this work are those obtained in this case, i.e., when the ring is in the mixed-valence or Kondo regime. Results for $\alpha = 0$ are presented in Fig. 2(a). Because of symmetry [20], it is enough to show G in the interval $0 \leq \Phi_{\rm AB} \leq \pi$.

For small enough $\Delta/|E^{(3)}_0 - E^{(4)}_0|$, where $\Delta = \pi\rho_0 V^2(|\alpha^0_{0\sigma}|^2 + |\alpha^1_{0\sigma}|^2)$, charge fluctuations are frozen and a clear signature of Kondo physics is displayed in the characteristic plateau in G at the ideal conductance G_0 (the unitary limit) [4]. This is shown in the figure for the smaller values of V at small fluxes. The dependence of G with flux, is related to the corresponding dependence of the energy levels and matrix elements with ϕ. For larger V and $\Phi_{\rm AB} \sim \pi$, the system is in the intermediate valence regime, as reflected in Fig. 2(b) in which the total occupancy of the configuration with three particles $n_f = \sum_{j\sigma}\langle f^\dagger_\sigma f_\sigma\rangle$ is shown. Therefore, the conductance deviates from the unitary limit.

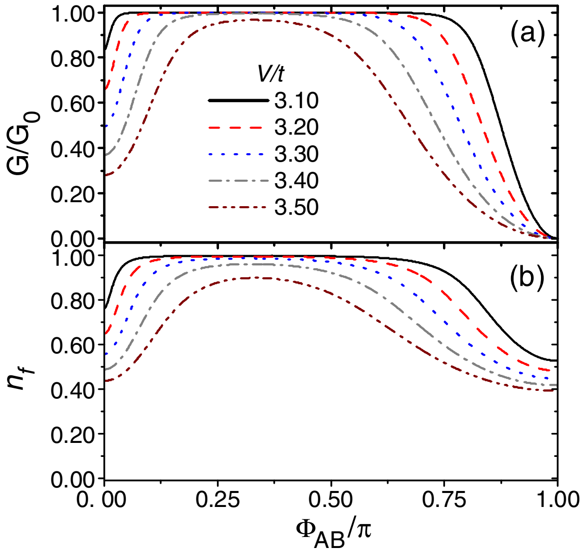

FIG. 2 (color online). (a) Conductance and (b) occupancy as a function of magnetic flux for $\alpha = 0$, $V_g = 0$, $U = 6t$, and several values of V.

Independently of the other parameters, G vanishes at $\Phi_{\rm AB} = \pi$. Within our formulation, at this point the states of the $n = 3$ configuration become doubly degenerate between states of different parity. The matrix elements $\alpha^\eta_{j\sigma}$ entering Eq. (9) have the same modulus but different sign, therefore producing a complete destructive interference inside the absolute value. To our knowledge, there are no calculations so far showing at the same time this destructive interference and ideal conductance in the Kondo regime. The JB expression gives values below $0.1G_0$ for all $\Phi_{\rm AB}$ and parameters of Fig. 2.

The effect of the SOC on the total conductance is dramatic in the Kondo regime. The results presented in Fig. 3 show dips (additional to that of $\Phi_{\rm AB} = \pi$), which are larger as α grows. The main difference with the case $\alpha = 0$ is that $n_{f\uparrow} \neq n_{f\downarrow}$, therefore producing a partial destruction of the Kondo resonance, mimicking the effect of a Zeeman term. This effect is larger for lower Δ (when the system is deeper inside the Kondo regime), which for the parameters of Fig. 3 corresponds to $\Phi_{\rm AB} \sim \pm 0.3\pi$. For $\Phi_{\rm AB} = \pi$, complete cancellation is not achieved due to the effect of the AC phase (see inset in Fig. 3).

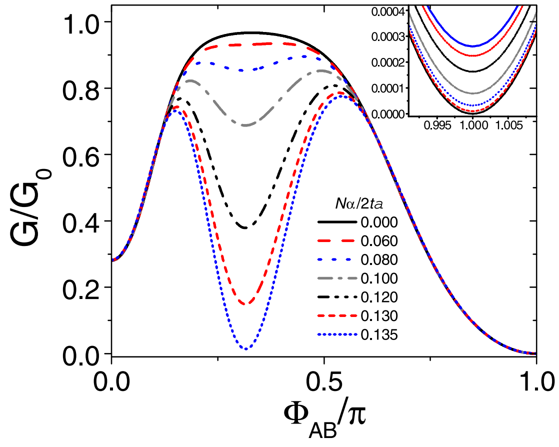

FIG. 3 (color online). Conductance as a function of flux for $V = 3.5t$ and different α. Other parameters as in Fig. 2.

Another important effect of the SOC in the Kondo regime is that it leads to currents with significant spin polarization. If a spin σ (up or down) in the quantization direction $\vec{n}_\theta$ is injected in the ring at the right lead ($\varphi = 0$) it comes out at the left lead ($\varphi = \pi$) with spin σ in the direction $\vec{n}'_\theta = (\sin\theta, 0, \cos\theta)$ or vice versa [17]. The corresponding conductance G_σ is spin dependent, as shown in Fig. 4. The ratio of the conductances can reach a factor 2 or larger with ideal $G_\uparrow$ ($G_\downarrow$) for flux $\Phi_{\rm AB} = 0.15\pi$ (-0.15π) and rather small α [20]. For these values, the z component of the quantization axis for any φ is larger than 0.99 [17].

In summary, we have presented an approach to calculate the conductance through a ring of interacting QDs threaded by a magnetic flux and with spin-orbit coupling α in the Kondo, mixed-valence, and nonmagnetic regimes. The effects of α are incorporated into Aharonov-Casher phases using a gauge transformation that leads to the Hubbard Hamiltonian Eqs. (4) and (5). Using a method based on a mapping of the relevant exact eigenstates of the ring onto an effective multilevel Anderson impurity and with the use of a slave-boson representation, we are able to describe the properties of the ring connected to the leads. The method is valid for small values of the coupling between rings and leads V and small values of magnetic field B, such that the Zeeman energy is much less than T_K. When the ring is in the Kondo regime, we obtain ideal conductance for $\alpha = 0$ and magnetic flux far from half a flux quantum, for which there is complete destructive interference. The effect of a small nonvanishing α is to produce a progressive destruction of the Kondo effect, decreasing the conductance and leading to a strong spin dependence of it. Extensions to include the Zeeman term or other interacting systems with local interactions are straightforward.

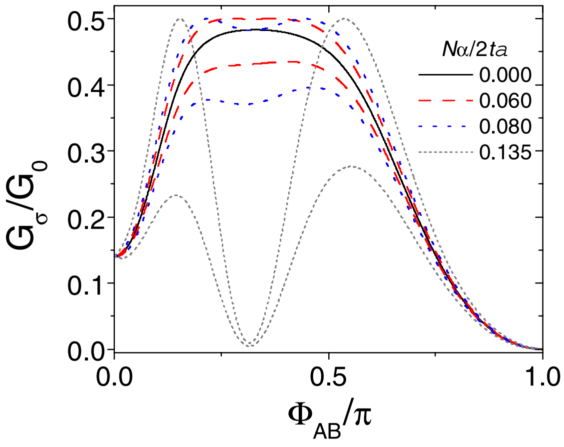

FIG. 4 (color online). Conductance for each spin as a function of flux for the parameters of Fig. 3 and different α.

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