Magnetoacoustic Spectroscopy in Superfluid ^{3}He-B

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We have used the acoustic Faraday effect in superfluid ^{3}He to perform high resolution spectroscopy of an excited state of the superfluid condensate, called the imaginary squashing mode. With acoustic cavity interferometry we measure the rotation of the plane of polarization of a transverse sound wave propagating in the direction of the magnetic field from which we determine the Zeeman energy of the mode. We interpret the Landé g factor, combined with the zero-field energies of this excited state, using the theory of Sauls and Serene, to calculate the strength of f-wave interactions in ^{3}He.

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Magnetoacoustic effects are less commonly known than their magneto-optical counterparts, but similar phenomena occur with transverse sound if there is magnetoelastic coupling. An acoustic Faraday effect (AFE) was first predicted for ferromagnetic crystals by Kittel [1] and subsequently was observed in magnetically ordered materials [2,3]. At low temperature in the superfluid state of ^{3}He, the spontaneously broken relative spin-orbit symmetry of the B phase provides the mechanism for such a coupling. An AFE was predicted by Moores and Sauls [4] and observed by Lee *et al.* [5] proving that collisionless transverse sound propagates in superfluid ^{3}He. With improvements in acoustic cavity techniques [6] we have achieved sufficiently high spectroscopic resolution over a wide range of frequency to measure the AFE and study the excited pair states of the condensate as a function of pressure. Pressure is an essential variable that permits one to vary the strength of quasiparticle and pairing interactions. Using the theory of Sauls and Serene [7] we relate our Faraday rotation results to these interactions.

The acoustic Faraday effect requires two coupled modes: a transverse sound mode, linearly dispersive in the absence of magnetoacoustic coupling, with phase velocity $c = \omega/q$ for frequency ω at wave vector q; the second mode must be magnetically active, $\omega^2 = \Omega_0^2(H) + bq^2$, having a magnetic field dependence in the long wavelength limit, usually with quadratic dispersion. For ordered materials Ω_0 is the ferro- or antiferromagnetic resonance frequency, proportional to the internal local field and shifted linearly by an applied field, H. For superfluid ^{3}He the magnetic mode, Ω_{2^-}, is an excited pair state of the superfluid condensate, having total angular momentum $J = 2$, called the *imaginary* squashing collective mode (ISQ), a reference to the nature of the order parameter distortions that characterize it, and to distinguish it from another $J = 2$ mode that involves *real* components [8]. The Zeeman splitting of the ISQ mode, linear in the magnetic field, leads to different couplings for right and left circularly polarized transverse sound.

The coupled-mode dispersion relation for superfluid ^{3}He in the long wavelength limit [4,9] can be written as

$$\frac{\omega^2}{q^2 v_F^2} = \Lambda_0 + \frac{\Lambda_{2^-}\omega^2}{\omega^2 - \Omega_{2^-}^2 - \frac{2}{5}q^2 v_F^2 - 2m_J g_{2^-}\gamma_{\rm eff} H\omega}, \quad (1)$$

where $\gamma_{\rm eff}$ is the effective gyromagnetic ratio of ^{3}He [7], m_J is the total angular momentum substate quantum number, and v_F is the Fermi velocity. The ISQ mode frequency follows the temperature and pressure dependence of the energy gap, $\Delta(T, P)$, shown in Fig. 1, or more precisely $\Omega_{2^-}^2 = a_{2^-}^2\Delta^2$, where $a_{2^-} \approx \sqrt{12/5}$. The first term in Eq. (1) is a quasiparticle restoring force, $\Lambda_0 = \frac{F_1^s}{15} \times (1-\lambda)(1+\frac{F_2^s}{5})/(1+\lambda\frac{F_2^s}{5})$. The magnetoacoustic coupling strength, for right and left circularly polarized sound waves ($m_J = \pm 1$), is $\Lambda_{2^-} = \frac{2F_1^s}{75}\lambda(1+\frac{F_2^s}{5})^2)/(1+\lambda\frac{F_2^s}{5})$. The Tsuneto function $\lambda(\omega, T)$ can be calculated from the energy gap [4]. The Fermi liquid parametrization of quasiparticle interactions is given in terms of $F_{1,2}^s$. Both a_{2^-} and the Landé g factor, g_{2^-}, are predicted to depend on the strength of quasiparticle and f-wave pairing interactions

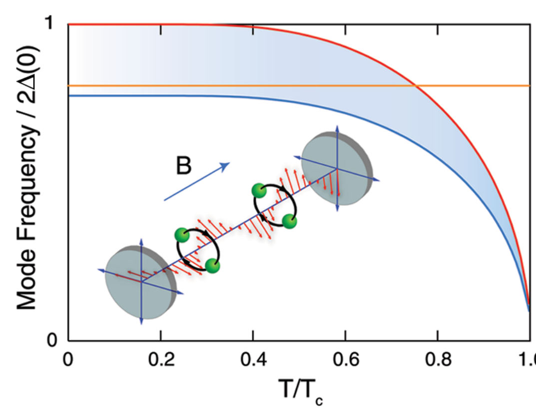

FIG. 1 (color online). The ISQ mode frequency (blue curve) and pair breaking (red curve) relative to the zero temperature gap versus temperature. Propagation of transverse sound, at constant frequency (orange line), is observed in the shaded (blue) region. The cartoon depicts the acoustic Faraday effect, in which linearly polarized transverse waves (red arrows) are coupled to ^{3}He Cooper pairs (green circles) and rotated by a magnetic field.

and consequently on temperature and pressure. In this Letter we present our measurements of the Faraday rotation angle from which we determine the g factor. We interpret our results, along with accurate measurements of a_{2^-} [6], in terms of the f-wave pairing interaction strength.

We cool the liquid ^{3}He by adiabatic nuclear demagnetization to temperatures $\approx 500\ \mu$K [10]. A cavity for liquid ^{3}He is formed using an ac-cut quartz transducer as one wall with an optically polished quartz reflector as the other. The spacing is defined by two wires with diameter $\approx 25\ \mu$m. The spacing was measured at room temperature using a confocal laser microscope at many places over its area. At 18 mK, $D = 31.6 \pm 0.1\ \mu$m was determined from the known dependence of the longitudinal sound velocity on pressure [8]. We use overtone frequencies, odd harmonics between 13 and 27, in the range 76 to 159 MHz.

As the phase velocity of transverse sound changes so does the number of half wavelengths in the cavity, altering the acoustic impedance at the surface of the piezoelectric transducer and producing a shift in the resonance spectrum which we detect with a resolution of $\approx 2 \times 10^{-6}$ using an rf bridge, FM modulation, and lock-in detection [6,10]. We detect the changing velocity as an oscillatory acoustic response, displayed in Fig. 2.

It is expected [4] that off-resonant coupling of transverse sound to the ISQ mode holds only in the shaded region of Fig. 1. Otherwise propagation is critically damped, either by pair breaking, $\omega > 2\Delta$, or if $\omega < \Omega_{2^-}$. In the latter case there are no real-valued solutions for q in Eq. (1). In our previous work [6], extended here to 27 bar, we used the acoustic signature for $\omega = \Omega_{2^-}(T, P)$ to obtain $\Omega_{2^-}(0, P) = \sqrt{12/5}(1.0018 + 0.00144P)\Delta^+(0, P) \pm 0.3\%$. These results are expressed in terms of the weak-coupling-plus gap [8,11], $\Delta^+(T, P)$, fixed to the Greywall temperature scale [12]. Additionally, we find that some harmonics of ac-cut transducers can generate *longitudinal* acoustic cavity resonances giving oscillatory acoustic response immediately below the shaded region in Fig. 1. We reported this earlier [6] although, at the time, we were unaware of its origin.

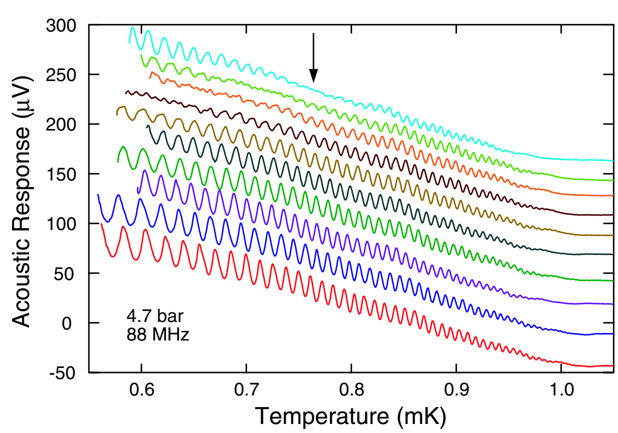

FIG. 2 (color online). Acoustic cavity response as a function of temperature for 4.7 bar and 88 MHz. Each trace (offset for clarity) is for a different magnetic field, bottom to top: 0, 61, 91, 122, 152, 183, 244, 274, 305, and 366 G. As the magnetic field is increased the linear polarization of transverse sound is rotated, decreasing the amplitude of the interference oscillations. At 366 G there is a minimum (arrow) in the acoustic response at 765 μK, which corresponds to a $\pi/2$ rotation of the polarization of the reflected wave with respect to the detection and generation direction.

The acoustic Faraday effect is schematically represented in Fig. 1. Application of a magnetic field splits the ISQ mode into five components. One of these, $m_J = 1$, couples to right circularly polarized sound and its frequency is increased by field. A second, $m_J = -1$, couples to left circular sound and is decreased by field. Consequently, these circularly polarized waves have different phase velocities. They interfere and thus rotate the plane of linear polarization proportional to path length and to the applied magnetic field, directly analogous to the magneto-optic effect discovered by Michael Faraday.

To accurately measure the AFE-rotation angle, we slowly sweep either the temperature or pressure at constant frequency in various magnetic fields applied along the direction of propagation. For temperature sweeps this corresponds to the horizontal line in Fig. 1, typically in the range $\approx$500 to 900 μK. Acoustic response data for temperature sweeps in different magnetic fields are shown in Fig. 2, with $A = A_0 + A_1 \cos\theta \sin(2D\omega/c + \phi)$, where $\theta = \frac{\pi}{2}(H/H_{\pi/2})$ is the Faraday rotation angle, $H_{\pi/2}$ is the field that rotates the polarization by $\pi/2$, and ϕ is a phase. The $\cos\theta$ factor is the projection of the rotated polarization onto a fixed direction of linearly polarized transverse sound, characteristic of the transducer. A_0 is a smoothly varying background signal in the absence of cavity interference; A_1 is the maximum signal modulation from acoustic interference in the cavity. The rapid oscillatory behavior in Fig. 2 comes from the temperature dependence of the phase velocity, modulated by the field dependence of the Faraday rotation angle. Typical results for θ are shown in Fig. 3.

The amplitude of the oscillations in the acoustic response in Fig. 2 decreases as the magnetic field is increased at fixed temperature, passing through a minimum indicated, for example, by an arrow in Fig. 2 at 366 G and 765 μK. This corresponds to a $\pi/2$ rotation of the linear polarization as a wave traverses a round-trip path in the cavity. From the data, Fig. 2, we find θ at constant temperature and plot its dependence on magnetic field, Fig. 3. We find that the rotation angle is proportional to magnetic field for all pressures.

It is convenient to work at the lowest temperatures to minimize temperature dependences, so we extrapolate our measurements of $H_{\pi/2}(T)$ to $T = 0$ using a phenomenological expression that fits our data well, $H_{\pi/2}(T) = H_{\pi/2}(0) + Be^{-2\Delta(0)/k_BT}$, where B is a fit parameter. An example of these results, inset to Fig. 3, shows that

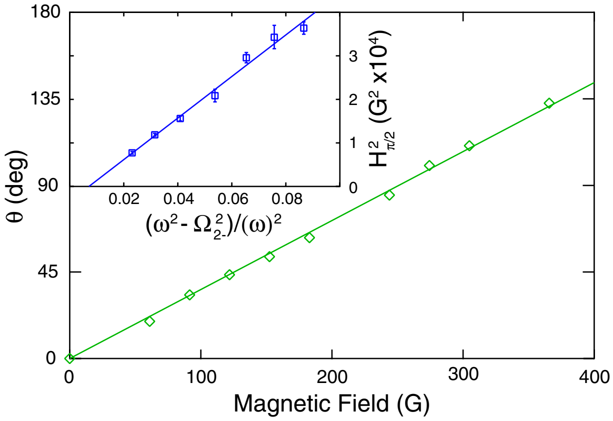

FIG. 3 (color online). The Faraday rotation angle as a function of applied field H for representative data at $P = 4.7$ bar, $T = 626$ μK, and frequency 88 MHz. (Inset) The square of the field for a $\pi/2$ Faraday rotation angle, $H^2_{\pi/2}$, is shown as a function of proximity to the ISQ mode, $\Omega_{2^-}(P)$, extrapolated to $T = 0$, with a frequency of 135.1 MHz and centered around 17.3 bar.

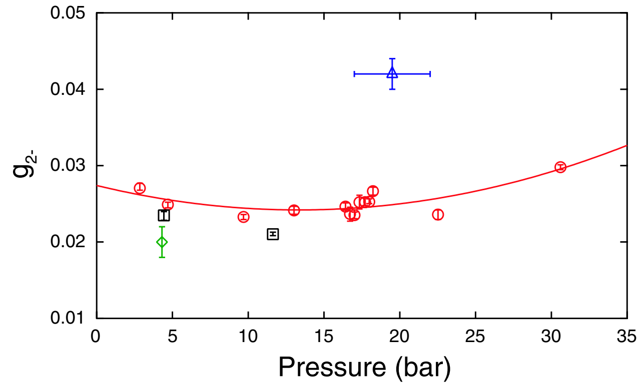

FIG. 4 (color online). The Landé g factor, g_{2^-}, as a function of pressure at $T = 0$. The red circles are from Faraday rotation measurements as a function of temperature, extrapolated to zero temperature. The black squares are measurements performed via pressure sweeps at our lowest temperatures. The blue triangle is from Movshovich *et al.* [14] and the green diamond is from Lee *et al.* [5]. The curve is given by $g_{2^-} = 0.0274 - 4.8 \times 10^{-4}P + 1.8 \times 10^{-5}P^2$.

$H_{\pi/2}(0)$ becomes smaller for acoustic frequencies approaching the ISQ mode.

Comparison with theory [9] requires that we calculate the g factor in terms of $H_{\pi/2}$, using the condition for $\pi/2$ rotation, $q_+ - q_- = \frac{\pi}{2D}$, where $q_\pm$ corresponds to $m_J = \pm 1$ in Eq. (1). We can show that sufficiently close to the ISQ mode,

$$g_{2^-} = \sqrt{\frac{\omega^2 - \Omega^2_{2^-}}{\omega^2}} \frac{v_F \Lambda^{1/2}_{2^-}}{\gamma_{\rm eff} H_{\pi/2} 8D}. \qquad (2)$$

Determination of g_{2^-} depends on precise knowledge of the ISQ mode frequency, which we have measured independently in zero field. This scaling of the frequency dependence of g_{2^-} follows from the dispersion relation, Eq. (1), which can be verified experimentally. We have measured $H_{\pi/2}(0)$ at a single acoustic frequency of 135.1 MHz within a small pressure range from $P = 16.4$ to 18.2 bar, that tunes the ISQ mode frequency from 129.1 to 133.5 MHz (variation in g_{2^-} is small). Our results in the inset of Fig. 3 are a validation of the predicted frequency behavior in Eq. (2). An unconstrained linear fit to the data has a small offset that corresponds to a shift in ISQ mode frequency of 0.3%, which is within experimental error.

Equation (2) is accurate when the difference between the acoustic frequency and the ISQ mode frequency is small. Outside of this limit the quasiparticle term, Λ_0, as well as the dispersion term, $\frac{2}{5}q^2v_F^2$, become non-negligible. Accordingly, we have calculated g_{2^-} from $H_{\pi/2}(0)$ using the full dispersion relation in Eq. (1), which is solved numerically. This is performed for all of our data and the resulting g_{2^-} values are presented in Fig. 4 as red circles.

A second method for data acquisition is to sweep the pressure at our lowest temperature, ≈ 500 μK, for various applied magnetic fields. In this case the pressure dependence of the transverse velocity is responsible for oscillatory acoustic response similar to that shown for temperature sweeps. From these measurements we obtain $H_{\pi/2}(0)$ and, as described above, determine the values for g_{2^-}, black squares in Fig. 4. The two methods are in good agreement.

Our results for g_{2^-} cover the full pressure range of superfluid ^{3}He-B. The theoretical value for $g_{2^-}(T = 0)$ without the effects of F_2^s or f-wave interactions is 0.0372 [7,13]. A previous result, shown as a green diamond in Fig. 4, by Lee *et al.* [5], agrees qualitatively with our work, where the difference can be ascribed to less accuracy in their determination of Ω_{2^-}. Movshovich *et al.* [14] used longitudinal acoustics at much higher magnetic fields to directly determine the ISQ mode splitting. Their considerably higher result, shown as a blue triangle, is likely due to nonlinear magnetic field dependence of the mode frequencies at large magnetic fields that arise from gap distortion.

The deviation of g_{2^-} from its weak-coupling value is an indication of the important role of ^{3}He Fermi liquid and pairing interactions. According to the theory of Sauls and Serene [7] the two relevant parameters are F_2^s and the strength of f-wave pairing interactions, $x_3^{-1} = 1/(v_1^{-1} - v_3^{-1})$. Here v_1 and v_3 are the pairing potentials due to p-wave and f-wave interactions, respectively. The theory allows calculation of x_3^{-1} from g_{2^-} and Ω_{2^-}, with F_2^s and the energy gap as inputs. We use values for F_2^s and Δ^+ as tabulated in Ref. [8]. The theoretical expression for g_{2^-} is complex [7] and is not reproduced here. The green circles in Fig. 5 show the result of this calculation, including all predicted dependences of Λ_{2^-} and Λ_0 on F_2^s.

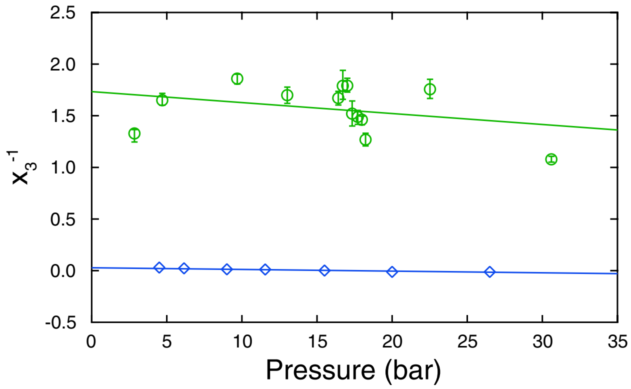

FIG. 5 (color online). The green circles are the f-wave interaction strength x_3^{-1} versus pressure from measurements of the ISQ mode frequencies and g factor. The blue diamonds are from ISQ mode frequencies alone [6]. Their difference is discussed in the text.

There is an extensive literature on f-wave interaction effects in superfluid ^{3}He [8,15] intertwined with determination of the Landau parameters F_2^a and F_2^s. From our work we find that x_3^{-1} is positive, which means that f-wave interactions are repulsive. These values are in disagreement with analysis [16] of our measurements of the ISQ mode frequencies [6], blue diamonds in Fig. 5. There are three possibilities for this discrepancy within the framework of the theory [7,16]: tabulated values of F_2^s are incorrect [8], there are nontrivial strong coupling corrections to the ISQ mode frequencies or g factor, and the values of the gap function are inaccurate. The estimated accuracy [12] of the absolute temperature scale of 1%, and hence the gap amplitude, or the variation of F_2^s by 0.5, have small effects on the calculation of the g factor and change x_3^{-1} within the scatter of the data. But F_2^s is least well known at low pressure and might exceed the range we have estimated. We note that even at low pressure, $\approx$5 bar, the Fermi liquid and f-wave corrections to the mode frequencies are $\lesssim$0.9%, less than the strong coupling corrections to $\Delta \approx 2.5\%$. This suggests that calculations of the mode frequencies beyond the weak-coupling model [7] might be required as was pointed out in the case of the A phase collective modes [15]. On the other hand, the g factor is more strongly modified by interaction effects as compared with the mode-frequency and so x_3^{-1} derived from the g factor should be more robust. A full resolution of this problem will require further work.

Sauls and Serene [16] predicted that for a combination of either negative values of x_3^{-1} or a negative Fermi liquid interaction term, F_4^s, there is a new order parameter collective mode close to the gap edge with total angular momentum $J = 4$. It should be possible to use high resolution acoustic cavity methods, as in the present work, to look for these modes although our determination of the repulsive interaction in the f-wave channel is not encouraging.

In summary, we have investigated the magnetoacoustic Faraday effect in superfluid ^{3}He using high resolution transverse sound techniques. Our spectroscopic data for the ISQ mode frequency and g factor provide a detailed characterization of this excited state of the order parameter in superfluid ^{3}He. We find from our results, combined with the theory, that f-wave pairing interactions are repulsive. But there is a significant discrepancy between our different experiments interpreted within this theoretical framework which will require additional work to be resolved.

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