Possible Realization of Directional Optical Waveguides in Photonic Crystals with Broken Time-Reversal Symmetry

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We show how, in principle, to construct analogs of quantum Hall edge states in "photonic crystals" made with nonreciprocal (Faraday-effect) media. These form "one-way waveguides" that allow electromagnetic energy to flow in one direction only.

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In this Letter, we describe a novel effect involving an interface between two magneto-optic photonic crystals (periodic "metamaterials" that transmit electromagnetic waves) which can theoretically act as a "one-way waveguide", i.e., a channel along which electromagnetic energy can propagate in only a single direction, with no possibility of being backscattered at bends or imperfections. The unidirectional photonic modes confined to such interfaces are the direct analogs of the "chiral edge states" of electrons in the quantum Hall effect (QHE) [1,2]. The key enabling ingredient is the presence of "nonreciprocal" (Faraday-effect) media that breaks time-reversal symmetry in the metamaterial.

Just as in the electronic case, every two-dimensional photonic band is characterized by a topological invariant known as the Chern number [3], an integer that vanishes identically unless time-reversal symmetry is broken. If the material contains a photonic band gap (PBG), the Chern number, summed over all bands below the gap, plays a role similar to that of the same quantity summed over all *occupied* bands in the electronic case. In particular, if the total Chern number *changes* across an interface separating two PBG media, there necessarily will occur states localized to the interface having a nonzero net current along the interface [1,2]. In the photonic case, such states would comprise our "one-way waveguide".

Such an interface between two PBG media can be realized as a domain wall in a 2D periodic photonic metamaterial, across which the direction of the Faraday axis reverses. Unidirectional edge states are guaranteed in this system provided that the Faraday effect generates photonic bands with nonzero Chern numbers. Here, we construct photonic bands with nonzero Chern invariants in a hexagonal array of dielctric rods with a Faraday effect. We then show that as a consequence of topology of the single-particle photon bands in the Brillouin zone, the edge states of light occur along domain walls (where the Faraday effect vanishes).

It may seem surprising that the physics of the QHE can have analogs in photonic systems. The QHE is exhibited by incompressible quantum fluid states of electrons—conserved strongly interacting charged fermions—in high magnetic fields, while photons are nonconserved neutral bosons which do not interact in linear media; furthermore, photonic bands can be described classically, in terms of Maxwell's equations. However, the integer QHE can, in principle, occur without any uniform magnetic flux density (just with broken time-reversal symmetry) as has explicitly shown by one of us in a graphenelike model of noninteracting Bloch electrons [4]; thus Landau-level quantization is not an essential requirement for the quantum Hall effect.

We have transcribed the key features of the electronic model of Ref. [4] to the photonic context. The edge states are a property of a one-particle eigenstate problem similar to the Maxwell normal-mode problem, so they are replicated in the photonics problem. (The QHE itself has no photonic analog, as it follows from the Pauli principle of filling all one-particle states below the Fermi level.)

The Maxwell normal-mode problem in loss-free linear media with spatially periodic local frequency-dependent constitutive relations is a generalized self-consistent Hermitian eigenproblem, somewhat different from the standard Hermitian eigenproblem. The nonreciprocal parts of the local Hermitian permittivity and permeability tensors $\boldsymbol{\epsilon}(\boldsymbol{r}, \omega)$ and $\boldsymbol{\mu}(\boldsymbol{r}, \omega)$ are odd imaginary functions of frequency, so frequency dependence is unavoidable. The generalized eigenproblem has the structure

$$\boldsymbol{U}^{\dagger}(\boldsymbol{k})\boldsymbol{A}\boldsymbol{U}(\boldsymbol{k})|u_n(\boldsymbol{k})\rangle = \omega_n(\boldsymbol{k})\boldsymbol{B}(\omega_n(\boldsymbol{k}))|u_n(\boldsymbol{k})\rangle, \qquad (1)$$

where $\boldsymbol{U}(\boldsymbol{k})$ is a unitary operator that defines the Bloch vector $\boldsymbol{k}$; $\boldsymbol{A}$ and $\boldsymbol{B}(\omega)$ are Hermitian operators, with the real-eigenvalue stability condition that the Hermitian operator $\boldsymbol{B}_0(\omega) \equiv (\partial/\partial\omega)[\omega\boldsymbol{B}(\omega)]$ is positive definite [this assumes that the periodic medium coupled to the electromagnetic fields has a linear response described by harmonic-oscillator modes, none of which have natural frequency $\omega_n(\boldsymbol{k})$—a detailed derivation has been presented in Ref. [5]]. The eigenfunctions $\langle\boldsymbol{r}|u_n(\boldsymbol{k})\rangle$ are the spatially periodic factors of the Bloch states. The electronic band-structure problem is a simplification of Eq. (1), with $\boldsymbol{A}$ replaced by the one-electron Hamiltonian, $\boldsymbol{B}$ by the identity operator $\mathbb{1}$, and ω_n by the energy eigenvalue.

In this formulation of Maxwell's equations, the eigenfunction $\boldsymbol{u}_n(\boldsymbol{k},\boldsymbol{r}) \equiv \langle \boldsymbol{r}|u_n(\boldsymbol{k})\rangle$ is the 6-component vector of complex electromagnetic fields

$$\boldsymbol{u}_n(\boldsymbol{k},\boldsymbol{r}) = \begin{pmatrix} \boldsymbol{E}_n(\boldsymbol{k},\boldsymbol{r}) \\ \boldsymbol{H}_n(\boldsymbol{k},\boldsymbol{r}) \end{pmatrix}. \quad (2)$$

In this basis, $\boldsymbol{U}(\boldsymbol{k},\boldsymbol{r}) = \exp i\boldsymbol{k}\cdot\boldsymbol{r}$ and $\boldsymbol{A} = -i\boldsymbol{J}^a\nabla_a$ (with $\nabla_a \equiv \partial/\partial r^a$, and repeated indices summed), where

$$\boldsymbol{J}^a = \begin{pmatrix} 0 & i\boldsymbol{L}^a \\ -i\boldsymbol{L}^a & 0 \end{pmatrix}, \qquad \boldsymbol{B} = \begin{pmatrix} \boldsymbol{\epsilon}(\boldsymbol{r},\omega) & 0 \\ 0 & \boldsymbol{\mu}(\boldsymbol{r},\omega) \end{pmatrix}; \quad (3)$$

here $\boldsymbol{L}^a$ are the 3×3 spin-1 matrices, $(L^b)^{ac} = i\epsilon^{abc}$. If the physical electromagnetic fields are given by the real parts of $\boldsymbol{u}_n(\boldsymbol{k},\boldsymbol{r})\exp i[\boldsymbol{k}\cdot\boldsymbol{r} - \omega_n(\boldsymbol{k})t]$, the spatially periodic time-averaged energy density and energy current are the quadratic forms $[\boldsymbol{u}_n^*, \boldsymbol{B}_0(\omega_n)\boldsymbol{u}_n]$, $(\boldsymbol{u}_n^*, \boldsymbol{J}^a\boldsymbol{u}_n)$. For our purposes, the key photonic band-structure quantity is the *Berry connection* $\mathcal{A}_n^a(\boldsymbol{k})$, a real function of $\boldsymbol{k}$ given by

$$\mathcal{A}_n^a = \frac{\langle u_n|\boldsymbol{B}_0(\omega_n)|\nabla_k^a u_n\rangle - \langle \nabla_k^a u_n|\boldsymbol{B}_0(\omega_n)|u_n\rangle}{2i\langle u_n|\boldsymbol{B}_0(\omega_n)|u_n\rangle}, \quad (4)$$

where $\nabla_k^a \equiv \partial/\partial k_a$ is the k-space derivative. We obtained (4) as a generalization of the $\boldsymbol{B} = \mathbb{1}$ expression [3] by deriving (1) from a standard Hermitian eigenproblem where electromagnetic fields are explicitly coupled to harmonic-oscillator degrees of freedom of the medium [5].

The solution of the normal-mode eigenproblem only determines $\boldsymbol{u}_n(\boldsymbol{k},\boldsymbol{r})$ up to an arbitrary $\boldsymbol{k}$-dependent phase factor; if the replacement $\boldsymbol{u}_n(\boldsymbol{k},\boldsymbol{r}) \to \boldsymbol{u}_n(\boldsymbol{k},\boldsymbol{r})\exp i\chi_n(\boldsymbol{k})$ is made, $\mathcal{A}_n^a(\boldsymbol{k}) \to \mathcal{A}_n^a(\boldsymbol{k}) + \nabla_k^a\chi_n(\boldsymbol{k})$. The Berry connection is a "gauge-dependent" analog of the electromagnetic vector potential; the associated gauge-invariant function (analogous to the magnetic flux density) is the k-space *Berry curvature* $\mathcal{F}_n^{ab}(\boldsymbol{k}) = \nabla_k^a\mathcal{A}_n^b - \nabla_k^b\mathcal{A}_n^a$. The *Berry phase* $\exp i\phi_n(\Gamma) = \exp i\oint \mathcal{A}_n^a dk_a$ associated [6] with adiabatic evolution around a closed path Γ (here in k space) is the gauge-invariant analog of the Bohm-Aharonov phase factor and can be expressed in terms of the integral of $\mathcal{F}_n^{ab}$ over a surface bounded by Γ [3].

The Berry curvature satisfies a k-space analog of the Gauss law, except that "monopole" singularities emitting total "Berry flux" $\pm 2\pi$ can occur at k-space points where there are "accidental degeneracies" between bands (this quantization of the monopole charge ensures that the expression for the Berry phase in terms of Berry curvature on a surface bounded by Γ is independent of how that surface is chosen [3]). The integer Chern invariant associated with any compact surface (2-manifold) Σ in k space is

$$C_n^{(1)}(\Sigma) = \frac{1}{2\pi}\iint_\Sigma dk_a \wedge dk_b \mathcal{F}_n^{ab}. \quad (5)$$

In the case of a 2D band structure, Σ may be taken to be the 2D Brillouin zone (BZ) itself, and $C_n^{(1)}$ is a property of the 2D band [7]. If time-reversal symmetry is unbroken, $\mathcal{F}_n^{ab}(-\boldsymbol{k}) = -\mathcal{F}_n^{ab}(\boldsymbol{k})$, and Chern numbers vanish.

We now wish to construct a 2D photonic band structure where some bands have a nonzero Chern number. The key idea is to start with a band structure that has both time-reversal symmetry *and* inversion symmetry, which allows the existence of pairs of "Dirac points" in the 2D BZ. These are isolated points where two bands become degenerate, but split apart with a linear dispersion (resembling that of the massless Dirac equation) for nearby Bloch vectors. Dirac points are generically allowed because if inversion symmetry is present, $\mathcal{F}_n^{ab}(-\boldsymbol{k}) = \mathcal{F}_n^{ab}(\boldsymbol{k})$; in combination with time-reversal symmetry, this means that $\mathcal{F}_n^{ab}(\boldsymbol{k}) = 0$. It is then possible to choose a phase convention such that the eigenfunctions $\boldsymbol{u}_n(\boldsymbol{k},\boldsymbol{r})$ of (1) are real for all $\boldsymbol{k}$. The eigenproblem is then of the real-symmetric type, where it is possible to find an "accidental" degeneracy between two bands by varying just two parameters (in this case, the 2D Bloch vector $\boldsymbol{k}$); in contrast, in the general complex-Hermitian case, *three* parameters must be varied to find a degeneracy, which cannot be done by merely "fine-tuning" a 2D $\boldsymbol{k}$.

Dirac points can exist in a 2D band structure with both spatial-inversion and time-reversal symmetry, but a gap opens if either symmetry is broken. While breaking of inversion symmetry leads to nonzero Berry curvature (and hence corrections to the "semiclassical" equations for the trajectories of light rays in adiabatically varying media [8]), it does not lead to nontrivial topology of the bands. In contrast, when a gap opens at Dirac points due to time-reversal breaking, the two bands that split apart inevitably acquire nonzero Chern numbers.

We can now give an in-principle demonstration that "one-way waveguides" can be constructed using nonreciprocal photonic crystals. Consider a system with a uniform isotropic permeability tensor $\mu_0\delta^{ab}$, and an isotropic but spatially varying permittivity tensor $\epsilon(\boldsymbol{r})\epsilon_0\delta^{ab}$, with

$$\epsilon(\boldsymbol{r}) = \epsilon[1 + \lambda V_G(\boldsymbol{r})], \qquad V_G(\boldsymbol{r}) = 2\sum_{n=1}^{3}\cos(\boldsymbol{G}_n\cdot\boldsymbol{r}), \quad (6)$$

where $\boldsymbol{G}_n$, $n = 1, 2, 3$ are three equal-length reciprocal vectors in the xy plane, rotated 120° relative to each other. For small λ, this problem can be solved analytically in a "nearly-free-photon" approximation. This system has continuous translational invariance in the z direction, and we will restrict attention to wave numbers $k_z = 0$, with Bloch vector $\boldsymbol{k}$ in the xy plane. The electromagnetic fields then separate into decoupled "TE" and "TM" sets, $\{E_x, E_y, H_z\}$ and $\{H_x, H_y, E_z\}$; we specialize to the TE set. The six corners of the (first) BZ are at $\pm\boldsymbol{K}_n$, where $\boldsymbol{K}_1 = (\boldsymbol{G}_2 - \boldsymbol{G}_3)/3$, etc., and $|K| = |G|/\sqrt{3}$; since $\boldsymbol{K}_2 - \boldsymbol{K}_1 = \boldsymbol{G}_3$, the three wave vectors $\boldsymbol{K}_i$ are equivalent as Bloch vectors. To leading order in λ and the deviation $\delta\boldsymbol{k} = \boldsymbol{k} - \boldsymbol{K}_i$ of the 2D Bloch vector from the BZ corner, the three "free photon" TE plane waves with speed c_0 split into a "Dirac-point" doublet with $\omega = \omega_D \pm v_D|\delta \boldsymbol{k}| + O(|\delta \boldsymbol{k}|^2)$, where $\omega_D = c_0|K|[1 - \lambda/4 + O(\lambda)^2]$, $v_D = c_0/2 + O(\lambda)$, and a singlet $\omega = \omega_0 + O(|\delta \boldsymbol{k}|^2)$, $\omega_0 = c_0|K|[1 + \lambda/2 + O(\lambda^2)]$.

We now perturb the Dirac points by a Faraday term (which explicitly breaks time-reversal symmetry), with an axis normal to the xy plane, added to the permittivity tensor: $\epsilon^{xy} = -\epsilon^{yx} = i\epsilon_0\epsilon\eta(\boldsymbol{r}, \omega)$, where

$$\eta(\boldsymbol{r}, \omega) = \eta_0(\omega) + \eta_1(\omega)V_G(\boldsymbol{r}); \tag{7}$$

$\eta_0(\omega)$, $\eta_1(\omega)$ are real odd functions of ω. We assume that, for $\omega \approx \omega_D$, $|\eta_0(\omega)|, |\eta_1(\omega)| \ll |\lambda| \ll 1$, with negligible frequency dependence. The Dirac points now split, with dispersion $\omega = \omega_D \pm v_D(|\delta \boldsymbol{k}|^2 + \kappa^2)^{1/2}$, where, to leading order in η, $\kappa = |K|[\frac{3}{2}\eta_1(\omega_D) - 3\lambda\eta_0(\omega_D)]$.

For small κ, the Berry curvatures of the upper and lower $k_z = 0$ bands near the split Dirac points are

$$F^{xy}_{\pm}(\delta \boldsymbol{k}) = \pm\tfrac{1}{2}\kappa(|\delta \boldsymbol{k}|^2 + \kappa^2)^{-3/2}. \tag{8}$$

There is a total integrated Berry curvature of $\pm\pi$ near each Dirac point, giving total Chern numbers ± 1 for the split bands. By inversion symmetry, the Berry curvatures at the two Dirac points have the same sign; if the gap was opened by broken inversion symmetry, with unbroken time-reversal invariance, they would have opposite sign, and the Chern number would vanish.

We now consider an adiabatically spatially varying Faraday term parametrized by a $\kappa(\boldsymbol{r})$ that is positive in some regions and negative in other regions. The splitting of the Dirac points vanishes locally on the line where $\kappa(\boldsymbol{r}) = 0$. It is necessary that, in the perfectly periodic structure with $\kappa = 0$, there are no photonic modes at other Bloch vectors that are degenerate with the modes at the Dirac points.

Such frequency isolation of the Dirac points cannot occur in the weak-coupling "nearly-free photon" limit, but can be achieved, at least for $k_z = 0$ modes, in hexagonal arrays of infinitely long dielectric rods parallel to the z axis. An example can be seen in Fig. 1(a) of Ref. [9]. That figure was exhibited to demonstrate a frequency gap between the first and second TE bands, but incidentally also shows that the second and third TE bands are separated by a substantial gap except in the vicinity of the BZ corners, where they touch at Dirac points. The corresponding TM bands were not given in Ref. [9], but we found that the Dirac-point frequency ω_D is also inside a large gap of the TM spectrum (see Fig. 1). When a Faraday term is added, the bands forming the Dirac point in Fig. 1 split apart, and each now nondegenerate band will have associated with it a nonzero Chern number (see Ref. [5]).

The Faraday effect incorporated to the hexagonal array of rods explicitly breaks time-reversal symmetry on the scale of the unit cell of the metamaterial: the permittivity tensor acquires an imaginary off-diagonal component having the periodicity of the unit cell, as described above. A hexagonal array consisting of a material having a large Verdet coefficient, such as a rare-earth garnet with ferromagnetically ordered domains, would give rise to such an effect.

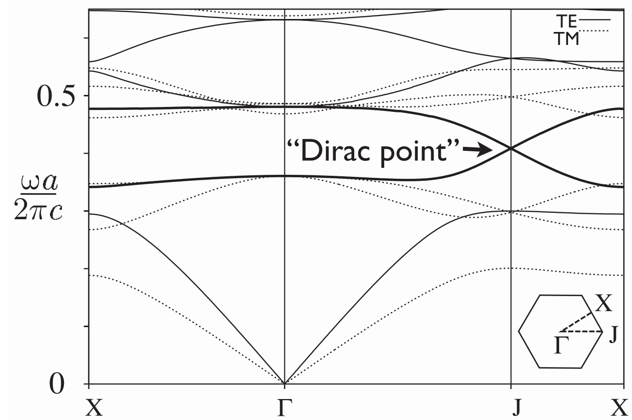

FIG. 1. Photon bands for $k_z = 0$ electromagnetic waves propagating normal to the axis of a hexagonal 2D array of cylindrical dielectric rods; a is the lattice constant. As in Fig. 1(a) of Ref. [9], the rods fill a fraction $f = 0.431$ of the volume, with dielectric constant $\epsilon = 14$, and they are embedded in an $\epsilon = 1$ background. The lowest five 2D bands are well separated from higher bands, except near a pair of "Dirac points" at the two distinct Brillouin zone corners (J).

While these $k_z = 0$ Dirac-point modes are not degenerate with any other $k_z = 0$ modes, they *are* degenerate with $k_z \neq 0$ modes. To fully achieve a "one-way" edge-mode structure, the light must also be confined in the z direction, with Dirac points at a frequency that is nondegenerate with *any* other modes. To design such structures, it will be necessary to vary the filling factor of the rods along the z direction so that light remains confined to regions of relatively larger filling factors. The technical challenge would be to vary the filling factors without introducing any modes into the bulk TE gaps surrounding the Dirac points.

Let $|u_\sigma(\pm \boldsymbol{k}_D)\rangle$, $\sigma = \pm$ be the degenerate solutions of Eq. (1) at a pair of isolated Dirac points, normalized so $\langle u_\sigma(\pm \boldsymbol{k}_D)|\boldsymbol{B}_0(\omega_D)|u_{\sigma'}(\pm \boldsymbol{k}_D)\rangle = B_0\delta_{\sigma\sigma'}$. Now add a Faraday perturbation $\delta \boldsymbol{B}(\boldsymbol{r}, \omega)$: in degenerate perturbation theory, normal modes with small $\delta\omega = \omega - \omega_D$ have the form $\sum_{\sigma,\pm}\psi^{\pm}_{\sigma}(\boldsymbol{r})\boldsymbol{U}(\pm \boldsymbol{k}_D, \boldsymbol{r})\boldsymbol{u}_\sigma(\pm \boldsymbol{k}_D, \boldsymbol{r})$. For slow spatial variation, there is negligible mixing between modes at different Dirac points, and $\psi^{\pm}_{\sigma}(\boldsymbol{r})$ is the solution of

$$\sum_{\sigma'}(-iJ^a_{\perp}\nabla_a - \omega_D\delta B(\boldsymbol{r}))^{\pm}_{\sigma\sigma'}\psi^{\pm}_{\sigma'}(\boldsymbol{r}) = \delta\omega B_0\psi^{\pm}_{\sigma}(\boldsymbol{r}), \tag{9}$$

where $J^a_{\perp}$ and $\delta B(\boldsymbol{r})$ are 2×2 matrices given by

$$(J^a_{\perp})^{\pm}_{\sigma\sigma'} = \langle u_\sigma(\pm \boldsymbol{k}_D)|\boldsymbol{J}^a|u_{\sigma'}(\pm \boldsymbol{k}_D)\rangle, \quad a = x, y, \; (\delta B(\boldsymbol{r})) = \langle u_\sigma(\pm \boldsymbol{k}_D)|\delta \boldsymbol{B}(\boldsymbol{r}, \omega_D)|u_{\sigma'}(\pm \boldsymbol{k}_D)\rangle. \tag{10}$$

For a straight line interface, this equation has the form $v_D \hat{K}|\psi\rangle = \delta\omega|\psi\rangle$, with $v_D > 0$, and

$$\hat{K} = -i\boldsymbol{\sigma}^x \nabla_x + \delta k_{\|}\boldsymbol{\sigma}^y + \kappa(x)\boldsymbol{\sigma}^z, \tag{11}$$

where $\boldsymbol{\sigma}^a$ are Pauli matrices. Here $k_{Dy} + \delta k_{\|}$ is the conserved Bloch vector parallel to the interface; we take $\kappa(x)$ to be monotonic, with $\kappa(x) \to \pm\kappa^\infty$ as $x \to \pm\infty$.

It is instructive to first consider the exactly-solvable case $\kappa(x) = \kappa^\infty \tanh(x/\xi)$, $\xi > 0$, where $\hat{K}^2$ is essentially the integrable Pöschl-Teller Hamiltonian [10]. The spectrum of modes bound to the interface is

$$\omega_0(\delta k_{\|}) = \omega_D + s_\kappa v_D \delta k_{\|}, \qquad s_\kappa \equiv \mathrm{sgn}\,(\kappa^\infty), \tag{12a}$$

$$\omega_{n\pm}(\delta k_{\|}) = \omega_D \pm v_D(\delta k_{\|}^2 + \kappa_n^2)^{1/2}, \qquad n > 0, \tag{12b}$$

with $|\kappa_n| < |\kappa^\infty|$; for the integrable model, κ_n^2 is given by $2n|\kappa^\infty|/\xi$, $n < |\kappa^\infty|\xi/2$. There is always a unidirectional $n = 0$ mode with speed v_D and a direction determined by the sign of κ^∞; in the small-ξ (or sharp-wall) limit $|\kappa^\infty|\xi < 2$, this is the only interface mode.

Let $\phi(\kappa^2)$ be the dimensionless area in the $x - k_x$ phase plane enclosed by a closed constant-frequency orbit $(k_x)^2 + [\kappa(x)]^2 = \kappa^2 < |\kappa^\infty|^2$, corresponding to a bound state. For the integrable model, this has the simple form $\phi(\kappa^2) = \pi\kappa^2\xi/|\kappa^\infty|$; the $n > 0$ bidirectional modes thus satisfy a constructive-interference condition

$$\phi(\kappa_n^2) = 2\pi n. \tag{13}$$

This contrasts with the usual semiclassical condition $\phi = 2\pi(n + \frac{1}{2})$; the change is needed for the $n = 0$ "zero mode" (12a) to exist, and it can be interpreted as deriving from an extra Berry phase factor of -1 because the orbit encloses a Dirac degeneracy point at $(x, k_x) = (0, 0)$. For general $\kappa(x)$, the $n = 0$ eigenfunction is

$$\psi_\sigma^0(\boldsymbol{r}) \propto \varphi_\sigma(s_\kappa)\exp\left[i\delta k_{\|} y - s_\kappa \int^x \kappa(x')dx'\right], \tag{14}$$

$\boldsymbol{\sigma}^y \boldsymbol{\varphi}(s) = s\boldsymbol{\varphi}(s)$. For slowly varying $\kappa(x)$, the condition (13) will determine κ_n^2 for any $n > 0$ interface modes.

Since there are two Dirac points, there are two such unidirectional edge modes at a boundary across which the Faraday axis reverses. The crucial feature is that both modes propagate in the *same* direction and cannot disappear, even if the interface becomes sharp, bent, or disordered. As in the QHE, the difference between the number of modes moving in the two directions along the interface is topologically determined by the difference of the total Chern number of bands at frequencies below the bulk photonic band gap in the regions on either side of the interface; in this case $|\Delta C^{(1)}| = 2$.

For $|\delta\omega| < v_D|\kappa^\infty|$ a Faraday interface has no counterpropagating modes into which elastic backscattering can take place, so the "one-way waveguide" that it forms is immune to localization effects, just like electronic transport in the QHE. In the QHE, the number of electrons is strictly conserved; in photonics, the photons only propagate ballistically if absorption and nonlinear effects are absent. These effects do allow degradation of the electromagnetic energy current flowing along the interface, so the analogy with the QHE is not perfect.

Even if a 2D metamaterial with isolated Dirac points can be designed, the problem of finding a suitable magneto-optic material to provide the Faraday effect must be addressed. The effect must be large enough to induce a gap that overcomes the effect of any inversion-symmetry breaking. The parameter $|\kappa^\infty|$ is the inverse length that controls the width of the unidirectionally propagating channel (and the unidirectional frequency range); in order to keep the wave confined to the interface and prevent leakage, the Faraday coupling must be strong enough so that this width is significantly smaller than the physical dimensions of the sample of metamaterial.

In summary, we have shown that analogs of quantum Hall effect edge modes can, in principle, occur in two-dimensional photonic crystals with broken time-reversal symmetry. The electromagnetic energy in these modes travel in a single direction. Explicit theoretical examples of such modes have been constructed in Ref. [5]. Such quasilossless unidirectional channels are a novel possibility that might one day be physically realized in "photonic metamaterials" with nonreciprocal constituents.

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[1] B. I. Halperin, Phys. Rev. B **25**, 2185 (1982).
[2] X.-G. Wen, Phys. Rev. B **43**, 11 025 (1991).
[3] B. Simon, Phys. Rev. Lett. **51**, 2167 (1983).
[4] F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988).
[5] S. Raghu and F. D. M. Haldane, arXiv:cond-mat/0602501.
[6] M. V. Berry, Proc. R. Soc. A **392**, 45 (1984).
[7] D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs, Phys. Rev. Lett. **49**, 405 (1982).
[8] M. Onoda, S. Murakami, and N. Nagaosa, Phys. Rev. Lett. **93**, 083901 (2004).
[9] M. Plihal and A. A. Maradudin, Phys. Rev. B **44**, 8565 (1991).
[10] See, e.g., L. D. Landau and E. M. Lifschitz, *Quantum Mechanics: Non-Relativistic Theory* (Pergamon, New York, 1977), p. 73, Problem 5.