Nonperturbative Vacuum-Polarization Effects in Proton-Laser Collisions

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In the collision of a high-energy proton beam and a strong laser field, merging of laser photons can occur due to the polarization of vacuum. The probability of photon merging is calculated by exactly accounting for the laser field which involves a highly nonperturbative dependence on the laser intensity and frequency. It is shown that the nonperturbative vacuum-polarization effects can be experimentally measured by combining the next generation of tabletop petawatt lasers with proton accelerators presently available.

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In classical electrodynamics the superposition principle holds for electromagnetic fields of arbitrary strength in vacuum. Instead, quantum electrodynamics (QED) predicts that under the influence of electromagnetic fields of the order of the ''critical'' fields $E_{\rm cr} = m^2/e = 1.3 \times 10^{16}$ V/cm and $B_{\rm cr} = m^2/e = 4.4 \times 10^{13}$ G, vacuum shows nonlinear electromagnetic features ($-e < 0$ and m are the electron charge and mass, respectively, and natural units with $\hbar = c = 1$ are used) [1]. The experimental observation of nonlinear vacuum-polarization effects (VPEs) is of fundamental interest, especially to test the validity of QED in fields of such high strength. Highly charged ions with a charge number Z, such that $Z\alpha \lesssim 1$ with $\alpha = e^2/4\pi$ being the fine-structure constant, provide electric fields of the order of $E_{\rm cr}$ at the typical QED length scale $\lambda_c = 1/m$ (Compton length). This has facilitated the observation of VPEs induced by the strong Coulomb field of heavy ions via the measurement of Delbrück scattering, i.e., the scattering of a γ photon by an atomic field [2] and of the related process of γ-photon splitting in atomic fields [3]. Noticeably, in both cases effects of high-order corrections in $Z\alpha$ have been observed in the cross sections [4].

The continuous progress in laser technology opens unique opportunities to test QED in the presence of intense electromagnetic waves [5,6]. Petawatt-class lasers are currently under development, based on the optical parametric chirped pulse amplification technique in several laboratories [6]. The envisaged intensities of the order of 10^{24}–10^{26} W/cm^2 are likely to be attained at the Extreme Light Infrastructure (ELI) in the near future [7]. The first theoretical investigations of QED in the presence of a strong background laser field go back to the 1960s [8,9]. Later, the calculations of higher-order QED diagrams like the mass operator [10], the polarization operator [11], and the photon splitting [12] were performed. Refractive VPEs induced by laser fields have never been experimentally observed, essentially because attained laser intensities are much smaller than the critical intensity $I_{\rm cr} = E_{\rm cr}^2/8\pi = 2.3 \times 10^{29}$ W/cm^2 and laser-photon energies are much smaller than $m = 0.5$ MeV. To the best of our knowledge, the only feasible proposals to observe refractive VPEs in a laser field explored so far in the literature are restricted to perturbative VPEs in which the leading contributions in the laser field are the only, at least theoretically, observable effects while all higher-order effects are strongly suppressed [13].

In this Letter we put forward a scheme by which it is experimentally feasible to observe nonperturbative refractive VPEs in laser fields. We calculate the probability for an even number of laser photons to merge due to VPEs in the head-on collision of a high-energy proton beam and a strong laser field, by accounting exactly for the laser field. The nonperturbative QED is manifest in the opening of multiphoton vacuum-polarization channels as well as in the scaling of the probability.

To this end the proton beam has unique features. On the one hand, the proton is light enough to be accelerated up to very high energies, namely, to 980 GeV at the Fermilab Tevatron, or even to 7 TeV at the Large Hadron Collider (LHC) [14]. Then, the laser field amplitude and frequency in the proton rest frame are enhanced by the Lorentz γ factor of the proton so that the nonlinear QED regime can be attained, allowing in principle for the experimental observation of high-order nonperturbative VPEs. On the other hand, the proton mass $M = 938$ MeV is large enough that the background radiation arising from multiphoton Thomson scattering of the laser photons is, as we will see, very low. Moreover, since the proton charge number is $Z = 1$, here the VPEs are essentially driven by the strong laser field.

Without loss of generality we can consider a proton moving along the negative y direction with velocity β and a counterpropagating laser beam with amplitude E_0 and frequency ω_0. The laser is assumed to be linearly polarized along the z direction. The process of laser photons merging due to VPEs in a proton field is represented by the Feynman diagram in Fig. 1 where the right photon leg represents the outgoing photon resulting from the merging, the thick fermion loop indicates that the fermion propagators are calculated by accounting exactly for the laser field, and the crossed photon leg indicates the electromagnetic field of the proton which is taken into account

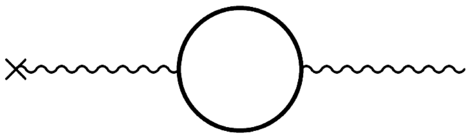

FIG. 1. Feynman diagram corresponding to the process of laser-photon merging induced by VPEs in a proton field.

as a perturbation since $Z = 1$ and is described by the four-potential $A^\mu(t, \mathbf{r}) = eu^\mu/4\pi\sqrt{x^2 + \gamma^2(y + \beta t)^2 + z^2}$ with $u^\mu = (\gamma, 0, -\beta\gamma, 0)$ being the proton four-velocity and $\gamma = 1/\sqrt{1 - \beta^2}$ its relativistic Lorentz factor. The probability amplitude of the laser-photon merging process can be calculated as $M(k) = (2\pi)^{-4} \times \int d^4q A_\mu(q)\Pi^{\mu\nu}(q, k)e^*_\nu(k)/\sqrt{2\omega}$ (see Fig. 1), where $A_\mu(q)$ is the four-dimensional Fourier transform of $A_\mu(t, \mathbf{r})$, $\Pi^{\mu\nu}(q, k)$ is the polarization tensor in a laser field [11], and $e_\nu(k)$ is the polarization four-vector of the outgoing real photon with four-momentum $k^\mu = (\omega, \mathbf{k})$ [15]. Starting from the above amplitude, one obtains the differential rate $d\mathcal{R}$ by applying Fermi's golden rule: $d\mathcal{R} = |M(k)|^2 d\mathbf{k}/(2\pi)^3$ (the quantization space volume and time interval are set equal to unity here) [15]. In the considered case of a monochromatic plane wave, $d\mathcal{R}$ can be written as $d\mathcal{R} = \sum_{n=1}^\infty d\mathcal{R}_{2n}$, where $d\mathcal{R}_{2n}$ is the differential rate of photons resulting from the merging of $2n$ laser photons (Furry's theorem prevents the merging of an odd number of laser photons in this process [15]). The integration over $\mathbf{k}$ is easier when employing spherical coordinates (ω, ϑ, φ) with the y-axis as the polar axis. The integrals over ω and φ can be carried out and the resulting differential rate $d\mathcal{R}_{2n}/d\vartheta$ written as

$$\frac{d\mathcal{R}_{2n}}{d\vartheta} = \frac{\alpha^3}{64\pi^2}\frac{(1+\beta)m^4}{\omega_0^3}\frac{\sin^3\vartheta}{(1-\cos\vartheta)^4} \times \frac{|c_{1,2n}|^2 + |c_{2,2n}|^2}{n^3}. \quad (1)$$

In this expression ϑ is the angle between the momentum of the emitted photon and the y axis and

$$c_{j,2n} = i^n \int_{-1}^{1} dv \int_0^\infty \frac{d\rho}{\rho} e^{-i\Phi_{2n}}\left\{\xi^2\left[\frac{A}{2}\mathcal{J}_n^*(z_{2n}) - \sin^2\rho J_n(z_{2n})\right]\delta_{j,1} + \xi^2\frac{\sin^2\rho}{1-v^2}\mathcal{J}_n(z_{2n}) + \frac{\eta_{2n}}{2}\left(n - i\frac{1-v^2}{4\rho}\right)J_n(z_{2n})\right\}, \quad (2)$$

where $j \in \{1, 2\}$, $\delta_{j,j'}$ is the Kronecker δ function, $\Phi_{2n} = 2n\rho + 4\rho\{1 + \xi^2[1 - \sin^2(\rho)/\rho^2]/2\}/\eta_{2n}(1 - v^2)$, $z_{2n} = 2\rho\xi^2[\sin^2(\rho)/\rho^2 - \sin(2\rho)/2\rho]/\eta_{2n}(1 - v^2)$, $A = 1 + \sin^2(\rho)/\rho^2 - \sin(2\rho)/\rho$, and $\mathcal{J}_n(z) = J_n(z) + iJ'_n(z)$ with $J_n(z)$ being the ordinary Bessel function of order n and $J'_n(z)$ its derivative. The coefficients $c_{j,2n}$ depend on the two Lorentz- and gauge-invariant quantities $\xi = eE_0/m\omega_0$ and $\eta_{2n} = (k_0k_{2n})/m^2 = \omega_0\omega_{2n}(1 - \cos\vartheta)/m^2$, where $k_0^\mu = (\omega_0, 0, \omega_0, 0)$ is the four-momentum of the laser photons and k_{2n}^μ is the four-momentum of the outgoing photon whose energy $\omega_{2n} = 2n\omega_0(1 + \beta)/(1 + \beta\cos\vartheta)$ depends on the number of laser photons merged, according to the energy conservation and the Doppler shift. When the proton is highly relativistic, ω_{2n} is $4\gamma^2$ times larger than $2n\omega_0$ for backscattered photons. The quantity $\sqrt{2\eta_{2n}}$ is the total energy of a laser photon and of an emitted photon with four-momentum k_{2n} in their center-of-momentum system in units of m.

In the asymptotic limit $\xi \ll 1$ of weak laser fields, the multiphoton vacuum processes are suppressed, and the leading rate in Eq. (1) is that with $n = 1$ and it coincides with the result of the Feynman box diagram corresponding to the perturbative treatment of the laser field. The rate increases with increasing ξ. Moreover, present optical lasers easily exceed the electron relativistic threshold $\xi \approx 1$ (laser intensity $I_0 = E_0^2/8\pi \approx 10^{18}$ W/cm^2 at $\omega_0 \approx 1$ eV). Therefore, we discuss here only the most practical regime $\xi \gg 1$. In this asymptotic limit the dynamics is determined by the parameter χ_{2n}:

$$\chi_{2n} \equiv \xi\eta_{2n} = \sqrt{\frac{I_0}{I_{\rm cr}}}\frac{2n(1+\beta)\omega_0}{m}\frac{1-\cos\vartheta}{1+\beta\cos\vartheta}. \quad (3)$$

The leading-order contribution to the coefficients $c_{j,2n}$ depends only on χ_{2n} and is given by

$$c_{j,2n} = e^{-i\pi/3}\int_0^1 dv \int_0^\infty \frac{d\lambda}{\lambda} e^{-\exp(i\pi/3)\lambda - x_{2n}} b_{j,2n}, \quad (4)$$

where $x_{2n} = \chi_{2n}^2\lambda^3(1 - v^2)^2/96$ and

$$b_{j,2n} = j\chi_{2n}^2\lambda^2\frac{1 - v^{4/j}}{16}[I_n(x_{2n}) - I'_n(x_{2n})] + \frac{I_n(x_{2n})}{\lambda}, \quad (5)$$

with $I_n(x)$ being the modified Bessel function of order n and $I'_n(x)$ its derivative. The dependence of the differential rate $d\mathcal{R}_{2n}/d\vartheta$ on the parameter χ_{2n} is highly nonperturbative. χ_{2n} is the so-called quantum intensity parameter responsible for the magnitude of the nonperturbative effects in the present regime. The perturbative results are obtained only if $\chi_{2n} \ll 1$ (the leading contributions at $\xi \gg 1$ and $\chi_{2n} \ll 1$ coincide here with those at $\xi \ll 1$ and $\eta_{2n} \ll 1$). In this case, the leading-order asymptotics of the coefficients $c_{j,2n}$ have the typical perturbative scaling as χ_{2n}^{2n}:

$$c_{j,2n} = (-1)^{n+1}\sqrt{\pi}\frac{(3n-2)!}{\Gamma(2n+3/2)}\frac{(2n)!}{n!}\frac{3jn+1}{192^n}\chi_{2n}^{2n}, \quad (6)$$

with the gamma function $\Gamma(x)$. The higher-order terms in the $2n$-photon merging amplitude are proportional to

$\chi_{2n}^{2(n+l)}$, with l being a positive integer, and stem from the merging of $2n$ laser photons accompanied by the exchange of l laser photons without net absorption. In the particular case of $n = 1$ and $\beta = 0$ our expression of the total rate $\mathcal{R}_2 = \int_0^\pi d\vartheta d\mathcal{R}_2/d\vartheta$ is in agreement with the results in the paper by Milstein *et al.* in [13].

In the opposite limit of large χ_{2n} we obtain

$$c_{j,2n} = -\frac{2e^{-i\pi/3}}{21}\frac{j+1}{6^{1/3}}\frac{\Gamma(2/3)\Gamma(5/6)}{\Gamma(7/6)}\frac{\Gamma(n-1/3)}{\Gamma(n+4/3)}\chi_{2n}^{2/3}. \tag{7}$$

In this regime of ultrastrong laser fields where $\chi_{2n} \gg 1$, the differential rate scales nonperturbatively with χ_{2n}. However, the rate *slowly* decreases with n and behaves at large n as $d\mathcal{R}_{2n}/d\vartheta \sim 1/n^5$.

We consider below two numerical examples corresponding to different possible experimental conditions in which the process of photon merging is observable. In the first one we consider the interaction between a petawatt laser pulse and a proton bunch with parameters available at the LHC. We use the following laser parameters [7,16]: a pulse energy of 3 J, a pulse duration of 5 fs at 10 Hz repetition rate, and, the beam being focused onto one wavelength of 0.8 μm, an intensity of $I_0 = 3 \times 10^{22}$ W/cm^2. The main parameters of the proton bunch are [14] a proton energy of 7 TeV, a number of protons per bunch of 11.5×10^{10}, a bunch transversal radius of 16.6 μm, and a bunch length of 7.55 cm. In this regime there is a competing process to the process of $(2n)$-photon merging ($2n\gamma$M): the $(2n)$-photon Thomson scattering ($2n\gamma$TS) of the laser photons by the proton beam. In fact, the energies of the photons produced via $2n\gamma$M and via $2n\gamma$TS are equal. The two processes interfere with each other and the total photon rate has to be calculated accordingly by summing up the two amplitudes. In Fig. 2 we compare the differential rate $d\mathcal{W}^{(2)}/d\vartheta$ of the photons emitted only via 2γTS (dashed line, result via [9]) with the total differential rate $d\mathcal{T}^{(2)}/d\vartheta$ including also the photons resulting from the 2γM (continuous line).

The differential yields with and without the VPEs contribution are clearly distinct. This becomes more apparent if we compare the total rate of photons emitted only via 2γTS with the total rate of the two processes included. By integrating $d\mathcal{W}^{(2)}/d\vartheta$ and $d\mathcal{T}^{(2)}/d\vartheta$ with respect to ϑ and by multiplying the resulting quantities with the laser pulse duration and repetition rate as well as the effective number of protons which interacts with the laser beam, we obtain approximately 320 events per hour without including the photons arising from 2γM and about 670 events per hour when including them. It is interesting to observe that the total rate of photons resulting only from 2γM is about 390 photons per hour, showing a destructive interference effect between the two processes of about 5%–6%. On the upper horizontal axis in Fig. 2 we have indicated the value of the parameter χ_2 at the corresponding angle ϑ: it is evident that in the relevant region of the spectrum $\chi_2 \sim 1$

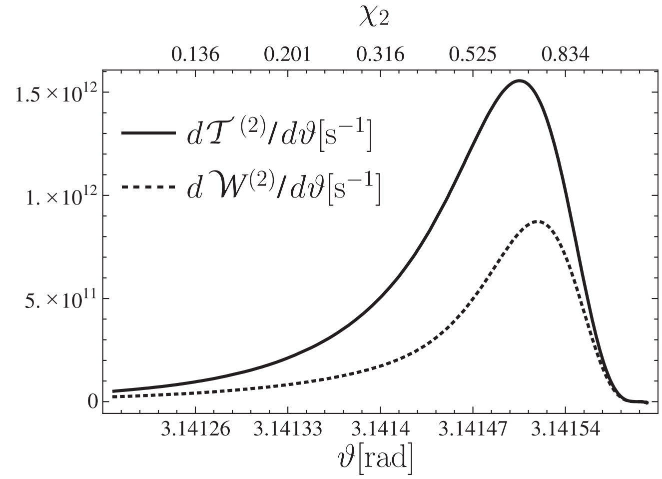

FIG. 2. The rate per unit angle ϑ of photons emitted only via 2γTS (dashed line) and via both 2γTS and 2γM (continuous line). The upper horizontal axis shows the values of the parameter χ_2 as a function of ϑ [see Eq. (3)].

and the perturbative approach is inadequate [17]. Finally, due to the nonperturbative nature of the process in this regime, the rate of photons resulting from the merging of four laser photons is not small either: 5.4 events per hour. In this case the 4γTS is safely negligible due to its scaling as ξ_p^8, with $\xi_p = (m/M)\xi \approx 6.4 \times 10^{-2}$. In the previous proposals [13], the corresponding multiphoton channels involving more laser photons than the minimum required are suppressed by several orders of magnitude.

In the following example we consider the possibility of employing the already operative Tevatron accelerator. The relevant parameters of the proton bunches at the Tevatron are [14] as follows: a proton energy of 980 GeV, a number of protons per bunch of 24×10^{10}, a bunch transversal radius of 29 μm, and a bunch length of 50 cm. The relatively small proton energy here can be compensated by employing a strong attosecond pulse of extreme ultraviolet (XUV) radiation (see [18]): intensity of $I_0 = 3.8 \times 10^{22}$ W/cm^2, photon energy of 70 eV, pulse duration of 60 as, and repetition rate of 10 Hz [19]. In this setup, the photon rate produced via 2γTS and 4γTS is completely negligible. We report only the average number of photons emitted in 1 h by the merging of two and of four laser photons, which is approximately equal to 490 and 6.6, respectively. Also in this case the values of the parameter χ_{2n} near the maximum of the photon angular spectrum does not permit the use of perturbation theory as $\chi_2 = \chi_4/2 \approx 0.7$ at $\vartheta = 3.1410$ rad. At the same angle the energy of the emitted photon is about 440 MeV. By employing the laser parameters of ELI (70 PW Ti:sapphire laser focused onto 5 μm, 5 fs pulse duration at 1 Hz repetition rate; see [18,19]) we would obtain up to two photons per pulse from 2γM and even the merging of ten laser photons would be experimentally accessible (two photons per hour).

In the collision of a laser pulse with a proton beam, Delbrück scattering can occur as well. While this process is clearly distinguishable from the process at hand insofar that the frequency of the scattered photon is different from ω_{2n} for any n, the question arises if the Delbrück scattering can compete with the photon merging process in terms of probability. For the considered setup, the answer is no. In fact, starting from the Born-approximated cross section of the Delbrück scattering which is certainly suitable at $Z = 1$ (see, e.g., [4]), it can be shown that the order of magnitude of the ratio between the number of photons scattered per unit time via Delbrück scattering and the rate $\mathcal{R}_2$ is given by $(\alpha/\xi)^2$. This quantity is very small in the situations considered above.

In the nonperturbative regime of laser-proton interaction when $\chi_{2n} \sim 1$, electron-positron pair creation may occur. By estimating the pair production rate as $\alpha^2 m/\gamma$ [20], one can show that the depletion of the laser energy due to this process is negligible by several orders of magnitude. The photons backscattered by these pairs in the laser field represent a background for our process. We have shown that in the first example considered, the photons produced in the relevant energy range of 450–500 MeV represent at about 1% of those produced only by the merging. In the second example, however, the electrons and positrons are created on average with an energy of about 250 MeV which is too small to produce photons with an energy of 440 MeV.

The produced high-energy photons still travel through the laser beam almost in the opposite direction and could decay via the production of electron-positron pairs. The rate of pair production by a photon with energy ω_2 in a laser field with $\xi \gg 1$ and $\chi_2 \sim 1$ can be estimated as $\sim 10^{-2}\alpha m^2/\omega_2$ by employing the results in [11]. Then, the lifetime in the laser field of a photon with an energy $\sim$400–500 MeV is around 14–17 fs, i.e., longer than the considered laser beam durations. Therefore, the produced photons are able to escape from the region where the strong laser field is present and can, in principle, be detected. A more quantitative estimate of the photon yield also requires an accurate knowledge of the beams' characteristics. Qualitatively we can argue that the spatiotemporal shape of the beams will not have a significant impact on our estimate of the total photon yield as the latter takes into account the reduction of the interaction volume due to the beams' space-time confinement and overlapping.

In conclusion, we have shown that existing proton accelerators combined with next generation tabletop petawatt lasers allow for measuring nonperturbative refractive VPEs in a laser field. These become manifest in the opening of multiphoton channels of interaction as well as in the enhanced contribution of high-order effects.

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