servable effects.

The quantitative feature of the experimental RW[2] results which does not agree with the AG theory is c_S/c_0. Here c_S is the impurity concentration for which $T_S = 0$, while c_0 is the concentration for which $\omega_0 = 0$. According to AG, $c_S/c_0 = 1.1$, whereas RW find $c_S/c_0 \simeq 2$.

We believe that this discrepancy is due to the limitations of the optical model, which does not allow for adiabatic correlations of electron pairs. The fraction of excited states mixed into the ground state[10] by magnetic scattering is of order $f = \exp(-\Delta\tau_S)$. For such states the sign of Δ should be reversed, just as it is above θ, the Debye frequency.[11] Within the framework of the optical model this is not feasible. In principle, at least, we can construct separated wave packets of basis states in the magnetic impurity bands consisting wholly of "ground" states or wholly of "excited" states (relative to the superconductor with only orbital scattering). The fractional energy gained by reversing the sign of Δ for the "excited" states would then be $2f$, which accounts qualitatively for the RW value of c_S/c_0.

I am grateful to Professor F. Reif for communicating further details of his experiment prior to publication, and to the Sloan Foundation for a generous grant.

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[1]A. A. Abrikosov and L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. 39, 1781 (1960) [translation: Soviet Phys.—JETP 12, 1243 (1961)].

[2]F. Reif and M. A. Woolf, Phys. Rev. Letters 9, 315 (1962), hereafter referred to as RW.

[3]M. Tinkham and R. A. Ferrell, Phys. Rev. Letters 2, 331 (1959).

[4]P. W. Anderson, J. Phys. Chem. Solids 11, 26 (1959).

[5]In the Gor'kov formalism the Anderson states are responsible for the isotropic one-dimensional $(\omega - \Delta)^{-1/2}$ singularity in $N_S(\omega)$ in the presence of nonmagnetic scattering. If our conjecture about wave packets in the presence of magnetic scattering is correct, there should be a threshold in $N_S(\omega)$ of the three-dimensional type, i.e., proportional to $(\omega - \omega_0)^{1/2}$. This is exhibited implicitly in Eq. (49d) of AG. The wave packet at $\omega = \omega_0$ is distinguished as that combination of quasi-particle states which maximizes the expectation value of $|\vec{\sigma}\cdot\vec{S}|$. There $\vec{\sigma}$ is the spin density of the wave packet and $\vec{S}$ is the total paramagnetic spin density.

[6]According to reference 3, half of the supercurrent is due to differences in the density of states and half to matrix elements (coherence factors). According to the principle of spectroscopic stability, the formation of wave packets in the presence of magnetic scattering should not alter this greatly. I have therefore added a factor of 2 to the right-hand side of (12) to take account of coherence factors.

[7]M. Lax and J. C. Phillips, Phys. Rev. 110, 41 (1958).

[8]R. E. B. Makinson and A. P. Roberts, Australian J. Phys. 13, 437 (1960).

[9]H. Suhl, D. R. Fredkin, J. S. Langer, and B. T. Matthias, Phys. Rev. Letters 9, 63 (1962).

[10]E. O. Kane, J. Appl. Phys. 32, 83 (1961).

[11]P. Morel and P. W. Anderson, Phys. Rev. 125, 1263 (1962).

THERMAL CONDUCTIVITY OF SUPERCONDUCTORS OF THE SECOND KIND*

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(Received 27 December 1962)

We have measured the thermal conductivity as a function of magnetic field of a superconducting alloy with sufficiently short mean free path to have a negative interphase surface energy.[1] We used a cylindrical specimen of indium with about 3 at. % bismuth in a longitudinal field. At temperatures low compared to the superconducting transition temperature T_c, the conductivity goes through a pronounced minimum, similar to that observed by Sladek in indium-thallium alloys.[2] We have been able to explain this behavior in terms of an energy-gap-dependent variation of the lattice and electronic thermal conductivities, without having to postulate any additional scattering mechanism. The variation of the effective energy gap with field in the mixed state which is indicated by the experiment is in agreement with that expected from Abrikosov's model[3] for superconductors with negative surface energy, now generally called superconductors of the second kind.

Our specimen was annealed for about six weeks, and had a residual resistivity of 5.94 $\mu\Omega$-cm, and a transition temperature of 4.21°K. The measurements were made in an apparatus described previously.[4] A typical curve of the thermal conductivity as a function of the external field H_e is shown in Fig. 1 for 2.16°K. The conductivity is seen to drop rapidly near a field H_{c2}, after which it goes through a minimum before reaching a limiting value at a field H_{c1}. When the field is decreased the curve is closely reproduced to H_{c2}, where it rises to a value lower than the initial one, indicating a considerable fraction of frozen-in flux.

The existence of two critical fields, $H_{c2} < H_c < H_{c1}$ (where H_c is the thermodynamic critical field), between which the specimen is in a so-called mixed state, is characteristic of superconductors of the second kind.[3,5] The detailed magnetic behavior of these materials has been calculated by Abrikosov[3] in a treatment based on the Ginzburg-Landau theory,[6] and has recently been verified by magnetization measurements on a specimen of very similar composition to that used in this experiment.[7] The ratio of H_{c1} to H_{c2} near the transition temperature is related to the Ginzburg-Landau parameter κ,[5] which may also be deduced from the residual resistivity.[8] For our specimen the measured residual resistivity indicates a value $\kappa = 1.41$; the ratio of H_{c1} to H_{c2} near T_c is consistent with this value, and their temperature dependence is also as expected.[7] This justifies their interpretation as the limiting fields of the mixed state of the specimen.

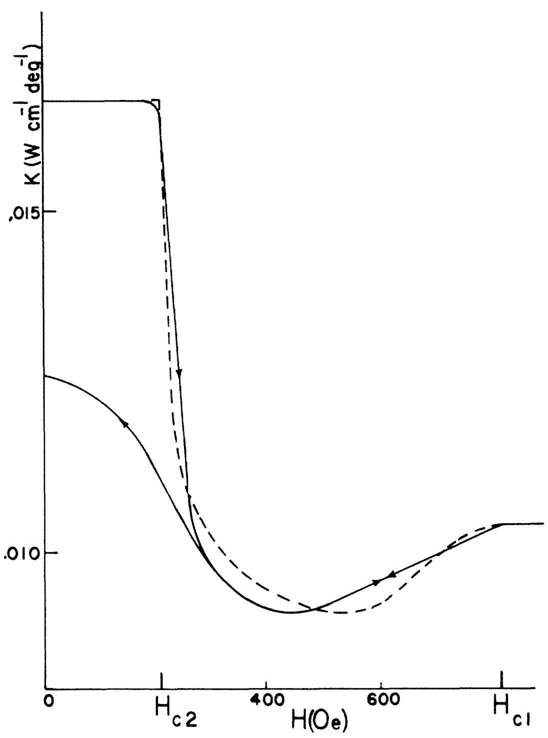

FIG. 1. The variation of the thermal conductivity K (W cm^{-1} deg^{-1}) of an In + 3 at. % Bi specimen with magnetic field H (Oe) at $T = 2.16$°K. The solid lines show the experimental results in increasing and decreasing field. The dashed line shows the thermal conductivity calculated from the magnetization.

Our results at several temperatures, as well as those of Sladek for an In + 20 at. % Tl sample,[2] are summarized in Table I, the second through fourth columns of which list the observed values of the conductivity in the superconducting phase, K_s, that in the normal phase, K_n, and the value at the minimum, $(K_{\rm min})_{\rm exp}$.

The success of the Ginzburg-Landau-Abrikosov (GLA) model in describing the magnetization of superconductors of the second kind[7,9] leads one to accept one of its fundamental statements, which is that in the mixed state the specimen is still entirely superconducting except along filaments of negligible volume. The specific heat measurements on V_3Ga also support this view.[10,11] We therefore characterize the specimen as a whole in the mixed state by an average, field-dependent energy gap $\bar{\epsilon}(H)$. According to the theory of Bardeen, Rickayzen, and Tewordt (BRT),[12] the electronic conductivity ratio K_{es}/K_{en} and the lattice conductivity ratio K_{gs}/K_{gn} are functions of the single parameter $y(T) = \epsilon(T)/kT$, where $\epsilon(T)$ is the energy gap in the pure superconducting phase. We now make the assumption that $\bar{\epsilon}(H)$, characterizing the mixed state at some temperature, determines the field variation of K_{em}/K_{en} and of K_{gm}/K_{gn} (where the subscript m denotes the mixed state) in the same way in which the temperature dependence of $\epsilon(T)$ affects the ratios K_{es}/K_{en} and K_{gs}/K_{gn}. In other words, we define a parameter $y(H) = \bar{\epsilon}(H)/kT$ which we substitute for $y(T)$ in the BRT expressions for the conductivity ratios.[13]

To carry out the calculations we used the separation of the heat conductivities in the superconducting and normal phases into lattice and electronic contributions, as well as the temperature dependence of K_{es}/K_{en} and of K_{gs}/K_{gn}, obtained in recent measurements of K_s and K_n of different indium alloys.[14] The separated conductivities are listed in Table I. The formulas of BRT are known to be reasonably successful in describing the ratio[15] K_{es}/K_{en} when the electron conductivities are primarily limited by im-

Table I. Values at different temperatures for our In + 3 at. % Bi and Sladek's In + 20 at. % Tl specimens of the following thermal conductivities, all in mW cm^{-1} deg^{-1}: the observed total conductivity in the superconducting phase, K_s, in the normal phase, K_n, and at the minimum, $(K_{\min})_{\exp}$, the calculated value at the minimum $(K_{\min})_{\text{calc}}$, as well as the separated components, K_{es}, K_{gs}, K_{en}, and K_{gn}.

T (°K)	K_n	K_s	$(K_{\min})_{\exp}$ (mW cm^{-1} deg^{-1})	$(K_{\min})_{\text{calc}}$	K_{es}	K_{gs}	K_{en}	K_{gn}
			In + 3 % Bi					
3.31	16.8	18.6	16.7	16.8	10.6	8.0	13.5	3.3
2.72	13.5	17.0	12.9	13.1	6.2	10.8	11.1	2.4
2.16	10.3	16.6	9.0	9.1	3.0	13.6	8.9	1.4
2.05	9.6	16.9	8.2	8.2	2.3	14.6	8.3	1.3
1.85	8.6	17.3	7.2	7.6	1.7	15.6	7.6	1.0
1.62	7.4	18.1	5.8	6.3	1.0	17.1	6.6	0.8
			In + 20 % Tl					
2.03	14.9	15.7	13.8	13.6	6.8	8.9	13.3	1.6
1.77	12.9	14.4	11.4	11.6	4.4	10.0	11.6	1.3
1.61	11.8	14.0	9.9	10.0	3.1	10.9	10.6	1.2
1.45	10.6	13.9	8.7	9.1	2.1	11.8	9.5	1.0

purity scattering, and the ratio K_{gs}/K_{gn} when the lattice conductivities are determined only by electron scattering.[14,16] The measurements just mentioned[14] also provide the corrections which have to be made in our analysis for the phonon-impurity and phonon-boundary scattering, which we assume to be the same in the normal and superconducting phases and hence field-independent in the mixed state. It might be noted that in the superconducting phase at our lowest temperatures the resistances to lattice conduction due to these additional scattering mechanisms are together roughly equal to the resistance caused by the electrons.

The results of these calculations yield K_{em} and K_{gm}, as well as $K_m = K_{em} + K_{gm}$, at a given temperature as a function of $\bar{\epsilon}(H)$. These conductivities at 2.16°K are plotted in Fig. 2, showing a minimum very nearly equal to that actually measured at this temperature. In Table I we list under $(K_{\min})_{\text{calc}}$ the values of the conductivity minima obtained by this analysis at other temperatures both for our specimen and for Sladek's; the general agreement is very satisfactory. The depth of the minimum increases with decreasing temperature because the conductivity ratios do; when the conductivity components do not change very much in going from one phase to another the minimum may be very shallow or even absent.

We turn again to the GLA model to investigate the details of the variation of the conductivity with magnetic field, in particular the rapid drop

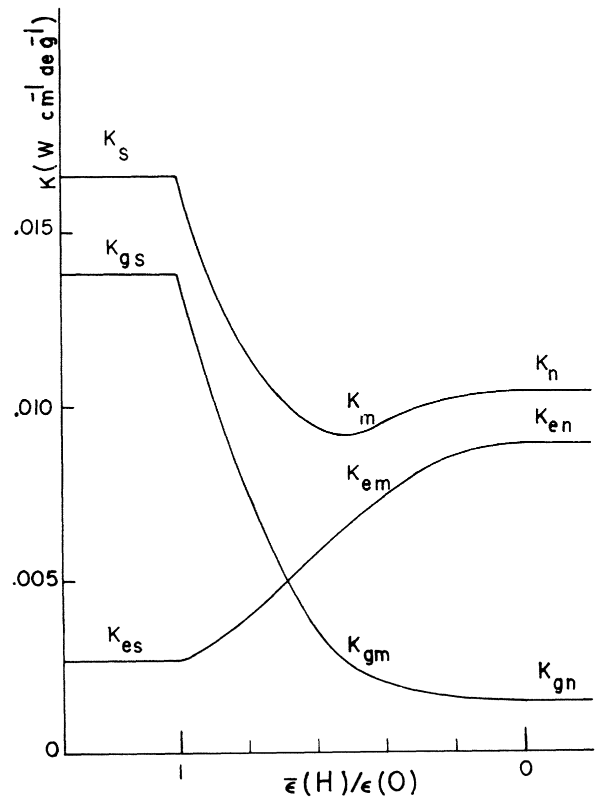

FIG. 2. The calculated variation of the electric conductivity, K_{em}, the lattice conductivity, K_{gm}, and the total conductivity, K_m, in the mixed state as a function of the average energy gap $\bar{\epsilon}(H)$ in units of the zero-field gap $\epsilon(0)$. The figure also shows the total conductivities and their electronic and lattice components in the superconducting and normal phases.

near H_{c2} observed in our measurements. Abrikosov[3] characterizes the mixed state by the mean square of the order parameter Ψ, and shows this to vary linearly with the magnitization near H_{c1} and T_c. As the order parameter is proportional to the energy gap,[17] the GLA model thus suggests that the average energy gap in the mixed state is proportional to the square root of the magnetization, at least near H_{c1} and T_c. As a first approximation we extend this to all temperatures and throughout the mixed state, and use it and our knowledge of the dependence of the magnetization on field and on[3,7] κ to find $\bar{\epsilon}(H)$ as a function of H_e. This, in turn, allows us to calculate K_m as a function of H_e. The dashed line in Fig. 1 shows this calculated variation of K_m at 2.16°K. The main features of the experimental curve are seen to be reproduced.

According to the GLA model, the actual order parameter, and hence the energy gap, varies periodically in a plane perpendicular to the applied field. The period is of the order of the coherence length which, in turn, approximately equals the electronic mean free path. It is thus conceivable that the spatial variation of the gap introduces a scattering mechanism for electrons further limiting their mean free path. This would affect both K_e and K_g.[4,14] However, we are unable to explain our results and Sladek's by a mean-free-path change in any consistent fashion.

We are very grateful to Professor B. Serin and Professor E. Abrahams for a number of illuminating conversations.

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*Work supported by the National Science Foundation.

[1]See, for example, E. A. Lynton, Superconductivity (Methuen and Company, Ltd., London, England, and John Wiley & Sons, Inc., New York, 1962), Chap. VI.

[2]R. J. Sladek, Phys. Rev. 97, 902 (1955). The author used samples the magnetic behavior of which had been studied by J. W. Stout and L. Guttman [Phys. Rev. 88, 703 (1952)]. Some of these, in particular the single crystal of indium containing 20 at. % Tl, have recently been recognized as good examples of superconductors of the second kind [reference 7; T. G. Berlincourt (unpublished); Proceedings of the Eighth International Conference on Low-Temperature Physics, London, 1962 (Butterworths Scientific Publications, Ltd., London, 1962)].

[3]A. A. Abrikosov, Zh. Eksperim. i Teor. Fiz. 32, 1442 (1957) [translation: Soviet Phys.-JETP 5, 1174 (1957)]; J. Phys. Chem. Solids 2, 199 (1957).

[4]P. Lindenfeld and W. B. Pennebaker, Phys. Rev. 127, 1881 (1962).

[5]B. B. Goodman, Phys. Rev. Letters 6, 597 (1961); IBM J. Res. Develop. 6, 63 (1962).

[6]V. L. Ginzburg and L. D. Landau, Zh. Eksperim. i Teor. Fiz. 20, 1064 (1950); see, for example, reference 1, Chap. V.

[7]T. Kinsel, E. A. Lynton, and B. Serin, Phys. Letters 3, 30 (1962).

[8]L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. 37, 1407 (1959) [translation: Soviet Phys.-JETP 10, 998 (1960)].

[9]See, for example, reference 7 for a number of references.

[10]F. J. Morin, J. P. Maita, H. J. Williams, R. C. Sherwood, J. H. Wernick, and J. E. Kunzler, Phys. Rev. Letters 8, 275 (1962).

[11]B. B. Goodman, Phys. Letters 1, 215 (1962).

[12]J. Bardeen, G. Rickayzen, and L. Tewordt, Phys. Rev. 113, 982 (1959).

[13]We are grateful to Dr. R. E. Jones of the IBM Thomas J. Watson Research Center for sending us a table of $y(T)$ vs T calculated by Dr. J. C. Swihart.

[14]P. Lindenfeld and H. Rohrer (to be published).

[15]See, for example, C. B. Satterthwaite, Phys. Rev. 125, 873 (1962).

[16]A. Connolly and K. Mendelssohn, Proc. Roy. Soc. (London) A266, 429 (1962).

[17]L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. 36, 1918 (1959) [translation: Soviet Phys.-JETP 9, 1364 (1960)].