noise level that is due to the electron beam and its standard deviation, (c) the contribution from the "laser noise," and (d) an oscilloscope trace of the laser output.

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POINT-BY-POINT MAPPING OF THE FERMI SURFACE

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The recently discovered phenomenon of magnetoplasma oscillations or helicons in metals[1-5] suggests a new method to determine the shape of the Fermi surface which for simple shapes should permit a point-by-point mapping of the surface. The helicons are observed in the limit when $\omega_c\tau \gg 1$, where ω_c is the lowest cyclotron frequency of the electrons and τ is their relaxation lifetime. In this limit and for the wavelengths that have been experimentally excited, the attenuation of these oscillations is small. However, if frequencies higher than those already observed but still much less than ω_c are excited, a new mechanism for absorption of these oscillations should appear. The wavelength of the oscillations at which this new attenuation mechanism appears depends on the geometric property of the Fermi surface and, in particular, may depend on the geometric property of a single point of this surface.

The physical origin of this attenuation can be easily understood. If in the metal a helicon of frequency ω and wave number k_H propagating along the magnetic field is excited, then an electron of average velocity $\bar{v}_H$ along the magnetic field experiences, because of the usual Doppler shift, a frequency

$$\omega' = \omega \pm \bar{v}_H k_H. \tag{1}$$

If ω' coincides with the cyclotron frequency of the electron ω_c, the absorption of the energy of the helicon by the electron will occur. In metals an estimate of the frequency ω at which this absorption will occur indicates that $\omega \ll \omega_c$, and thus absorption occurs when

$$\omega_c = \bar{v}_H k_H \tag{1'}$$

is satisfied for some electrons in the metal. The theory of this Doppler-shifted absorption has already been given for simple models of solids[6,7] and plasmas.[8] The form in which (1′) can be expressed for an arbitrarily shaped Fermi surface will be given here.

Experimentally it is possible to measure both ω and k_H of the helicon. However, only k_H enters in (1′), and thus experimentally one obtains information only on the ratio $\omega_c/\bar{v}_H$. The effect of the magnetic field, in a semiclassical picture, is to cause the electrons to move in the orbits in reciprocal space (k space) determined by the intersection of planes normal to the magnetic field and the Fermi surface, and satisfying the equation[9]

$$\hbar(d\vec{k}/dt) = e\vec{v}\times\vec{H}. \tag{2}$$

The period of revolution of an electron T is obtained directly from (2) to be

$$T = \frac{\hbar}{eH}\oint \frac{dk_t}{v_\perp}, \tag{3}$$

where dk_t is an element in reciprocal space tangent to the orbit, $v_\perp$ is the instantaneous component of the velocity of the electron in the plane normal to H, and the integration is around the orbit. The average velocity of an electron along H, $\bar{v}_H$, is given by

$$\bar{v}_H = \frac{\hbar}{eHT}\oint \frac{v_H dk_t}{v_\perp}, \tag{4}$$

where v_H is the instantaneous component of velocity of the electron along H. Since $\omega_c = 2\pi/T$, a combination of (4) and (1′) gives that

$$k_H = 2\pi eH\left(\hbar \oint \frac{v_H}{v_\perp} dk_t\right)^{-1}. \tag{5}$$

Using the relations that

$$v_\perp = v\sin\theta,$$

$$v_H = v\cos\theta,$$

$$dk_t = \rho_\perp d\phi,$$

$$\rho_\perp = \rho\sin\theta, \tag{6}$$

where θ is the angle between the normal to the Fermi surface and H, ϕ is the angle in the plane of the orbit between the normal to the orbit and a fixed direction in the plane, ρ is the radius of curvature of a normal section of the Fermi surface formed by the intersection of the Fermi surface and the plane which includes the normal to the Fermi surface and the tangent to the orbit, (5) can be written in the form

$$k_H = 2\pi e H(\hbar \oint \rho\cos\theta d\phi)^{-1}. \tag{5$'$}$$

In this form it is seen that k_H depends only on the geometric properties of the Fermi surface. The onset of the Doppler-shifted absorptions will occur for that orbit where (5′) is a minimum. In the case of a sphere (5′) is a minimum at a point; in particular, at that point whose normal is along H. In general, if (5′) has a minimum at a point on the Fermi surface, it is always at that point whose normal is along H. In this case (5′) reduces to

$$k_H = eH[\hbar(\rho_1\rho_2)^{1/2}]^{-1}, \tag{7}$$

where ρ_1 and ρ_2 are the two principal radii of curvature of the Fermi surface at the point. In general, there will be more than one point whose normal is along H, but not all of these points will contribute to the absorption. The helicon is a circularly polarized wave, and only when the direction of the angular frequency of the electric field observed by the electron is in the same sense as its rotation in its orbit will it contribute to the absorption. Holes, for example, have to move in the opposite direction from electrons in order to contribute to the absorption. Thus, for the case of a sphere, only one of the two points will contribute to the onset of the absorption since they correspond to electrons moving in opposite directions.

One can imagine Fermi surfaces where (5′) is not a minimum at a point but on some finite-sized orbit. It is important to be able to distinguish between this case and that of a point in order to be able to interpret the experimental data unambiguously. The manner of the onset of the absorption permits this distinction. One can show that the real part of the conductivity σ_1 for a helicon, as a function of k_H has the general form, when $\omega_c\tau \gg 1$,

$$\sigma_1(k_H) = [B(k_H)\bar{v}_\perp \oint dk_t]\left(k_H \frac{d\bar{v}_H}{dk_H}\right)^{-1}, \tag{8}$$

where $B(k_H)$ is a complicated nonzero function of the shape of the Fermi surface and the variation of $v_\perp$ over an orbit, the integration is over the orbit that satisfies (5′), and $\bar{v}_\perp$ is an average of $v_\perp$ over this orbit. When the onset of the absorption occurs at a point, $\sigma_1(k_H) = 0$ because both $\bar{v}_\perp$ and $\oint dk_t$ are zero. It can be shown that $d\sigma_1(k_H)/dk_H$ is finite also in this case. However, when the onset of the absorption occurs for a finite-sized orbit, $\sigma_1(k_H)$ is nonzero and $d\sigma_1(k_H)/dk_H$ is infinite. Thus the case of a point or finite-sized orbit is clearly distinguishable by the manner of the onset of the absorption. For the special case where the Fermi surface is a sphere, (8) becomes[6]

$$\sigma_1(k_H) = \frac{3\pi n e^2}{4p_0 k_H}\left[1 - \left(\frac{\omega_c}{v_0 k_H}\right)^2\right], \quad \omega_c \le v_0 k_H,$$

$$= 0, \qquad \omega_c > v_0 k_H,$$

where p_0 is the Fermi momentum, v_0 is the constant Fermi velocity, and n is the number of electrons per unit volume.

In summary it has been shown that the wave number at which the onset of the Doppler-shifted absorption occurs is dependent on only the shape of the Fermi surface. If this onset is comparatively slow, it measures the geometric property given in (7) of a point on the Fermi surface whose normal is along H. If this onset is abrupt, it measures the geometric property given in (5′) of the finite-sized orbit for which (5′) is a minimum.

For simple convex shapes of the Fermi surface such as one expects to find in the alkali metals, it is expected that the onset of absorption will be caused almost entirely by points on the Fermi surface, and a point-by-point measurement of the Fermi surface should be possible. All present methods of measuring the shape of the Fermi surface measure a geometric property of a complete orbit and are not as sensitive to, say, small deviations from a spherical shape as a point-by-point measurement would be.

The method proposed here can also be used to determine the shape of the Fermi surface in semimetals and semiconductors. In fact, measurements of this effect in bismuth have already been made.[10] In these materials it is not always true that $\omega \ll \omega_c$, and the equations given here have to be modified accordingly. In particular, k_H in Eqs. (1′), (5′), and (7) has to be replaced by $k_H(1 - \omega/\omega_c)^{-1}$. In these materials, because $\omega \approx \omega_c$, the same equipment that measures the properties of the helicons (ω and k_H) can also measure ω_c and thus determine the same geometric properties as for metals.

It has been pointed out to the author after this paper was written that the analysis presented here for helicons has already been done for the case of circularly polarized transverse sound waves.[11,12] The general ideas in the analysis for both helicons and circularly polarized sound waves propagating along a magnetic field are the same, but the details are somewhat different. However, helicons appear to be a much more useful means than circularly polarized sound waves to study this onset of the Doppler-shifted absorption. It is not possible to produce circularly polarized transverse sound waves except along a few high-symmetry directions.[11,12] Helicons do not have this limitation and can propagate in all directions. It also appears that the distinction between the onset of absorption at a point or at a finite-sized orbit is more striking for helicons.

In conclusion, a caution should be inserted. The derivation given here has assumed an independent-particle model for the electrons in the metal. If many-body effects are important they may invalidate the results given here.

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OBSERVATION OF PERSISTENT CURRENT IN A SUPERCONDUCTING SOLENOID

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The classical experiment of Onnes,[1] in which a persistent current is induced in a closed superconducting circuit and the trapped magnetic field observed over a period of time, has been repeated several times.[2-6] From the length of the period of observation and the accuracy of the measurement, one can set a lower limit to the time constant of the circuit. Apparently the highest value to this limit was set in the experiment of Collins[3] at a value of approximately 250 years.

To extend this limit by several orders of magnitude, we undertook to apply modern nuclear magnetic resonance (NMR) techniques to the measurement of the field. A double layered solenoid of 984 turns of 0.020-in. diameter Nb-25% Zr alloy approximately 4 in. in diameter and 10 in. long as shown in Fig. 1 was constructed to provide a homogeneous field. The measured axial field profile of the central 0.5 in. is shown in Fig. 2. The terminals of the coil are permanently connected by spot welding.

After inducing a persistent current in the coil, its magnetic field was measured by NMR techniques and recorded with time. The first run ex-