in this paper.

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†On leave from the Weizmann Institute, Rehovoth, Israel.

[1]Our model is similar, in a sense, to the Koba-Takeda model [Progr. Theoret. Phys. (Kyoto) 19, 269 (1958)]. However, while they considered only pionic and $K\bar{K}$-pair clouds, we would like to emphasize the importance of the simple dissociations of the type $\bar{p} \longleftrightarrow \bar{K}+Y$ and $p \longleftrightarrow K+Y$, in a certain type of processes, as presented in Fig. 1.

[2]Report of G. R. Lynch, Rev. Modern Phys. 33, 395 (1961).

[3]N. Xuong, G. R. Lynch, and C. K. Hinrichs, Phys. Rev. 124, 575 (1961).

[4]G. Goldhaber, S. Goldhaber, W. Powell, and R. Silberberg, Phys. Rev. 121, 1525 (1961).

[5]C. K. Hinrichs, B. J. Moyer, J. A. Poirier, and P. M. Ogden, Phys. Rev. 127, 617 (1962).

[6]J. Button, P. Eberhard, G. Kalbfleisch, J. Lannutti, G. Lynch, B. Maglić, M. L. Stevenson, and N. Xuong, Phys. Rev. 121, 1788 (1961).

[7]Birmingham, CERN, Ecole Polytechnique, Imperial College and Saclay groups, International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN, Geneva, Switzerland, 1962), p. 236 (3- to 3.6-BeV/c $\bar{p}$'s).

[8]B. C. Maglić, G. R. Kalbfleisch, and M. L. Stevenson, Phys. Rev. Letters 7, 137 (1961).

[9]A. Stanjano [Nuovo Cimento 24, 774 (1962)] has tried to explain this asymmetry by using an extended Koba-Takeda model. He was not able, however, to explain the much larger asymmetry of the K^- mesons (see reference 1).

STUDY OF THE ANNIHILATION OF STOPPED ANTIPROTONS IN HYDROGEN: THE REACTION $\bar{p}+p \rightarrow \pi^+ + \pi^- + \pi^0$

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This report describes a study of the reaction

$$\bar{p}+p \rightarrow \pi^+ + \pi^- + \pi^0 \tag{1}$$

for antiprotons from the CERN synchrotron stopping in the Saclay 81-cm hydrogen bubble chamber. A total of 4148 annihilations giving two-prong events have been measured and analyzed. We estimate that about 93% of the events in the fiducial volume used were from the annihilation of protonium, the remaining 7% being annihilations in flight. An analysis of the entire sample of events will be given elsewhere.

All possible candidates for Reaction (1), selected by an inspection of the missing mass, were tried in a least-squares kinematic fitting program.[1] In the fitting program all possible combinations of two or three mesons (π and K) satisfying charge and strangeness conservation were tried as hypotheses. Events satisfying more than one hypothesis were rejected[2] and an upper χ^2 limit of 4 for the $\pi^+\pi^-\pi^0$ final state imposed. 605 events satisfied these criteria.

We have made an estimate of the contamination from $\pi^+\pi^-2\pi^0$ events by analyzing four-prong annihilations: 1100 such events have been put through the same selection and fitting procedure as used for the two-prong events. Of the 4400 $\pi^+\pi^-$ combinations 47 satisfied the single neutral pion hypothesis, i.e., could be represented kinematically by replacing one $\pi^+\pi^-$ pair by a single π^0. These "events" came predominantly from the type

$$p+\bar{p} \rightarrow 2\pi^+ + 2\pi^-, \tag{2}$$

which are about 12.5% of all four-prong events, according to our measurements. Using isotopic spin weights to estimate the rate of

$$p+\bar{p} \rightarrow \pi^+ + \pi^- + 2\pi^0 \tag{3}$$

from the measured branching ratio of Reaction (2), we estimate a contamination from this source of

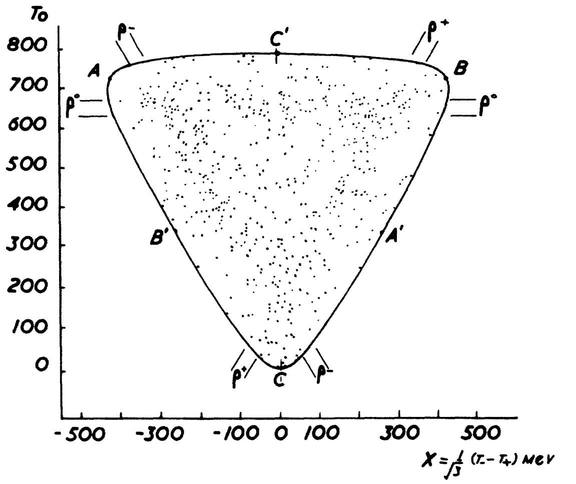


FIG. 1. The Dalitz plot for the events from Reaction (1). The kinetic energy of the positive pion is zero at B and a maximum at B', and similarly for the negative pion at C and C'. The bands labelled ρ^+, ρ^-, and ρ^0 indicate the two-body process $p+\bar{p}\rightarrow\rho+\pi$ with mass for the ρ of 755 MeV and with a width of ±55 MeV.

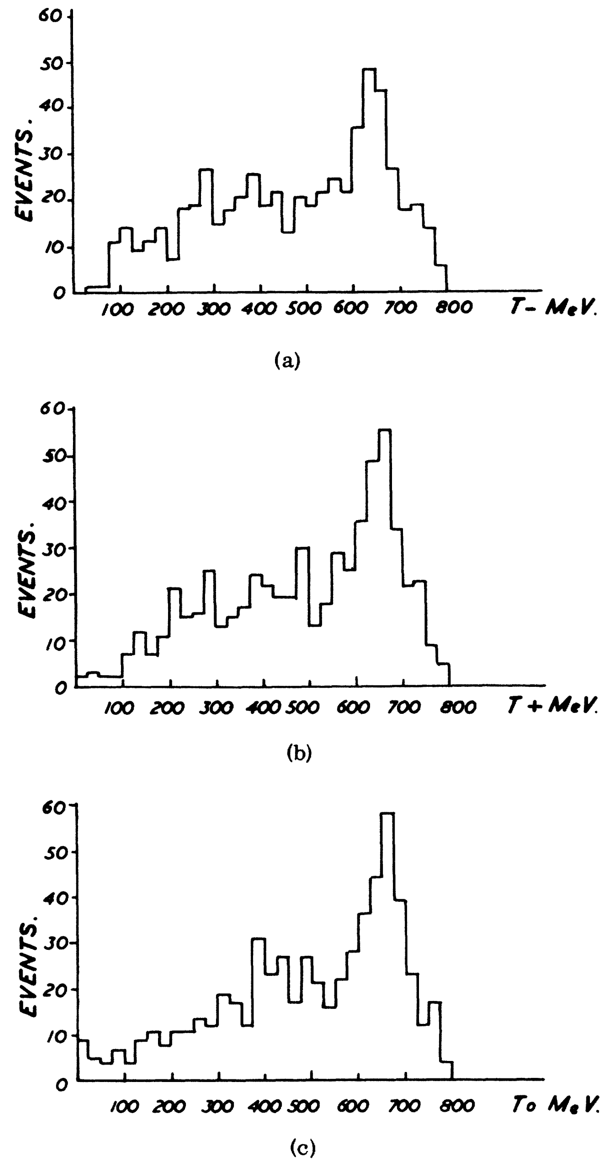


FIG. 2. The kinetic energy spectra of (a) the negative, (b) the positive, and (c) the neutral pions.

13.8%. This procedure neglects the presence of two- and three-pion resonances. An estimate including the rate of ρ^0 production in events of type (2) indicates that the contamination estimate could be as much as 20%. We have no way of estimating the effect of ω^0-meson production which can occur for Reaction (3) but not for Reaction (2). The branching ratio for the three-pion annihilation is then $(5.4\pm1.0)\times10^{-2}$.

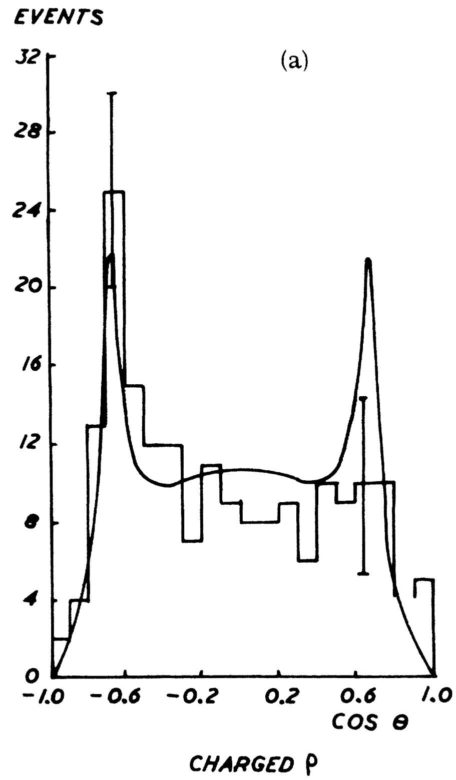
The results of this experiment are shown on the Dalitz plot in Fig. 1. The kinetic energy spectra of the π^-, π^+, and π^0 mesons are shown in Figs. 2(a), 2(b), and 2(c), respectively. The peaks at 650 MeV are due to the ρ mesons. The same effect is seen on the Dalitz plot as the accumulation of points within three bands parallel to the sides of the diagram, as indicated in Fig. 1. These bands contain events corresponding to the processes

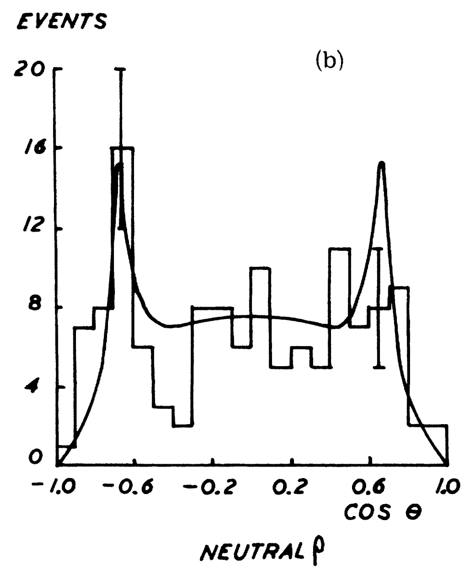
$$\bar{p}+p\rightarrow\rho+\pi. \tag{4}$$

The production of the ρ meson appears to be a dominant feature of the annihilation to three pions. By counting the number of points within the bands shown on the Dalitz plot the numbers obtained for ρ^+, ρ^-, and ρ^0 are 92, 100, and 100, respectively.

Table I gives the possible final states for protonium[3] annihilation into three pions from S or P initial states. The three-body final state is described as a two-body system: a dipion plus a pion. In the dipion rest system the momentum of either pion is q and the relative angular momentum l, and in the laboratory system the third pion has momentum P and angular momentum L relative to the dipion. The angle between $\vec{q}$ and $\vec{P}$ is θ. The final state $\rho^0\pi^0$ can only come from protonium states with $I=0$ ($^3S_1$ and $^1P_1$), and ρ production from these states only gives equal $\rho^+\rho^-\rho^0$ as is observed.

In Fig. 3 the angular distribution of the ρ-meson decays is shown. The experimental points fall off at $\cos\theta=\pm1$, whereas the simplest matrix element for $\rho+\pi$ production from the $^1P_1$ state does not vanish at these limits.[4] Charged ρ production from the $^1S_0$ state gives an angular distribution rising to a maximum at $\cos\theta = \pm 1$. The simplest interpretation of the results is then that the $\rho + \pi$ final state comes from the $^3S_1$ initial state.[5] In this case the resonant final state is represented uniquely by one configuration of angular momentum, (1, 1). Both the isotopic spin and angular





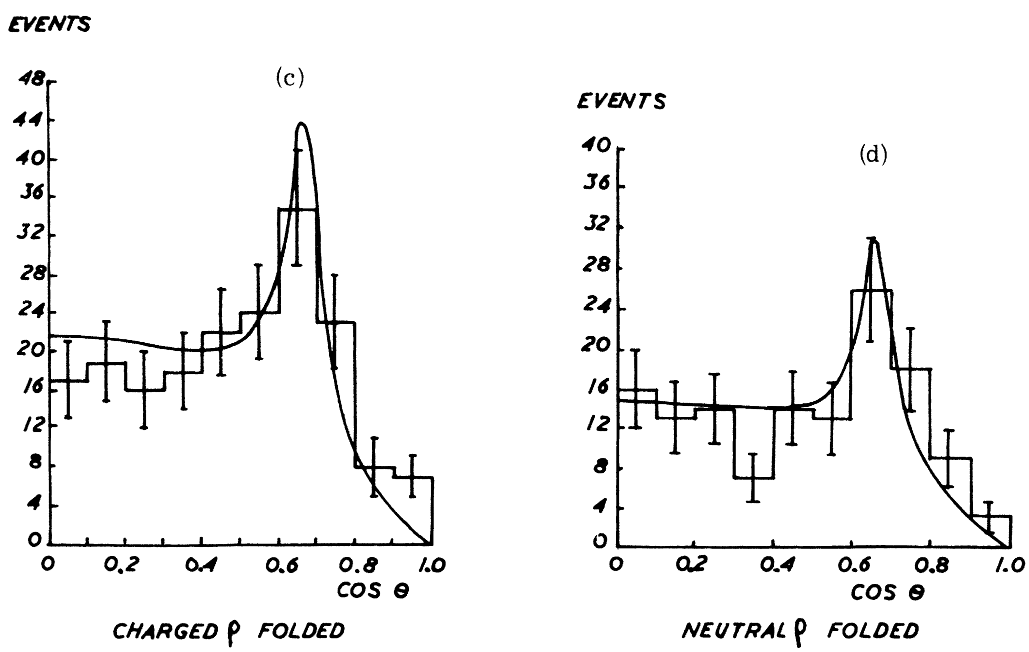


FIG. 3. The angular distribution of (a) the charged ρ and (b) the neutral ρ decays in their barycentric system. The distributions have been folded about $\cos\theta = 0$ in (c) and (d). The curves show the distributions expected for a $J = 1$ particle produced from the $^3S_1$ state. Events selected for these diagrams had values of the dipion mass between 700 and 820 MeV.

Table I. Initial and final states for protonium annihilation into three pions.

Initial state	Spin parity	I	C	G	Allowed final states (L, l)
$^1S_0$	0^-	1	+	-	(0,0), (1,1), (2,2), etc.
$^3S_1$	1^-	0	-	-	(1,1), (3,3), (5,5), etc.
$^1P_1$	1^+	0	-	-	(0,1), (2,1), (2,3), etc.
$^3P_0$	0^+	...	...	...	Annihilation to 3π forbidden (parity)
$^3P_1$	1^+	1	+	-	(0,1), (1,0), (1,2), etc.
$^3P_2$	2^+	1	+	-	(1,2), (2,1), (2,3), etc.

momentum wave functions of this system are totally antisymmetric. The consequences of this have been discussed by Bouchiat and Flamand,[6] who write the amplitude for process (4) in the following way:

$$A=\vec{S}[q_{+-}\times P_0 f(q_{+-})+q_{-0}\times P_+ f(q_{-0})+q_{0+}\times P_- f(q_{0+})], \tag{5}$$

where $\vec{S}$ represents the protonium spin and, e.g.,

$$f(q_{+-})=\frac{m_\pi^2\gamma}{q_{+-}^2-q_\gamma^2+i\gamma[q_{+-}^6/(q_{+-}^2+m_\pi^2)]^{1/2}}. \tag{5a}$$

$f(q)$ is an $l=1$ resonance factor and the parameters are the value of q at the resonance, q_γ, and the dimensionless quantity γ related to the width Γ_0.[7]

As expected for an $I=0$ state of three pions, the interference between the amplitudes for the different charge configurations is constructive everywhere, and in particular at the point where two resonance factors become equal, an enhancement occurs. This corresponds to the regions of the Dalitz plot where two bands cross and one pion can be in a resonant state with either of the other two.

This effect should appear in the angular distributions of Fig. 2, and the curves drawn were computed using the amplitude given by Eq. (5). The data are in reasonable agreement with the predictions of this model which assumes that the ρ meson has unit spin.[8]

We have therefore attempted to fit the distribution of M^*, the dipion mass, given in Fig. 4, by adding incoherently a contribution from the resonant state as described by Eq. (5) and a distribution corresponding to a simple phase space for three pions.[9] This treatment of the nonresonant contribution is an approximation, since in general it will be a combination of states (1, 1), (3, 3), (5, 5), etc., coherently added to the resonant amplitude. The two contributions have been added in the ratio 1:1 to obtain the curve shown in Fig. 4; the resonance parameters used are a ρ-meson mass of 755 MeV and a width of 110 MeV. The fit is not good in the region 900 to 1500 MeV; this could be due to the high-energy behavior of the p-wave $\pi\pi$ cross section or to the oversimplified treatment of the background.

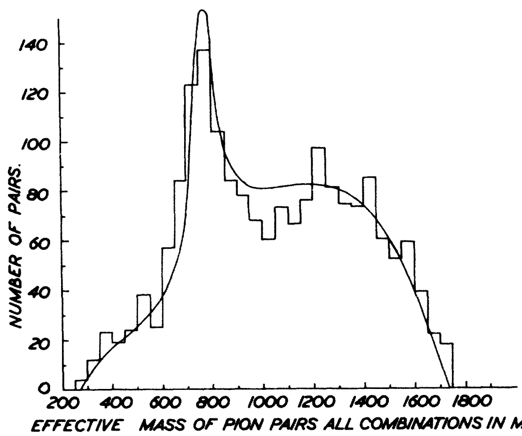


FIG. 4. The two-pion mass distribution for events from Reaction (1). The curve is an incoherent sum of equal proportions of the distribution calculated from Eq. (5) in the text, and a uniform phase-space background.

The branching ratio for annihilation to $\rho+\pi$ obtained by this means is $(2.7\pm0.6)10^{-2}$.

We wish to thank Professor B. P. Gregory and Dr. R. Armenteros whose efforts made the exposure such a success. Professor H. Steiner and G. Fidecaro were largely responsible for setting up the beam. The Oxford Group are indebted to Mrs. K. M. Derrick for efficient organization of the data, and Professor D. H. Wilkinson for constant encouragement. Mr. D. J. Crennell gave help in the early phases of the work. The Padua group wish to thank Dr. P. Kusstatscher and Dr.

A. Saladin for help with the electrostatic separators.

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[1]The fitting program was written by A. G. Wilson of the Rutherford Laboratory, National Institute for Research in Nuclear Science, Harwell, England. We are greatly indebted to Mr. Wilson for his help.

[2]The number rejected for this reason was about 12% of the number finally accepted as three-pion states.

[3]This description in terms of l and L is correct only in the nonrelativistic limit, and so the use of this representation in the present experiment where the ρ meson has a momentum of 780 MeV/c is an approximation.

[4]The lowest $^1P_1$ state leading to ρ^0 production is (0,1), $J=1^+$. It has been discussed in connection with the ω^0 spin analysis [M. L. Stevenson, L. W. Alvarez, B. C. Maglić, and A. H. Rosenfeld, Phys. Rev. 125, 687 (1962)].

[5]It has been suggested [S. Glashow, Phys. Rev. Letters 7, 469 (1961); J. Bernstein and G. Feinberg, Brookhaven National Laboratory report BNL 6122, 1962 (unpublished)] that the rate of decay of ω^0 into $\pi^+ + \pi^-$ by an electromagnetic interaction may be appreciable. The reaction $p+\bar{p} \to \omega^0 + \pi^-$ may proceed from S states only through the $^3S_1$ channel of isotopic spin 1, and so any contribution would be added coherently to the direct $^3S_1$ $\rho^0 + \pi^0$ amplitude. In this case equality of the rates of production of $\rho^0\rho^+\rho^-$ would in general imply some contribution of charged ρ's from, say, the $^1S_0$ state unless the interference in the $^3S_1$ channel was so arranged as to lead to the equality. The angular distribution of the charged ρ decay gives no evidence for charged ρ production from the $^1S_0$ state. Similarly it is not possible to exclude P-state contribution completely since interference between several (L,l) waves may occur in such a way as to allow the effect observed.

[6]C. Bouchiat and G. Flamand, Nuovo Cimento 23, 13 (1962).

[7]The relation used is $\gamma = \Gamma_0(q_\gamma^2 + m_\pi^2)/2q_\gamma^3$. Formula (5a) is a P-wave effective range approximation [B. Lee and M. T. Vaughn, Phys. Rev. Letters 4, 578 (1962)].

[8]D. D. Carmony and R. T. Van de Walle, Phys. Rev. Letters 8, 73 (1962).

[9]The contamination from the pseudo-three-pion events is included in this background

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ASYMPTOTIC PROPERTIES OF FIELDS AND SPACE-TIMES*

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(Received 14 December 1962)

This note outlines a new technique for studying asymptotic questions in (special or) general relativity whereby several new results are obtained. The questions dealt with here are the following: (1) a geometrical definition of asymptotic flatness, (2) covariant definitions of incoming and outgoing gravitational (and other) radiation fields, (3) simple deduction of detailed asymptotic behavior of the Riemann tensor (and other fields)—the "peeling off" property,[1-3] (4) definitions of total energy-momentum and its loss by radiation, with conservation laws, (5) unification of finite and asymptotic versions of the characteristic initial value problem,[2-7] and (6) geometrical derivation of the Bondi-Metzner-Sachs asymptotic symmetry group.[2,4,8] A longer term aim of this approach is for a covariant S-matrix theory incorporating gravitation.

The basic idea is as follows. Asymptotic questions are those relating to the "neighborhood of infinity." From the point of view of the metric structure of space-time, however, there is no such thing as a point at infinity, since such a point would be an infinite distance from its neighbors. But if we think only in terms of conformal structure of space-time (only ratios of neighboring infinitesimal distances are to have significance), then infinity can be treated as though it were simply an ordinary three-dimensional boundary $\mathscr{I}$ to a "finite" four-dimensional conformal region $\mathfrak{M}$. In fact, we may envisage a new "unphysical" metric $g_{\mu\nu}$ assigned (but perhaps only locally) to space-time, which is conformal to the original physical metric $\tilde{g}_{\mu\nu}$ with

$$g_{\mu\nu} = \Omega^2 \tilde{g}_{\mu\nu},$$

and according to which "infinity" is now finite and in most places regular. The boundary $\mathscr{I}$ of $\mathfrak{M}$ is given by $\Omega = 0$, with $\Omega_{;\mu} \neq 0$. ("Infinity" is given finite coordinate values, so $\tilde{g}_{\mu\nu}$ becomes infinite there.)

All covariant derivatives used here will be carried out according to the unphysical $g_{\mu\nu}$ metric so that properties of $\mathscr{I}$ and its neighborhood in $\mathfrak{M}$ may be studied. Any such property which is conformally invariant will then be a physically meaningful asymptotic property in the original space-time $\tilde{\mathfrak{M}}$. The basic reason for success with this approach is the conformal invariance of the zero rest-mass equations for each spin.[9,10] In particu-