histograms give essentially the same result: They do not favor the zero end of the scale appreciably and are consistent with a uniform distribution. Thus the mass spectrum predicted in reference 1 does not seem to have much experimental verification. The above procedure should be useful for testing any future mass-spectrum predictions.

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[1]Ernest E. H. Shin, Phys. Rev. Letters 10, 196 (1963).
[2]W. H. Barkas and A. H. Rosenfeld, Lawrence Radiation Laboratory Internal Report UCRL-8030 (unpublished), and revision by A. H. Rosenfeld, April 1963 (unpublished).

BOSON EIGENSTATES IN GENERALIZED FIELD THEORY*

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The generalized Kemmer and Dirac equations proposed in an earlier paper[1] have been examined for their possible relevance to the particles and resonances of spin 0, 1, and 2 observed in nature. The generalized Kemmer equation may be written in terms of two parameters m and m_0 in the form

$$[i\beta_\mu p_\mu + mc - m_0 c\beta_{\mu\nu}']\psi = 0, \qquad (1)$$

where $\beta_{\mu\nu} = [\beta_\mu, \beta_\nu]$, $\beta_{\mu\nu}' = [\beta_\mu', \beta_\nu']$, and $[\beta_\mu, \beta_\nu'] = 0$. The current density is assumed to be

$$j_\mu = \tfrac{1}{2} i e_0 c\psi^* \eta_4 \eta_4'(1+\eta_5')\beta_\mu \psi, \qquad (2)$$

where $\eta_\mu = 2\beta_\mu^2 - 1$ and $\eta_5' = \eta_1'\eta_2'\eta_3'\eta_4'$. It is easily verified that j_μ is conserved. The probability current

$$S_\mu = ic\psi^* \eta_4 \eta_4' \beta_\mu \psi \qquad (3)$$

is also conserved. For charged states ($\eta_5' = +1$) the total charge Q is proportional to the total probability, which in the classical theory is not positive definite:

$$Q = e_0 \int \psi^* \eta_4' \beta_4 \psi dV,$$

whereas for neutral states ($\eta_5' = -1$) we have $Q = 0$, as required.

The rest energies of the various states obtained from Eq. (1) are shown in Fig. 1 in terms of the unit mc^2 and as functions of the parameter $a = m_0/m$. Each curve in Fig. 1 is labeled by the spin of the state together with a notation (0 or ±) which indicates whether, according to Eq. (2), this state is neutral or charged. Algebraic expressions for these energy levels are exhibited in Table I.

These expressions represent the eigenstates of the bare Hamiltonian defined by Eq. (1), and it would be expected that the shape of the curves of Fig. 1 would be materially charged when interactions and radiative corrections are taken into account in the corresponding quantized field theory. Nevertheless, as an example of the states which are given by the classical field theory, Table I includes the numerical values of the rest energies for the particular choice $m = 940$ MeV ($= m_p$), $m_0 = am = 150$ MeV. It is apparent from Fig. 1 that, for this choice of the

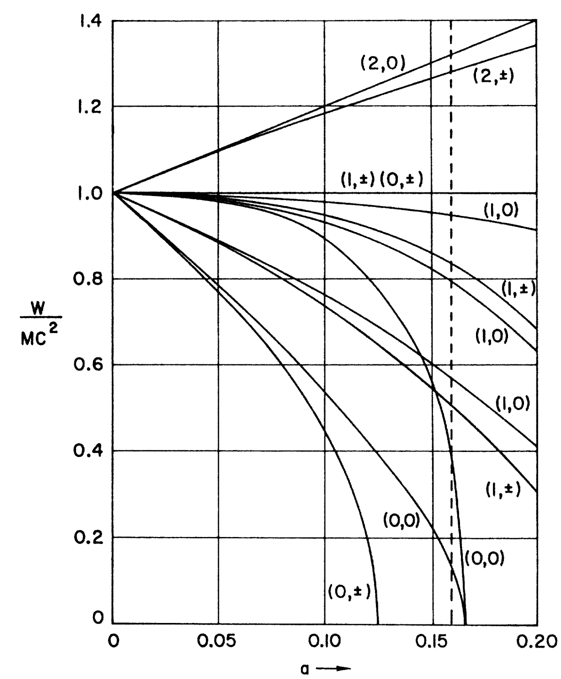

FIG. 1. Rest-energy eigenvalues of Eq. (1) in terms of the unit mc^2 and as functions of the parameter $a = m_0/m$. For each level the first number in parentheses gives the spin and the second denotes the electric charge. The vertical dashed line indicates the value of a used in Table I.

Table I. Spin charge and mass eigenstates of Eq. (1) and the numerical values of the rest energies for the particular choice $m = 940$ MeV, $m_0 = am = 150$ MeV.

Theory				Experiment			
Spin	Charge	Mass (units mc^2)	Mass (MeV)	Possible identification	Spin	Charge	Mass (MeV)
2	0	$1+2a$	1240	$f^{(0)}$	2(?)	0	1250
2	+-	$(1+4a)^{1/2}$	1203	[a]	?	...	1160
1	+-	1	940	[a]	?	...	990
0	+-	1	940				
1	0	$(1-4a^2)^{1/2}$	891	ω	1	0	790
1	+-	$\left[\frac{(1-4a)(1+2a)}{(1-2a)}\right]^{1/2}$	787	ρ^{+-}	1	+-	750
1	0	$(1+2a)(1-4a)^{1/2}$	746	ρ^0	1	0	
1	0	$[(1-4a)(1-4a^2)]^{1/2}$	536	ζ^0	?	0	550
1	+-	$(1-4a)\left[\frac{(1+2a)}{(1-2a)}\right]^{1/2}$	473	ζ^{+-}	?	+-	
0	0	$\left[\frac{(1-6a)(1+2a)}{(1-4a)}\right]^{1/2}$	370	η, ABC	0	0	560, 400
0	0	$[(1-4a)(1-6a)(1+2a)]^{1/2}$	134	π^0	0	0	135
0	+-	$(1-8a)^{1/2}$	imaginary	π^{+-}	0	+-	138

[a]See reference 4.

parameter a, the positions of the three lowest rest-energy levels are very sensitive to small changes either in the shape of the curves or in the value of a itself, and, in fact, for this value the lowest state has an imaginary rest energy. Apart from these states, the numerical order of the eigenvalues is independent of the parameters over a relatively wide range of values of a. For this reason, a possible identification with observed particles is also listed in Table I, but further experiment and theory are both necessary before it can be stated that such an identification is meaningful.

The particles listed in Table I have zero strangeness, but boson eigenstates of a different type arise from the generalized Dirac equation[2]

$$(i\gamma_\mu p_\mu + m'c - m_0'c\epsilon_{\mu\nu}\epsilon_{\mu\nu}')\psi = 0, \tag{4}$$

where

$$\epsilon_{\mu\nu} = \tfrac{1}{4}[\gamma_\mu, \gamma_\nu], \quad \epsilon_{\mu\nu}' = \tfrac{1}{4}[\gamma_\mu', \gamma_\nu'], \tag{5}$$

and $[\gamma_\mu, \gamma_\nu] = 0$. This equation leads to only two rest-energy eigenstates:

$$W = \pm c^2[m'(m'+m_0')]^{1/2} \text{ for spin } 1,$$

$$W = \pm c^2[m'(m'-3m_0')]^{1/2} \text{ for spin } 0.$$

With the same choice as before for the coefficient of the extra term[3] ($m_0' = m_0 \cong 150$ MeV) but with a smaller value ($m' = 800$ MeV) of the constant term in Eq. (4), this equation describes a particle which can exist in a spin-zero state with rest energy 530 MeV or in an excited spin-one state with rest energy 870 MeV. Although little significance can be attached to the similarity of these masses to those of the K and K^* mesons, the charges as well as the spins of these states appear to correspond to the observed results. The probability density S_μ and the current density j_μ which are conserved according to Eq. (4) may be assumed to be

$$S_\mu = ic\psi^*\gamma_4\gamma_4'\gamma_\mu\psi, \tag{6}$$

$$j_\mu = \tfrac{1}{2}ie_0c\psi^*\gamma_4\gamma_4'(1+i\gamma_5')\gamma_\mu\psi. \tag{7}$$

If we write $\gamma_4' = \rho_3'$, $\gamma_5' = \rho_2'$, it follows that the charge Q is given by

$$Q = e_0(\tfrac{1}{2}S + t_3), \tag{8}$$

where

$$\left.\begin{aligned} S &= \int\psi^*\rho_3'\psi dV \\ t_3 &= \tfrac{1}{2}\int\psi^*\rho_1'\psi dV \end{aligned}\right\}, \tag{9}$$

and where, according to Eq. (4), S and t_3 are separately conserved. With the Pauli representation of the operators ρ_1' and ρ_3', interchange of the two components of ψ changes the sign of S, and a change of the relative sign of the two compenents of ψ changes the sign of t_3. The strangeness thus appears simply as the total probability $[S=-(i/c)\int S_4 dV]$ which in the classical field theory can be normalized to +1 for particles and -1 for antiparticles. Equation (4) therefore leads to the correct spins, masses, and conjugate doublet structure for both K and K^* mesons. Associated production of a K and $\overline{K}$, a K^* and $\overline{K}$, or a K^* and $\overline{K}^*$ would then conserve probability, and hence strangeness, but the disintegration of a single K or K^* would not.

There are no rest-energy eigenstates of Eq. (1) or Eq. (4) other than those listed above, but states of spin greater than 2 or charge greater than 1 would occur in the theory in which $\beta_{\mu\nu}'$ of Eq. (1) were replaced by the sum of two commuting Kemmer spin operators. Such a theory would also include some new eigenstates of spin 0, 1, and 2.

Predictions of the present work are that the spin of the ζ particle is unity, and the spin of the $f^{(0)}$ is 2. In addition, the highest $\pi^0\pi^-$ resonance observed by Shalamov and Grashin[4] would have spin 2, and the second highest would be a mixture of spin-0 and spin-1 states.

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[1]H. C. Corben, Proc. Natl. Acad. Sci. U.S. 48, 1559 (1962).

[2]H. C. Corben, Proc. Natl. Acad. Sci. U.S. 48, 1746 (1962), Eq. (14).

[3]Approximately the same value of m_0 appears in the generalized Dirac equation for fermions: $(i\gamma_\mu p_\mu + m''c - m_0 c\epsilon_{\mu\nu}\beta_{\mu\nu})\psi = 0$, with $\epsilon_{\mu\nu}$ given by Eq. (5) [H. C. Corben, Nuovo Cimento 28, 202 (1963)].

[4]Ya. Ya. Shalamov and A. F. Grashin, Zh. Eksperim. i Teor. Fiz. 43, 726 (1962) [translation: Soviet Phys. – JETP 16, 515 (1963)].