As a result, if Eq. (10) is considered as an inhomogeneous integral equation for $\langle \kappa l | \chi \rangle$, we find that the Neumann series for the solution is convergent and hence the solution is unique. In order that the solution be consistent, we obtain the eigenvalue condition.

$$(\pi)^{3/2}\Gamma(l+1)/2k\Gamma(l+\tfrac{3}{2}) = -1, \qquad (12)$$

which leads to a single pole in the neighborhood of -1. Further, if the terms in Eq. (10) which we have proved to be negligible are omitted, a differentiation of Eq. (10) leads immediately to Eq. (4).

The solution of Eq. (1) for the case of the nonlocal potential may be discussed by using an approach similar to the Yukawa case. The essential feature of the latter problem is that in the vicinity of -1 the Legendre function becomes approximately constant as a function of its argument. Making this same approximation in the present case, we find Eq. (5). This then leads to an infinity of solutions in the vicinity of -1.

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LEPTONIC HYPERON DECAYS AND UNITARY SYMMETRY*

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It is known that leptonic decays of hyperons are very rare. Experimentally,[1] the leptonic branching ratios of Λ and Σ^- decays are about an order of magnitude less than would be expected on the basis of a simple V-A theory with a coupling constant equal to the neutron β-decay coupling constant.[2] We propose that this discrepancy can be resolved with the aid of the unitary symmetry scheme ("eightfold way") of Gell-Mann[3] and Ne'eman.[4]

It was originally suggested by Feynman and Gell-Mann,[2] and by Gershtein and Zeldovich,[5] that the weak lepton-baryon interaction for no strangeness change is described by the universal coupling of the charged components of the isotopic vector current to a lepton current. If unitary symmetry holds, the obvious extension of this scheme is that the lepton current is coupled almost universally to the charged components of the eight-component unitary baryon current.[3] By almost universally we mean that the universality of coupling is violated to about the extent that unitary symmetry itself fails to be exact. A measure of the symmetry breakdown is furnished by the baryon mass splitting, with the N-Ξ mass difference about 0.25 times the mean baryon mass.

There are two possible eightfold vector currents that can be formed from the direct product of the baryon octet with itself; we call these d and f currents, in Gell-Mann's notation.[3] In general, a mixture of these currents will be coupled to the lepton current, with the f current being conserved (in the limit of exact symmetry). Denote the baryon currents by $J_\mu^{\ i}$, $i = 1\cdots 8$. $J_\mu^{\ 1} + iJ_\mu^{\ 2}$ leads to $n \to p + \beta + \bar{\nu}$, while $J_\mu^{\ 4} + iJ_\mu^{\ 5}$ couples Λ with p, and Σ^- with n. Label the baryon states by I_3 and Y, where I_3 is the third component of isotopic spin and Y the hypercharge. Then the matrix elements $\langle I_3 Y | J_\mu^{\ i} | I_3' Y' \rangle$ are of the form $\langle \| J_\mu \| \rangle C(I_3 Y; I_3' Y'; i)$ (analogous to the Wigner-Eckart theorem), and the number C depend also on the representation d or f. Set $\langle \| J_\mu \| \rangle$ equal to 1; then

$$G\langle \bar{p} | J_\mu^{\ 1} + iJ_\mu^{\ 2} | n \rangle = \sqrt{2}(d+f)G,$$

$$G\langle \bar{p} | J_\mu^{\ 4} + iJ_\mu^{\ 5} | \Lambda \rangle = -(d/\sqrt{3} + \sqrt{3}f)G,$$

$$G\langle \bar{n} | J_\mu^{\ 4} + iJ_\mu^{\ 5} | \Sigma^- \rangle = \sqrt{2}(d-f)G,$$

where d and f are numbers representing the d-f mixing ratio. G is the universal vector weak coupling constant. The axial vector coupling constants are renormalized by strong interactions. For n decay, $G_A^{\ N} = 1.25\,G$. For Λ and

Σ^- decay, we use the Goldberger-Treiman relations as given by Gell-Mann[6,3] to find

$$\frac{G_A^{\Lambda}}{G}=\frac{G_A^{N}}{G}\frac{2M_N}{M_N+M_\Lambda}\frac{g_{\Lambda NK}}{g_{NN\pi}}\frac{f_\pi}{f_K},$$

$$\frac{G_A^{\Sigma}}{G}=\frac{G_A^{N}}{G}\frac{2M_N}{M_N+M_\Sigma}\frac{g_{\Sigma NK}}{g_{NN\pi}}\frac{f_\pi}{f_K}.$$

The axial vector coupling constants are small, since f_π/f_K and $g_{YNK}/g_{NN\pi}$ are both small. (That f_π/f_K is small is a violation of unitarity symmetry; Gell-Mann has pointed out[3] that this violation may be understood in terms of the large mass splitting in the pseudoscalar octet, with m_π/m_K also being small.[7]) Although the coupling constants g_{YNK} are not precisely known, photoproduction experiments[8] on $\gamma+p\rightarrow Y+K$ indicate that $g_{YNK}^2/g_{NN\pi}^2\sim 0.1$. For instance, Kuo[9] has made a fit to the photoproduction cross section, and finds $g_{\Lambda NK}^2/g_{NN\pi}^2\sim 0.13$; although no careful fit to the Σ^0 production data has yet been made, the cross sections are comparable to those for Λ,[8] and it is reasonable to suppose that $g_{\Sigma NK}^2/g_{NN\pi}^2\sim 0.1$. Then one has, with $f_\pi^2/f_K^2\sim 0.16$, $G^{-2}(G_A^{\Lambda,\Sigma})^2\sim 0.04$. The total baryonic decay rate is proportional to $G_V^2+3G_A^2$. Thus the Λ branching ratio $R_U(\Lambda)$, as calculated by unitary symmetry, stands in proportion to the ratio $R_{UFI}(\Lambda)$ calculated by Feynman and Gell-Mann[2] on the basis of a "universal" Fermi interaction, as

$$R_\Lambda(x)\equiv\frac{R_U(\Lambda)}{R_{UFI}(\Lambda)}=\frac{(G_V^{\Lambda})^2+3(G_A^{\Lambda})^2}{G_V^2+3G_A^2}$$

$$=\frac{(1/6)[(1+3x)/(1+x)]^2+0.04}{4.75},$$

where $x=f/d$.

Likewise, for Σ^- decay,

$$R_\Sigma(x)\equiv\frac{R_U(\Sigma)}{R_{UFI}(\Sigma)}=\frac{[(1-x)/(1+x)]^2+0.04}{4.75}.$$

It will be seen at once from these expressions that R_Y is ~0.1 for all positive x. For a more precise comparison with experiment, we plot in Fig. 1 $R_\Lambda(x)$ and $R_\Sigma(x)$ vs x for positive x (negative x makes the ratios too large). Also on the figure are the experimental points of Humphrey et al.[1]; it is important to note that these points

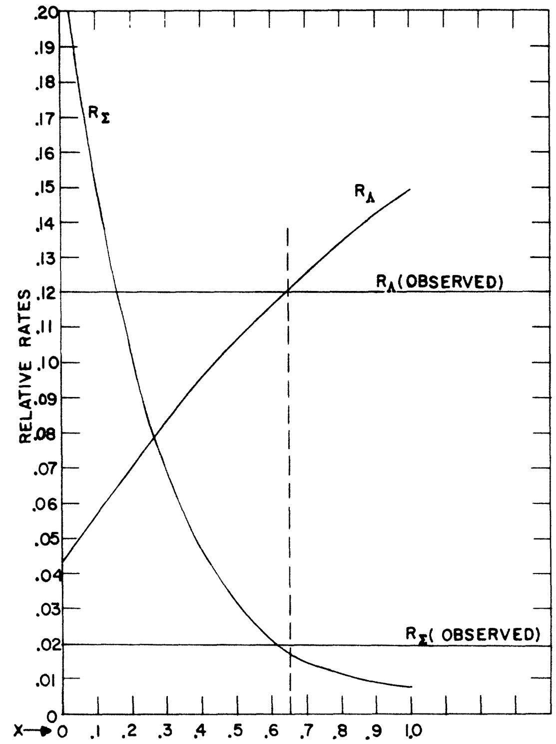

FIG. 1. Relative rates of Λ and Σ decay.

may be in error by a factor of two, although for clarity error bars are not shown in the figure. It will be seen that for $x=0.65$, one has a rather close fit to the experimental data. It is remarkable that a single value of x fits both the Λ and Σ data so well, especially since there is a factor of about 6 between the two ratios. This value of x indicates a slight predominance of d currents over f currents. It may be that in the coupling of the pseudoscalar meson current to the baryon current, d couplings are likewise predominant; certainly there must be some d couplings, or else $g_{\Sigma\Lambda\pi}$ is zero. If the f coupling is zero, then $g_{\Sigma\Sigma\pi}$ would vanish. For what it is worth, one might use the f/d ratio found above to predict the coupling constants of pions to baryons. The numbers turn out to be (with $g_{NN\pi}^2/4\pi=15$)

$$g_{\Sigma^0\Lambda\pi^0}^2/4\pi=7.5,$$

$$g_{\Sigma^+\Sigma^-\pi^0}^2/4\pi=10,$$

$$g_{\Xi\Xi\pi}^2/4\pi=1.$$

It is amusing to compare these figures to the phenomenological model of nonleptonic hyperon decay of Singh and Udgaonkar,[10] who find $g_{\Sigma\Lambda\pi}{}^2/4\pi \sim g_{\Sigma\Sigma\pi}{}^2/4\pi \approx 7$. As has been remarked by Gell-Mann,[3] no mixing ratio for the coupling of baryons to K is consistent with the photoproduction data. This unfortunate occurrence may be related to a large violation of unitary symmetry in the pseudoscalar meson octet, in association with a relatively small violation in the baryon octet.

Two other predictions can be made concerning the leptonic decays of Ξ particles. Of special interest is the predicted leptonic decay rate of $\Xi^0 \to \Sigma^+ + e^- + \bar{\nu}$, since, if unitary symmetry holds, this rate is independent of the mixing ratio x. We find $R_{\rm U}(\Xi^0)/R_{\rm UFI}(\Xi^0) = 0.2$, assuming $g_{\Xi\Sigma K}{}^2/g_{NN\pi}{}^2 \sim 0.1$. Since the branching ratio for the universal Fermi interaction is 2.4%,[2] we predict a branching ratio in Ξ^0 decay of ~0.5%. For $\Xi^- \to \Lambda + e^- + \bar{\nu}$, we find

$$\frac{R_{\rm U}(\Xi^-)}{R_{\rm UFI}(\Xi^-)} = \frac{(1/6)[(1-3x)/(1+x)]^2 + 0.04}{4.75} = 0.02$$

for $x = 0.65$. Recently, one decay event of the type $\Xi^- \to \Lambda + e^- + \bar{\nu}$ has been seen in 200 decays[11]; there are not yet enough events to establish a reliable branching ratio. It is clear, however, that the leptonic branching ratio is smaller than UFI, which would predict five leptonic Ξ^- decays in 200 events.

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TEST OF MASS SPECTRA*

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A recent Letter[1] displays a predicted mass spectrum for elementary particles and resonances. The values observed agree with the given values within one or two percent, leading the author to conclude that the fit is good. The given mass spectrum, $M(n)/m_{\pi^0} = \frac{2}{3}(n+\frac{1}{2})$, has a constant mass interval between successive predicted masses, namely, $0.66\,m_{\pi^0}$. We propose to test the prediction on the basis of the distribution of observed masses within this constant interval. The difference between an observed mass and the nearest predicted mass can never exceed $0.33\,m_{\pi^0}$; therefore we plot these differences on a scale from zero to $0.33\,m_{\pi^0}$. If the prediction is valid, the plotted points should favor the zero end of the scale. If uncorrelated, the points should be uniformly distributed. This test displays the fit (or lack of it) better than the simple observation of percent difference between observed and predicted mass, and may be used for predictions having a nonconstant mass interval by plotting the difference in terms of percentage of the interval in which the observed mass lies.

An isospin multiplet is considered to be one