tractive forces in each of these states.) It should be noted that these assignments are just those expected for the third members of Regge trajectories containing the P_{11}, F_{15} and P_{33}, F_{37} pairs already discussed. Another possibility is G_{17} (induced by D_{35} exchange) and G_{39} (induced by D_{13} exchange), although the nucleon-exchange terms are repulsive in G_{17} and G_{39}. In this case G_{17} (G_{39}) would be the second member of a trajectory beginning with D_{13} (D_{35}). The members of the pairs G_{17}, G_{39} and $H_{19}, H_{3,11}$ each have the reciprocal bootstrap relation.

Figure 1 summarizes the way in which the pairing of Regge trajectories arises through the action of the exchange forces. The regularity arises from the basic simplicity of the crossing matrix in the pion nucleon system. The results agree qualitatively with the isospin-dependent spin-orbit force proposed by Kycia and Riley.[17] Further details, numerical calculations, and a more careful analysis of complicating effects will be published elsewhere.

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ELASTIC $\pi^+ + p$ SCATTERING FROM 7 TO 17 BeV/c

K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan
Brookhaven National Laboratory, Upton, New York*
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Differential cross sections $d\sigma/dt$ for elastic $\pi^+ + p$ scattering in the momentum range 7 to 17 BeV/c have been measured. The range of 4-momentum transfer squared ($-t$) covered was 0.2 to 0.1 (BeV/c)^2. These measurements were made concurrently with those for $\pi^- + p$ and $p + p$ described in a previous publication.[1]

In the same momentum and t range we found earlier[1] a substantial shrinkage with increasing s (center-of-mass energy squared) of the curve of

$$\frac{d\sigma/dt}{(d\sigma/dt)_{\rm opt}}$$

for elastic scattering, of the type predicted by the Regge vacuum or three-pole theory,[2,3] in the $p + p$ case; but in the $\pi^- + p$ case we observed within errors either no shrinkage or only a very small fraction of the shrinkage occurring in $p + p$ scattering.

In the Regge-pole theory considering vacuum (P and P') and vector-meson (ω, ρ) trajectories, the $\pi^+ + p$ and $\pi^- + p$ scattering amplitudes are expected to be similar except for the ρ contribution which is coupled with opposite sign in $\pi^+ + p$ and $\pi^- + p$ cases. The $\pi^+ + p$ elastic-scattering data reported here also demonstrate either no shrinkage or within errors very little shirnkage with increasing s.

The experiment was performed at the Brookhaven AGS using the counter hodoscope, data handling, and on-line computing system previously described.[1] The beam setup was unchanged, the only difference being that the differential gas

Cherenkov counter in the incident particle telescope was tuned to select π^+. The data were treated in the same way as for the $\pi^- - p$ experiment.

The resulting differential cross sections are shown in Fig. 1 in the form and units used in reference 1,

$$X \equiv \left[\frac{\sigma_{\rm tot}(20\ {\rm BeV}/c)}{\sigma_{\rm tot}(p)}\right]^2 \frac{d\sigma}{dt} \text{ in } \frac{\rm mb}{({\rm BeV}/c)^2} \propto \frac{d\sigma/dt}{(d\sigma/dt)_{\rm opt}},$$

vs $-t$ $({\rm BeV}/c)^2$ in order to facilitate comparison with shrinkage predictions of the Regge-pole theory. This normalization is such that at 20 BeV/c the quantity plotted is the measured $d\sigma/dt$. Values of the $\pi^+ + p$ total cross section were taken to be $\sigma_{\rm tot} = 22.26 + (25.10/p)({\rm BeV}/c)$ from a previous fit.[4] The abscissa is the mean effective value of $(-t)$ for the relevant interval which was calculated using the shape of the differential cross sections and allowing for the beam shape, beam momentum spread, for the target size, and finite counter sizes.

K^+ and p contaminations in the π^+ beam were measured ($\lesssim \frac{1}{2}$ %); their effect on the measurements of $d\sigma/dt$ for $\pi^+ + p$ was estimated to be negligibly small. Muon and electron beam contaminations were estimated for each momentum and corrections (2-6%) applied. The background of inelastic events was one to a few percent for most of the t range, but for a few cases at the highest t values, it became as high as 30%.

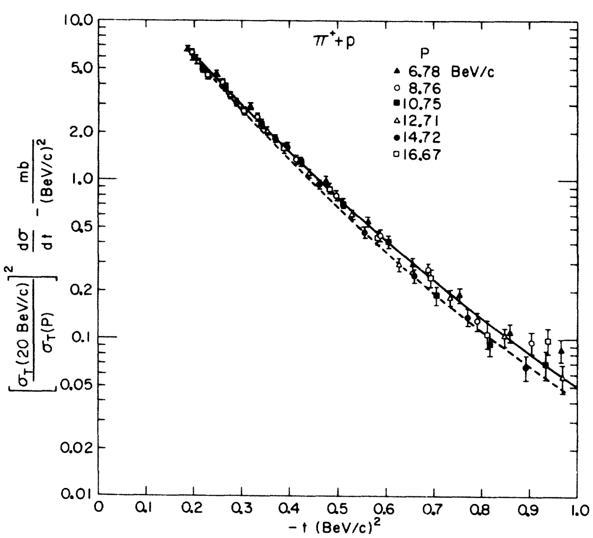

FIG. 1. $X \equiv [\sigma_{\rm tot}(p)]^{-2}[\sigma_{\rm tot}(20\ {\rm BeV}/c)]^2 d\sigma/dt$, which is proportional to $(d\sigma/dt)_{\rm opt}^{-1} d\sigma/dt$, versus t. The solid line is a least-squares fit to all the $\pi^+ + p$ data; the dashed line is a least-squares fit to all the $\pi^- + p$ data from reference 1.

The errors shown are compounded estimates of relative errors which affect conclusions regarding shrinkage. These include statistical errors (standard deviation), relative efficiency errors, relative normalization errors, uncertainty in background subtraction, and relative errors introduced in calculating mean t values due to momentum and angle uncertainties. The absolute normalization is uncertain in addition by 10-15%.

Figure 1 indicates no evidence for shrinkage of $(d\sigma/dt)_{\rm opt}^{-1} d\sigma/dt$ with increasing s.

A function of the form $\exp(a + bt + ct^2)$ fitted to all 53 $\pi^+ + p$ points gives a χ^2 of 62 where 50 is expected, which can still be considered as a reasonable fit. This fit is shown as a solid curve in Fig. 1. For comparison, a similar fit to the $\pi^- + p$ data is shown as a dashed curve. The $\pi^- + p$ fit yields a χ^2 of 51 where 47 is expected, and therefore is a good fit. As can be seen in Fig. 1, the $\pi^- + p$ and $\pi^+ + p$ fits are within errors about the same at low t, but there is a somewhat greater average slope in the $\pi^- + p$ fit.[5]

The predictions of the vacuum (P) Regge-pole (or one-pole) model[2] and the three-pole (P, P', and ω)[3] model, using spin-averaged amplitudes and approximately parallel straight-line Regge-pole trajectories, can both be represented by an equation of the form[1]

$$\log\left[\frac{d\sigma/dt}{(d\sigma/dt)_{\rm opt}}\right] = \log F(s,t) + [2\alpha(t) - 2]\log(s/s_0).$$

The results for α are independent of the choice of s_0. The s dependence of F is absent in the one-pole model and relatively weak for the three-pole model.[1,3] Of course, the one-pole analysis can very well demonstrate whether the over-all effect of all contributing poles leads to shrinkage, no shrinkage, or possible expansion of the curve of $(d\sigma/dt)_{\rm opt}^{-1} d\sigma/dt$ in the Regge-pole theory sense.

Figure 2 is a plot of $\log X$ at fixed t plotted versus $\log s$. Least-squares fits to the data of the form $d\sigma/dt = \exp(a + bt + ct^2)$ for each incident momentum were used to interpolate to fixed t values, since the original data (Fig. 1) had small t differences. The errors were taken from the adjacent experimental points, and as previously mentioned, include all effects relevant to a discussion of shrinkage. The points in Fig. 2 have been least-squares fitted to straight lines, which according to Eq. (1), have slopes $2\alpha(t) - 2$ (i.e., we neglect

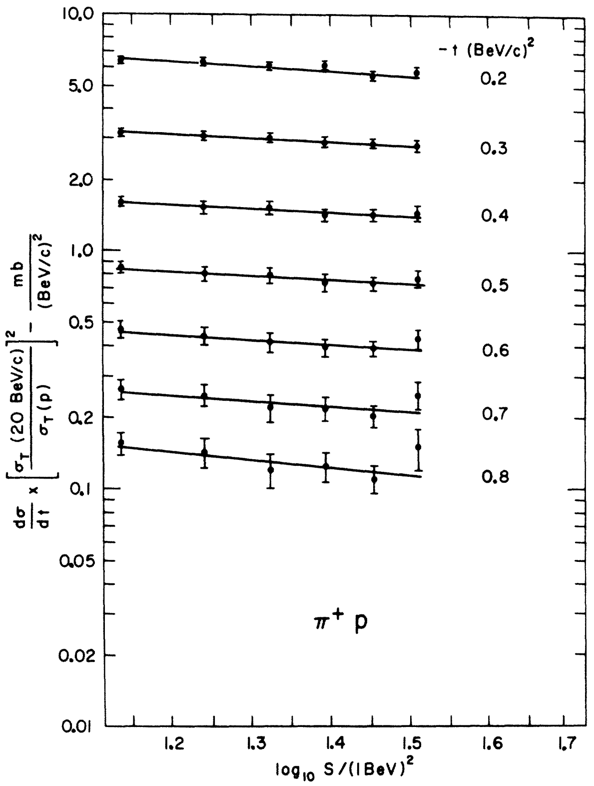

FIG. 2. Log X versus log$s$ for constant values of t. The small differences in t values of the original points were eliminated by interpolation of the least-squares fits to the data, the errors being taken from the nearest point.

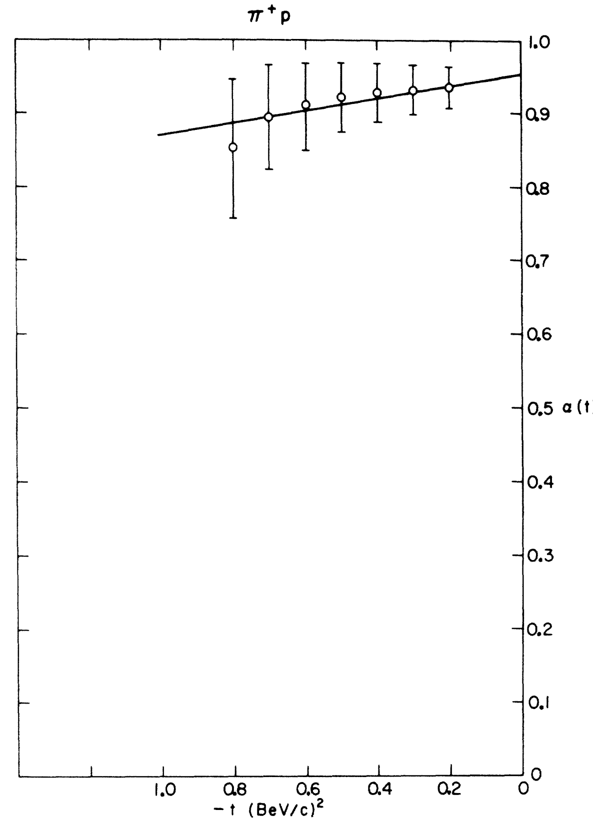

FIG. 3. $\alpha(t)$ versus t for $\pi^+ + p$ data (7-17 BeV/c).

the s dependence of F). The values of $\alpha(t)$ thus obtained are shown in Fig. 3. The least-squares fit of $\alpha(t) = a + bt$ yields $\alpha_{\pi^+ + p}(t) = (0.96 \pm 0.04) + (0.086 \pm 0.097)t$.

It is clear from both the above fit and Fig. 3 that for the $\pi^+ + p$ data, $\alpha(t)$ has no t dependence within the errors and, therefore, either there is no shrinkage of $(d\sigma/dt)_{\text{opt}}^{-1} d\sigma/dt$ for $\pi^+ + p$ or, at most, the shrinkage is a small fraction of that found for our 7- to 20-BeV/c $p+p$ data, for which $\alpha_{p+p}(t) = (1.07 \pm 0.03) + (0.83 \pm 0.07)t$, giving for the ratio for

$$\frac{d\alpha_{\pi^+ + p}(t)/dt}{d\alpha_{p+p}(t)/dt} = 0.10 \pm 0.11.$$

It was previously found[1] that over the same momentum and t range $\alpha_{\pi^- + p}(t) = (0.96 \pm 0.03) + (0.008 \pm 0.080)t$. Within the errors, both $\pi^- + p$ and $\pi^+ + p$ have the same $\alpha(t)$.

From the foregoing, it is established that there is no evidence for shrinkage of the elastic-scattering curve of either $\pi^+ + p$ or $\pi^- + p$ with increasing s (7- to 17-BeV/c incident momentum) in the t range 0.2-1 (BeV/c)^2, and within error limits the shrinkage is a small fraction of the effect in the $p+p$ case.

As was also found for the $\pi^- + p$ data, the $\pi^+ + p$ data contradict the dominance of the vacuum pole, or the more sophisticated three-pole model[3] (P, P', and ω), neglecting spin effects and assuming parallel straight-line Regge trajectories. As previously pointed out, the three-pole model neglects the ρ-pole contribution and reduces to a two-pole model for $\pi^\pm + p$ since the ω has wrong G-parity.[1] The fact that $\alpha(t)$ is the same within errors for $\pi^- + p$ and $\pi^+ + p$ also supports the assumption of a small effect due to the ρ pole since it appears with opposite signs in the $\pi^- + p$ and $\pi^+ + p$ scattering amplitudes.

Therefore, in view of the present $\pi^+ + p$ data, the previous $p+p$ and $\pi^- + p$ data[1] and the foregoing discussion, it seems more unlikely that a reasonably simple Regge-pole theory solution will be found to fit all our data.

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$$\frac{(d\sigma/dt)(\pi^- + p) - (d\sigma/dt)(\pi^+ + p)}{\frac{1}{2}[(d\sigma/dt)(\pi^- + p) + (d\sigma/dt)(\pi^+ + p)]}$$

at low t which increases somewhat with increasing t. Hence the data are consistent with the usual assignment of only a small effect due to the ρ pole at least at low t.

EXPERIMENTAL DETERMINATION OF THE NEUTRAL BRANCHING RATIOS OF THE η MESON*

Frank S. Crawford, Jr., and L. J. Lloyd
Lawrence Radiation Laboratory, University of California, Berkeley, California

and

Earle C. Fowler
Duke University, Durham, North Carolina

(Received 13 May 1963)

The generally accepted values for the quantum numbers of the η meson are given by $J^PI^G = 0^-0^+$.[1] These assignments have been established primarily through studies of the Dalitz-Fabri plot for the decay $\eta \to \pi^+\pi^-\pi^0$.[2] The existence of the expected mode $\eta \to \pi^+\pi^-\gamma$ has recently been established, with a branching ratio given by

$$\Gamma(\eta \to \pi^+\pi^-\gamma)/\Gamma(\eta \to \pi^+\pi^-\pi^0) = 0.26 \pm 0.08.^3 \quad (1)$$

The existence of the mode $\eta \to \gamma\gamma$ has been established by using etas produced in a heavy-liquid bubble chamber[4]; but its relative probability has not been determined prior to the experiment reported here. The expected decay mode $\eta \to 3\pi^0$ is difficult to observe[4] and has not been established by direct observation prior to this experiment. However, the ratio (neutral/charged) $= \Gamma(\eta \to \text{neutrals})/\Gamma(\eta \to \pi^+\pi^-x^0)$, with x^0 unresolved into π^0 and γ, has been determined in several experiments by counting "missing neutrals."[2] An average over these experiments gives (neutral/charged) $= 2.7 \pm 0.6$. The neutrals should correspond to $\eta \to \gamma\gamma$ plus $\eta \to 3\pi^0$ if the η quantum numbers are 0^-0^+.[5]

In the experiment described here we confirm the existence of the decay mode $\eta \to \gamma\gamma$, and find a branching ratio

$$\Gamma(\eta \to \gamma\gamma)/[\Gamma(\eta \to \pi^+\pi^-\pi^0) + \Gamma(\eta \to \pi^+\pi^-\gamma)] = 0.99 \pm 0.48. \quad (2)$$

We also establish directly the existence of the mode $\eta \to 3\pi^0$, and find

$$\Gamma(\eta \to 3\pi^0)/[\Gamma(\eta \to \pi^+\pi^-\pi^0) + \Gamma(\eta \to \pi^+\pi^-\gamma)] = 0.66 \pm 0.25. \quad (3)$$

The sum of our results (2) and (3) gives (neutral/charged) $= 1.65 \pm 0.53$, in only fair agreement with the result 2.7 ± 0.6 others have obtained by counting missing neutrals.[2]

Calculations based on the model $\eta \to \rho_1{}^0 + \rho_2{}^0$, $\rho_1{}^0 \to \gamma$, $\rho_2{}^0 \to \pi^+\pi^-$ predict $\Gamma(\eta \to \pi^+\pi^-\gamma)/\Gamma(\eta \to \gamma\gamma) \approx 1/4$.[6] If we combine our results (2) with our earlier result (1), we find

$$\Gamma(\eta \to \pi^+\pi^-\gamma)/\Gamma(\eta \to \gamma\gamma) = 0.21 \pm 0.12.$$

Since $\eta \to 3\pi$ should go into the 3π state with $I = 1$, one expects $\Gamma(\eta \to 3\pi^0)/\Gamma(\eta \to \pi^+\pi^-\pi^0) = 3/2$. Phase-