RESONANCE STRUCTURE OF THE PION-NUCLEON SYSTEM

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In a recent Letter, Kycia and Riley[1] suggested that the observed structure in the pion-nucleon cross section can be understood in terms of a spin-orbit force. For isospin $\frac{1}{2}$ this force is attractive for $j=l-\frac{1}{2}$, and for isospin $\frac{3}{2}$ the $j=l+\frac{1}{2}$ states are attractive. The purpose of this note is to propose a dynamical mechanism which gives rise to just such a force. In addition, we are able to understand in a natural way why the shape of the π^+p spectrum is similar to that of the π^-p but shifted to a higher energy. More precisely we show that, given the quantum numbers of a resonant $T=\frac{1}{2}$ state [e.g., nucleon N, $D_{3/2}$ (600 MeV), $F_{5/2}$ (900 MeV)], then the other three states of the same orbital momentum behave as suggested in reference 1. Thus the results do not depend on an understanding of the dynamics of the $T=\frac{1}{2}$ resonances.

The importance of forces generated by the crossed terms in π-N scattering has been emphasized by Chew,[2] who discussed the possibility that N^* exchange (N^* signifies the 3-3 resonance) produces the nucleon analogously to the induction of N^* via the N-exchange force. For the p-wave pion-nucleon system, the forces due to nucleon exchange have the appropriate spin-orbit character because of the structure of the crossing matrix in the static-nucleon theory.[3] In a similar way we consider the forces generated by the exchange of the second and third isospin-$\frac{1}{2}$ resonances. Such exchanges give rise to all partial waves, but if the exchanged state has orbital momentum l, then the states of higher l are down by a factor of order $(1/M)$, where M is the nucleon mass. (See, for example, CGLN.[4]) For simplicity we therefore consider the crossing matrix A for the heavy mass limit:

$$3(2l+1)A=\begin{bmatrix} 1 & -2l & -2 & 4l \\ -(2l+2) & -1 & 2(2l+2) & 2 \\ -4 & 8l & -1 & 2l \\ 4(2l+2) & 4 & (2l+2) & 1 \end{bmatrix}. \qquad (1)$$

The elements of $A_{\alpha\beta}$ are labeled as follows: $\alpha=1,2,3,4$ correspond to $T=\frac{1}{2}$, $j=l-\frac{1}{2}$; $T=\frac{1}{2}$, $j=l+\frac{1}{2}$; $T=\frac{3}{2}$, $j=l-\frac{1}{2}$; $T=\frac{3}{2}$, $j=l+\frac{1}{2}$, respectively.

We specialize now to the case of D_{13} exchange [notation: $(l)_{2T,2j}$]. From (1) we find the elements 1/15, -4/15, -2/15, and 8/15 for the states D_{13}, D_{15}, D_{33}, and D_{35}, respectively. Inspection of the rows of (1) shows that only D_{13} exchange leads to spin-orbit D-wave forces of the character proposed by Kycia and Riley. This is quite general: Only the exchange of a state of given parity (l) with the minimum T ($\frac{1}{2}$) and j ($j=l-\frac{1}{2}$) induces such a spin-orbit force. More important is the fact that the attractive force induced in the state of opposite T ($\frac{3}{2}$) and higher j ($j=l+\frac{1}{2}$) is by far the strongest of the exchange forces. Exchange of the third resonance (F_{15}) gives the following coefficients for F_{15}, F_{17}, F_{35}, and F_{37}: 1/21, -6/21, -2/21, and 12/21.

The essential features are shown by writing down a crude approximation to the D_{35} partial-wave dispersion relation:

$$D_{35}(W)\cong\frac{1}{\pi}\int_{M+\mu}^{\infty}\frac{dW'\,\mathrm{Im}D_{35}(W')}{W'-W-i\epsilon}+\left(\frac{8}{15}\right)\frac{1}{\pi}\int_{M+\mu}^{\infty}\frac{dW'\,\mathrm{Im}D_{13}(W')}{W'+W-2M}. \qquad (2)$$

In the crossed term we have kept only the resonant D_{13} state. W is $\omega+M$, ω the pion c.m. energy, and μ the pion mass. A good approximation to the crossed term is obtained by replacing $\mathrm{Im}D_{13}$ by a delta function. The resulting equation is of a familiar simple form and gives a resonant solution. The influence of inelasticity can be taken into account by Froissart's method.[5] The usual necessity of a cutoff prevents a prediction of the position of onset of D_{35} state, but the similarity of Eq. (2) to the Chew-Low theory[3] leads one to expect a displacement of D_{35} relative to D_{13} by a c.m. energy of the order of a pion mass. If we place the "shoulder" in the π^+-p cross section at 850-MeV pion lab energy, then the c.m. energy of the shoulder is 1.09 pion masses greater than that of the second resonance. The 1.35-BeV maximum, which should contain a substantial F_{37} contribution according to our model, has a c.m. energy 1.67 pion masses greater than the 900-MeV resonance (F_{15}). This is hardly alarming in view of the more distant position of the left-hand singularities in Eq. (2) for the F state. The new maxima found by Diddens et al.[6] are separated by 1.2 pion masses in the c.m. system. Though nothing firm is known about the properties of these phenomena, it seems likely that they are related in the same way as are the pairs D_{13}-D_{35}, F_{15}-F_{37}

discussed above. As remarked in reference 1, the present scheme is entirely compatible with (though certainly no proof of) the prophecy of Regge-pole theory.[7] We can understand how F_{37} arises from forces due to F_{15} exchange but cannot explain why F_{37} should be genetically related to the 3-3 resonance (P_{33}) in the absence of a dynamical understanding of the mutual relation of the nucleon and the F_{15} resonance.

It should be remarked that the relation between D_{13} and D_{35}, etc., is reciprocal in the sense of Chew,[2] in that D_{35} exchange gives an attraction in the D_{13} state. However, it seems quite unlikely that such a simple bootstrap can operate here as in the (N,N^*) system because the forces responsible for the $T=\frac{1}{2}$ resonances are more likely due to inelastic processes than to exchange of D_{35} and F_{37}.

Helland et al.[8] have recently published accurate angular distributions for the energy region under consideration. The behavior of their coefficient a_5 in π^--p scattering can be understood easily by the interference of F_{15} and D_{35} states (these states can be somewhat inelastic and out of phase). It is not really necessary that the phase of D_{35} exceed 90°. a_3 has the qualitative behavior of the same interference term but is slightly smaller than expected. In a previous paper[9] the assignment of D_{33} was recommended for the 850-MeV π^+-p "shoulder." That analysis was mainly based on data extrapolated from the vicinity of the second resonance. A careful analysis of the new data[8] is necessary before any firm conclusions can be drawn.

In summary, the exchange of D_{13} gives rise to repulsive D_{33} and D_{15} forces and probably a resonance in D_{35}. Similarly F_{15} gives rise to repulsive F_{17} and F_{35} forces and probably an F_{37} resonance. These predictions will perhaps be modified somewhat by the effects of inelasticity. The effect of the above pattern of forces is to enhance the spin-flip amplitude at the expense of the direct amplitude. Negative (and small) phase shifts are expected in D_{15}, D_{33}, F_{17}, and F_{35}. Except for the possible reciprocal bootstrap relation[2] between N and N^*, one may say that the isospin-$\frac{3}{2}$ spectrum is in a certain sense merely an "image" of the isospin-$\frac{1}{2}$ spectrum. It is tempting to call the D_{35}, F_{37}, and the new π^+-p phenomenon[6] "image resonances" to distinguish them from their more distinguished progenitors of isospin $\frac{1}{2}$.

If the proposed mechanism is correct, then it seems urgent to construct a convincing theory of the D_{13} and F_{15} resonances in order to obtain a dynamical understanding of Regge-pole ideas in the pion-nucleon system. Clearly, other boson-baryon systems are susceptible to a similar discussion. The most important theoretical task remaining seems to be to verify that the simple ideas survive in an analysis based on the complicated complete partial-wave dispersion relations.

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