

The correction is, however, too large, leaving about 2% to be explained.<sup>10</sup>

<sup>1</sup>M. Gell-Mann, California Institute of Technology Report CTSL-20, 1961 (unpublished); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).

<sup>2</sup>R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

<sup>3</sup>R. E. Marshak and E. C. G. Sudarshan, Proceedings of the Padua-Venice Conference on Mesons and Recently Discovered Particles, September, 1957 (Società Italiana di Fisica, Padua-Venice, 1958); Phys. Rev. **109**, 1860 (1958).

<sup>4</sup>Similar considerations are forwarded in M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1958).

<sup>5</sup>The lifetimes from W. H. Barkas and A. H. Rosenfeld, Proceedings of the Tenth Annual International Rochester Conference on High-Energy Physics, 1960 (Interscience Publishers, Inc., New York, 1960), p. 878. The branching ratio for  $K^+ \rightarrow \mu^+ + \nu$  is taken as 57.4%. W. Becker, M. Goldberg, E. Hart, J. Leitner, and S. Lichtman (to be published).

<sup>6</sup>B. P. Roe, D. Sinclair, J. L. Brown, D. A. Glaser, J. A. Kadyk, and G. H. Trilling, Phys. Rev. Letters **7**, 346 (1961). These authors give the branching ratio for  $K^+ \rightarrow \mu^+ + \nu$  as 64%, from which  $\theta = 0.269$ . Also this value agrees with that from  $K^+ \rightarrow \pi^0 + e^+ + \nu$  within experimental errors.

<sup>7</sup>N. Cabibbo and R. Gatto, Nuovo Cimento **21**, 872 (1961). Our notation for the currents is different from the one used in this reference and by Gell-Mann; the connection is  $j_\mu^{(0)} = j_\mu^1 + ij_\mu^2$ ,  $j_\mu^{(1)} = j_\mu^4 + ij_\mu^5$ .

<sup>8</sup>W. Willis et al. reported at the Washington meeting of the American Physical Society, 1963 [W. Willis et al., Bull. Am. Phys. Soc. **8**, 349 (1963)] this branching ratio as  $(0.9_{-0.4}^{+0.5}) \times 10^{-4}$ . If it is allowed to vary between these limits, our predictions for the  $\Sigma^- \rightarrow ne^- \bar{\nu}$  varies between  $0.8 \times 10^{-3}$  and  $4 \times 10^{-3}$ , and that for  $\Lambda^0 \rightarrow pe^- \bar{\nu}$  between  $1.05 \times 10^{-3}$  and  $0.56 \times 10^{-3}$ . I am grateful to the members of this group for prepublication communication of their results.

<sup>9</sup>R. P. Ely, G. Gidal, L. Oswald, W. Singleton, W. M. Powell, F. W. Bullock, G. E. Kalmus, C. Henderson, and R. F. Stannard [Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 445] give the branching ratio for  $\Lambda \rightarrow p + e^- + \bar{\nu}$  as  $(0.85 \pm 0.3) \times 10^{-3}$ , while that for  $\Sigma^- \rightarrow n + e^- + \bar{\nu}$  is given (see preceding reference) as  $(1.9 \pm 0.9) \times 10^{-3}$ .

<sup>10</sup>R. P. Feynman, Proceedings of the Tenth Annual International Rochester Conference on High-Energy Physics, 1960 (Interscience Publishers, Inc., New York, 1960), p. 501. Recent measurements of the muon lifetime have slightly increased the discrepancy. We think that more information will be needed to decide whether our 3rd assumption can be maintained.

## EXPERIMENTAL EVIDENCE ON $\pi - \pi$ SCATTERING NEAR THE $\rho$ AND $f^0$ RESONANCES, FROM $\pi^- + p \rightarrow \pi + \pi + \text{NUCLEON}$ , AT 3 BeV/c<sup>†</sup>

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This note reports some preliminary results on  $\pi - \pi$  scattering, near the 770-MeV  $\rho$  and 1250-MeV  $f^0$  resonances. The experiment is the one reported earlier<sup>1</sup>; with more data measured (now about 75% of the two-prong events), we have examined the data to see to what extent they seem analyzable in terms of  $\pi - \pi$  scattering. We give a brief summary of the results, and then a few details. A more detailed report will be available later.

(1) There is evidence of a major contribution from the one-pion-exchange mechanism ("peripheral collision"), for low nucleon recoil momentum. We take the region of  $\Delta^2 < \Delta_{\min}^2 + 10$  to be interpretable in terms of  $\pi - \pi$  scattering. ( $\Delta^2$  is the square of the four-momentum transfer to the nucleon, in units of the pion mass squared;  $\Delta_{\min}^2$  is the lower kinematic limit, which is a function of the  $\pi - \pi$  "mass" and the incident energy )

(2) We then consider these "peripheral" (i. e., peripheral-collision) events to be representative of the angular distribution of  $\pi - \pi$  scattering. Two obvious points of caution must be mentioned here: (a) Interference effects arise from nucleon isobar production, and (b) the effective  $\pi - \pi$  scattering is off the energy shell. From detailed examination of the data, we believe neither of these effects is so severe as to grossly affect the further conclusions below. A third possible complicating effect is interference from two-pion decay of the  $\omega$ , into  $\pi^+ \pi^-$ ; the possible magnitude of this effect is at present difficult for us to estimate.

(3) The spin of the  $f^0$  is greater than zero, as reported earlier by Veillet et al.<sup>2</sup> We believe it is difficult to draw any conclusion from these data as to whether the spin is 2 or greater than 2. (Isospin arguments, and the data directly, exclude spin 1.)

(4) The  $\pi^- - \pi^0$  scattering in the  $\rho$  region is con-

sistent with a dominant  $p$ -wave resonance, and a small repulsive  $T=2$   $s$ -wave phase shift,  $\delta_{02} = -14^\circ \pm 8^\circ$ ; similar results have been reported by a number of earlier workers.<sup>3</sup> The  $T=2$   $d$ -wave phase shift  $\delta_{22}$  is very small:  $\delta_{22} = 0^\circ \pm 6^\circ$ .

(5) The  $\pi^-\pi^+$  data in the vicinity of the  $\rho$  mass show a strong forward-backward asymmetry [see Fig. 2(b)]. If interpreted crudely in terms of  $\pi-\pi$  scattering, with the complications of item 2 above ignored, the data indicate a strong and possibly resonant  $T=0$   $s$  wave, and, in addition, an appreciable  $T=0$   $d$  wave. The  $d$ -wave phase shift appears to have a magnitude  $\leq 20^\circ$  to  $30^\circ$  in the mass range 600-900 MeV.<sup>4</sup>

(6) In energy regions other than the  $\rho$  and  $f^0$  regions our data are statistically too meager to enable us to draw any firm conclusions as to  $\pi-\pi$  phase shifts or other possible  $\pi-\pi$  resonances.

We now give a few details of the data and the analysis. We put some of the explanation in the figure captions.

Figure 1 shows the  $\pi-\pi$  mass plots for the events measured so far. A study of the distribution in the  $\pi-\pi$  "scattering angle"  $\theta_{\pi\pi}$  ( $\theta_{\pi\pi}$  is the angle between incoming and outgoing  $\pi^-$ , in the final di- $\pi$  c.m. system) for the three resonance regions  $\rho^-$ ,  $\rho^0$ , and  $f^0$  as a function of  $\Delta^2$  shows that for small  $\Delta^2$  the angular distribution is roughly independent of  $\Delta^2$ ; for larger  $\Delta^2$ , the angular distribution is no longer the same. For  $\rho^-$  and  $\rho^0$ , a clear change occurs for  $\Delta^2$  not much greater than 10; for the  $f^0$ , the change occurs only at considerably higher values of  $\Delta^2$ .

Examination of the  $\Delta^2$  distribution in these three resonance groups, and comparison with the theoretical distribution for one-pion-exchange (OPE) in perturbation theory, also indicates that for low values of  $\Delta^2$  the fit to the shape of the OPE distribution is not bad. Again, for the  $\rho^-$  and  $\rho^0$ , strong deviations from a smooth fit are found for  $\Delta^2 > 10$ , and for the  $f^0$  no such strong deviation occurs (for the  $f^0$ , there is no strong deviation for any  $\Delta^2$ ).

The Treiman-Yang test<sup>5</sup> gives no clear indication against OPE, for any  $\Delta^2$ , in our data for the three resonance groups.

From this and other analysis, we select the events in the range  $\Delta^2 = \Delta_{\min}^2$  to  $\Delta_{\min}^2 + 10$  as "peripheral" (OPE) events.

In Fig. 2 we show plots of  $M_{\pi\pi}$  vs  $\cos\theta_{\pi\pi}$ , for peripheral events. In this plot, the  $f^0$  stands out rather strongly, and the  $\cos\theta_{\pi\pi}$  distribution for the  $f^0$  is clearly incompatible with  $l=0$  (or even, disregarding the isotopic-spin restriction,

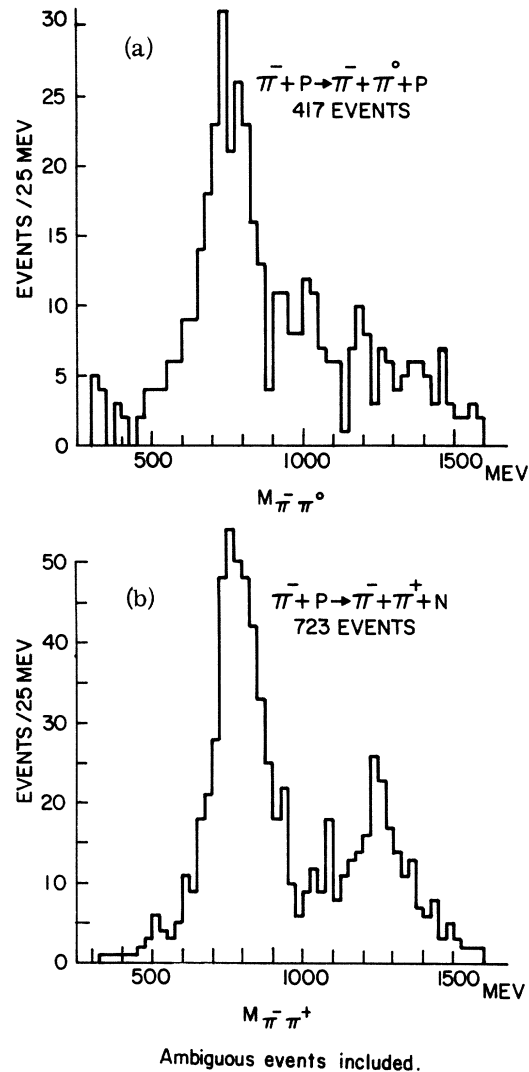


FIG. 1.  $\pi-\pi$  mass plots. (a)  $\pi^-\pi^0$ . A fraction of the order of 10% may be ambiguous with respect to whether one  $\pi^0$  or two  $\pi^0$ 's are actually present. (b)  $\pi^-\pi^+$ , including 132 ambiguous events, ambiguous with respect to distinguishing the  $\pi^+$  from a proton; detailed study leads us to believe that most of these ambiguous events are actually of the  $\pi^-\pi^+n$  type.

$l=1$ ).

We believe it is difficult to draw any conclusions from the data of Fig. 2 as to whether the spin of the  $f^0$  is 2, or instead greater than 2. The reason is that both the off-energy-shell nature of the  $\pi-\pi$  scattering and pion-nucleon interactions produce a finite "resolution width" in the  $\cos\theta_{\pi\pi}$  distribution. We have attempted to estimate the magnitude of this resolution effect. The off-energy-shell effect essentially introduces an uncertainty in the definition of  $\cos\theta$ . For real  $\pi-\pi$  scattering,  $\cos\theta_{\pi\pi}$  is simply related to  $q^2$ , the

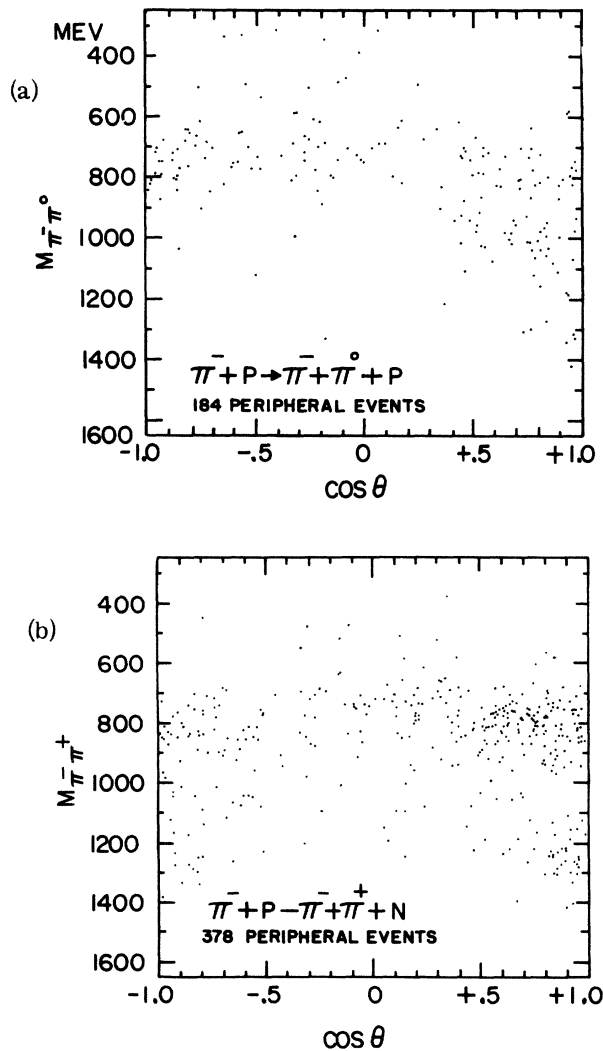


FIG. 2.  $\pi-\pi$  mass vs  $\cos\theta_{\pi\pi}$ , for peripheral events. (a)  $\pi^-\pi^0$ , (b)  $\pi^-\pi^+$ .

square of the invariant momentum transfer to the  $\pi^-$ . For scattering from a virtual  $\pi$ , as in the present experiment, there is an uncertainty in defining an equivalent  $\cos\theta_{\pi\pi}$ , i.e., there is no single clear prescription for generalizing the relation between  $\cos\theta$  and  $q^2$ , to the off-energy-shell case. Upon trying to make reasonable assumptions as to appropriate generalization procedures, we find that the corresponding uncertain-

ty in  $\cos\theta$  is of order of magnitude

$$\Delta\cos\theta|_{\text{off-energy-shell}} \sim [(\Delta^2 + 1)/M^2(\pi\pi)](1 - \cos\theta).$$

We have also made a rough estimate of the effect of pion-nucleon interactions on the "resolution" in  $\cos\theta_{\pi\pi}$ , by considering the transverse momentum transfer in  $\pi$ -nucleon diffraction scattering. When we combine that estimated effect with the off-energy-shell effect, the result leads us to believe that our present data on the  $f^0$  are insufficient to distinguish between spin 2 and higher values. When a considerably larger amount of data is available (preferably at higher incident beam energy, to reduce the off-energy-shell effects), then it may be possible to distinguish between these possibilities.

Finally, we remark that all of our analysis as to  $\pi-\pi$  phase shifts from the data reported here is based on the approximation that the  $\pi-\pi$  scattering is dominantly elastic for the  $\rho$  and for the  $f^0$ . This approximation may not be bad, since neither the  $\rho$  nor the  $f^0$  shows up detectably in four-pion states.

We wish to thank warmly Anne Baker, E. Leboy, T. McGrath, and our group of scanning and measuring technicians for their help in the measurement and analysis of these data, and we thank H. Brody for helpful discussions. Our thanks also go to the staff of the New York University 7090 Computer Center, for their assistance.

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<sup>1</sup>W. Selove, V. Hagopian, H. Brody, A. Baker, and E. Leboy, Phys. Rev. Letters **9**, 272 (1962).

<sup>2</sup>J. J. Veillet, J. Hennessy, H. Bingham, M. Bloch, D. Drijard, A. Lagarrigue, P. Mittner, A. Rousset, G. Bellini, M. diCorato, E. Fiorini, and P. Negri, Phys. Rev. Letters **10**, 29 (1963).

<sup>3</sup>D. Carmony and R. Van de Walle, Phys. Rev. Letters **8**, 73 (1962); Saclay-Orsay-Bari-Bologna collaboration (to be published); J. Naisse, Phys. Letters **1**, 247 (1962).

<sup>4</sup>A more detailed account of the apparent phase shifts obtained from an unrefined analysis of the data is available from the authors.

<sup>5</sup>S. Treiman and C. Yang, Phys. Rev. Letters **8**, 140 (1962).