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(p, 2p) ANGULAR CORRELATIONS IN THE DISTORTED-WAVE BORN APPROXIMATION*

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The angular correlation in a (p, 2p) experiment involving a proton in a definite shell-model state contains information about both the shell-model state and the distortion of the wave functions of the unbound particles in the entrance and exit channels.[1] In order to see what information can be reliably identified, it is necessary to do a distorted-wave Born-approximation calculation. Previous calculations[2] of angular correlations have used plane waves modified by space weighting factors calculated semiclassically.

A fully distorted-wave calculation of the angular correlation in the case where the momentum vectors of the final-state particles are symmetrical about the incident direction and coplanar with it has been coded for the IBM 7090. The struck proton may be initially in either an s state or a p state. A parameter study on the results of the 155-MeV $C^{12}(p, 2p)B^{11}$ experiment of Garron et al.[3] is reported here.

The approximations used are as follows:

$$d^3\sigma/d\Omega_L d\Omega_R dE_L = \frac{2\pi}{\hbar}\frac{m_0}{\hbar k_0}\frac{2m_0^3}{(2\pi\hbar)^6}(E_L E_R)^{1/2}$$
$$\times \sum_{M, M_1, m} C(J_1 jJ; M_1 M-M_1)^2$$
$$\times C(lsj; mM-M_1-m)^2 |M_l^m|^2.$$

This formula involves the extreme single-particle model with j-j coupling. J_1, j, and J are the angular momenta of the residual nucleus, the struck particle, and the initial nucleus; M_1, m, and M are the magnetic quantum numbers; l and s are the orbital angular momentum and spin of the struck particle. m_0 is the proton mass.

$$M_l^m = \iint d^3r_1 d^3r_2 \chi^{(+)}(\vec{k}_0, \vec{r}_1)\chi^{(-)*}(\vec{k}_L, \vec{r}_1)$$
$$\times \chi^{(-)*}(\vec{k}_R, \vec{r}_2)\psi_l^m(\vec{r}_2)v_{12}\delta(\vec{r}_1-\vec{r}_2).$$

$\chi^{(+)}$ and $\chi^{(-)*}$ are optical-model wave functions for ingoing and outgoing particles. $\hbar\vec{k}_0$ is the momentum of the incident proton, and $\hbar\vec{k}_L$ and $\hbar\vec{k}_R$ are the momenta of final-state particles scattered to the left and right, respectively. $k_L = k_R$, and they are on opposite sides of the $\vec{k}_0$ direction at an angle θ. All quantities are in the center-of-mass system. The optical-model parameters used were V_1, W_1, R, and a for the initial state and V_2, W_2, R, and a for the final state describing Woods-Saxon form factors.

For the shell-model wave function ψ_l^m of the struck proton, we have used finite square-well wave functions treating the radius R_b as a parameter and adjusting the binding energy to be equal to the experimentally observed removal energy, thus neglecting rearrangement energy.

The impulse approximation has been shown[2] to give the right order of magnitude. We feel that this is all it can do, so we have retained one arbitrary normalizing parameter v_{12} and have not attempted to describe the absolute magnitude of the cross sections. Provision has been made for finite-range interaction, and a more realistic description of the potential and an attempt to fit magnitudes will be described later.

The kinematics of the final-state three-body problem is not trivial and requires an approximation. After separating off the equation of motion of the center of mass, the two final-state particles are described by

$$\left(-\frac{1}{2\mu}\nabla_L{}^2+\mathcal{V}_L-\frac{1}{2\mu}\nabla_R{}^2+\mathcal{V}_R-\frac{1}{M}\vec{\nabla}_L\cdot\vec{\nabla}_R\right)\psi_{LR}=E\psi_{LR}.$$

The coordinate systems for the left and right particle relative to the residual nucleus are described by the subscripts L and R, E is the total energy in the center-of-mass system, $\mathcal{V}_L$ and $\mathcal{V}_R$ are the optical-model potentials, and $\mu = mM/(M+m)$, where m and M are the masses of the proton and residual nucleus, respectively. If the term in $\vec{\nabla}_L\cdot\vec{\nabla}_R$ is treated as a perturbation, then to first order ψ_{LR} is separable and $\vec{\nabla}_L\cdot\vec{\nabla}_R$ can be replaced by its eigenvalue $-\vec{k}_L\cdot\vec{k}_R$.

Neglecting terms of order m^2/M^2 and $\eta^2/E_{\mathrm{lab}}{}^2$, where η is the difference between E_{lab}, the incident energy in the laboratory system, and the summed energies of the final-state particles in the laboratory system, the wave function of each of the final-state particles is a solution of

$$\left[-\frac{1}{2\mu}\nabla^2+\mathcal{V}-\frac{1}{2}\left(E_{\mathrm{lab}}\frac{M+m}{M+2m}+\eta\frac{M+m}{M}\right)\right.$$
$$\left.-E_{\mathrm{lab}}\frac{m}{M}(2\cos^2\theta-2^{1/2}\cos\theta)\right]\psi=0.$$

The order of magnitude of the effect of relativistic kinematics has been estimated in the approximation $\eta=0$. For 155-MeV incident energy the difference between θ calculated with relativistic and nonrelativistic kinematics is about 1° in the region of interest. If we consider this as a linear perturbation on realistic nonrelativistic results, we would say that relativistic kinematics would give values of θ about 1° smaller than nonrelativistic kinematics. This is marginally significant in the present context and has been neglected.

The experimental features which we have taken into account in fitting the angular correlation curves for s and p states are (1) the location of the curve on the θ axis, (2) the over-all width of the curve, and (3) the ratio of magnitudes of s-state to p-state curves. The depth of the minimum and ratio of peak heights in the p-state curve are not considered to be sufficiently well defined experimentally to fit.

The best fit is shown in Fig. 1. The parameters used were as follows: $V_1=5$ MeV, $W_1=15$ MeV,

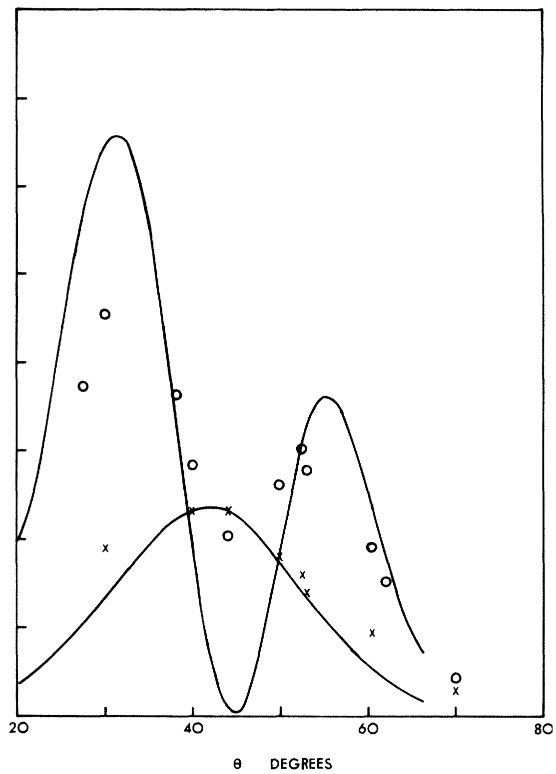

FIG. 1. The $(p,2p)$ angular correlation in the laboratory system for 155-MeV protons on C^{12} for $\eta=-16$ MeV (circles) and -36 MeV (crosses). The data are those of reference 3. Parameters are given in the text. The units on the ordinate scale are arbitrary but the same for each curve.

$V_2=40$ MeV, $W_2=10$ MeV, $R=3$ F, $a=0.65$ F, $R_b=2.4$ F (s state), and 3.5 F (p state). The average values of η were used as the binding energy for the square-well wave functions. They are -16 MeV for the s state and -36 MeV for the p state.

In fitting the experimental data, we first obtained a good fit to the p-state curve by adjusting the parameters V_1, W_1, V_2, W_2, R, a, v_{12}, and R_b. For the s state it was found that we only needed to adjust R_b to fit both the over-all width and the relative magnitude of s-state to p-state cross sections simultaneously.

A study of the effect of the parameters on the curves showed that the effect of R_b, the only parameter representing properties of the shell-model wave functions, was very different from the effects of the optical-model parameters. Thus R_b can be determined quite accurately for both the s and p states independently of the optical model.

The location of the curve on the θ axis can be shifted to larger angles by increasing V_2 and R (thus maintaining the well-known VR ambiguity in the optical model) and to smaller angles by increasing V_1 and $|\eta|$, the energy difference between entrance and exit channels, which is determined experimentally and not treated as a parameter. The effect of varying V_2 is much larger than that of varying V_1, since V_2 determines two optical-model wave functions, V_1 only determines one. It was found that a large difference between V_2 and V_1 was necessary to locate the curves properly. The values quoted are not unique.

The over-all width is determined almost exclusively by R_b. Increasing R_b decreases the over-all width and increases the magnitude of the cross section at the center of the curve. It is found that when the best value of R_b is used in each state, the relative magnitudes are automatically fitted well.

The effects of increasing W_1, W_2, and a are small. Increasing W_1 and W_2 decreases the magnitude of both curves slightly. In fitting the p-state curve, V_2 and V_1 have opposite effects on the ratio of peak heights. Increasing V_2 increases the ratio. Increasing both V_1 and V_2 reduces the depth of the minimum by a very small amount.

The physical conclusions which we tentatively draw from this calculation are rather significant. For finite potentials there cannot be significant differences between single-particle wave functions whose principal quantum number, angular momentum, binding energy, and rms radius are given. Hence it seems that a distorted-wave analysis of $(p, 2p)$ experiments determines the single-particle wave functions very well.

The rms radius of the charge distribution in C^{12} given by our empirical values of R_b is 2.5 F. The experimental value obtained from electron scattering is 2.4 F. The rms radius for s-state protons is 1.7 F, which is the experimental value for the α particle. Whether this is true for s states in other light nuclei is, at present, being investigated by a systematic study of the available data. Finer points concerning curve fitting are also being investigated.

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UNITARY SYMMETRY AND LEPTONIC DECAYS

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We present here an analysis of leptonic decays based on the unitary symmetry for strong interactions, in the version known as "eightfold way,"[1] and the V-A theory for weak interactions.[2,3] Our basic assumptions on J_μ, the weak current of strong interacting particles, are as follows:

(1) J_μ transforms according to the eightfold representation of SU_3. This means that we neglect currents with $\Delta S = -\Delta Q$, or $\Delta I = 3/2$, which should belong to other representations. This limits the scope of the analysis, and we are not able to treat the complex of K^0 leptonic decays, or $\Sigma^+ \to n + e^+ + \nu$ in which $\Delta S = -\Delta Q$ currents play a role. For the other processes we make the hypothesis that the main contributions come from that part of J_μ which is in the eightfold representation.

(2) The vector part of J_μ is in the same octet as the electromagnetic current. The vector contribution can then be deduced from the electromagnetic properties of strong interacting particles. For $\Delta S = 0$, this assumption is equivalent to vector-