of Eq. (5):

$$\lim_{T\to 0}\delta_T(R_i)\simeq\frac{2}{N}\sum_{k\in \mathrm{B.Z.}}\frac{1-\frac{1}{2}(\vec{k}\cdot\vec{R}_i)^2}{\exp[\beta\epsilon_k(T)]-1}+O\left(\left[\frac{R_i}{R_0(T)}\right]^4\right). \tag{11}$$

The self-consistent solution of Eqs. (4) and (11) leads to the terms $T^{3/2}$, $T^{5/2}$, $T^{7/2}$, and finally T^4, in the power-series expansion of $M(T)-M(0)$ near absolute zero. The coefficients must be obtained numerically from the function $J(R_i)$, but aside from this the present results are sufficiently similar to those of references 3, 4, and 6 that we need not elaborate further.

On the other hand, the Green function method[5] gives for the spin-wave energy in the absence of an external field,

$$\epsilon_k(T)=[M(T)/M(0)]\epsilon_k, \tag{12}$$

a *too simple* relation which although similar at first sight to what we have found [see Eq. (7)], is incorrect in detail, and is, in fact, tantamount to assuming that the thermal length *vanishes* (instead of which it actually becomes *infinite* at $T=0$). Moreover, this relation leads to a T^3 term in the low-temperature expansion of the magnetization[5] referred to previously, in disaccord with current notions about this state of affairs; all of which does not prevent the Green function method from perhaps giving a satisfactory account[9] of the thermal properties near to and above T_c, where spin-wave theory is sure to fail; but this remains to be shown.

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[1] With a view to applying nonlinear spin-wave theory to the study of the thermal properties of rare earth metals, and the calculation of T_c.

[2] For example, the Ruderman-Kittel interaction: M. A. Ruderman and C. Kittel, Phys. Rev. 96, 99 (1954). See also D. Mattis and W. Donath, Phys. Rev. 128, 1618 (1962), and further references therein. The possibility that spin-wave theory may not be applicable has been discussed [D. Mattis, Phys. Rev. 130, 76 (1963)], and such cases are, of course, excluded from the present considerations.

[3] F. J. Dyson, Phys. Rev. 102, 1217, 1230 (1956). Only nearest-neighbor interactions are considered therein.

[4] F. Keffer and R. Loudon, Suppl. J. Appl. Phys. 32, 2 (1961).

[5] V. L. Bonch-Bruevich and S. V. Tyablikov, *The Green Function Method in Statistical Mechanics* (North-Holland Publishing Company, Amsterdam, 1962), Chap. VII, and references therein.

[6] Micheline Bloch, Phys. Rev. Letters 9, 286 (1962).

[7] $\epsilon_k(T)$, $\mu(T)$, and $J_T(R_i)$ are the generalizations to finite temperature of the corresponding ground-state quantities ϵ_k, μ, and $J(R_i)$.

[8] Therefore the constant shielding effect we have found is probably not valid at small distances (except near the Curie temperature), whereas at large separation it may be observable at low (but always nonvanishing) temperature.

[9] The Green function method is being extended by many investigators beyond the results[5] of Bonch-Bruevich and Tyablikov. In fact, K. Kawasaki and H. Mori, Progr. Theoret. Phys. (Kyoto) 28, 690 (1962), have shown how a higher order decoupling leads to Eqs. (4) and (5) of the text.

MEASUREMENTS OF ELECTRON-ION RECOMBINATION IN A THERMAL CESIUM PLASMA*

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A new method for measuring the electron-ion recombination coefficient (α) in a dense thermal plasma in steady state has been reported in earlier publications.[1,2] In the first measurements[2] using this method, the volume recombination coefficient of cesium plasmas up to densities over 4×10^{12} ions/cm^3 was found to be approximately equal to or smaller than 3×10^{-11} cm^3/sec. This is at least an order of magnitude smaller than the lowest value reported in previous publications.[3,4] More recently, we have extended the measurement of α to higher plasma densities. This paper describes these experimental results and presents the results of the comparison between the measured values and those predicted by the theory of recombination.

Since a detailed account of the method of measurement has been given elsewhere,[2] only a brief description is presented here. This measurement technique utilizes a thermal cesium plasma generated by contact ionization of cesium vapor and thermionic emission of electrons. A schematic representation of the cesium-plasma apparatus is shown in Fig. 1. The cesium plasma

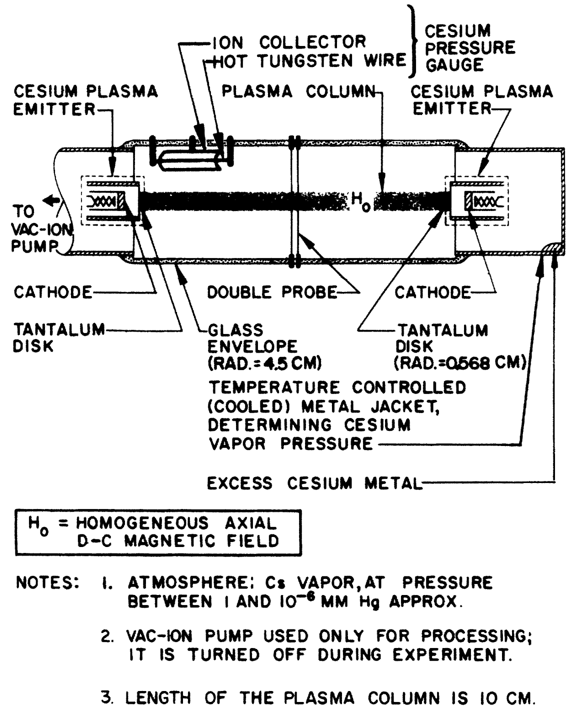

FIG. 1. Cesium plasma tube for measurement of recombination coefficient.

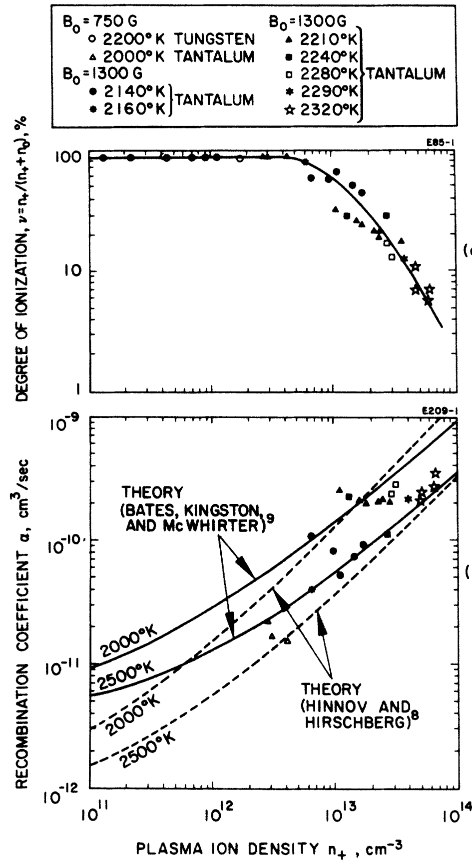

FIG. 2. (a) Degree of ionization as a function of plasma density. (b) Electron-ion recombination coefficient as a function of plasma density.

is generated at two 1.27-cm diameter tantalum disks which are exposed to cesium vapor and heated to thermionically emitting temperatures. This plasma is confined magnetically into a well-defined plasma column between the two emitters which are separated by 10 cm. The volume recombination coefficient is obtained from measurements of the degree of ionization $n_+/(n_++n_0)$ as a function of ion density. The results of these measurements are shown in Fig. 2(a). The decrease of the degree of ionization with increasing density is caused by the volume recombination.

To determine the recombination coefficient from these experimental results we equate the rates of ion generation and of ion loss, the latter being predominantly governed by recombination at the higher densities ($n_+>4\times10^{12}$ cm^{-3}). This leads to the following equation[2] for α in terms of measured parameters:

$$\alpha=\frac{\bar{v}_0}{2n_+L}\left[\frac{n_0}{n_+}-\left(\frac{A_p}{2A_e}+K_n\right)\frac{\bar{v}_+}{\bar{v}_0}\right], \qquad (1)$$

where n_+ is the plasma density, n_0 is the cesium neutral density, and $2A_e$ represents the combined area of the two emitters (A_e is assumed approximately equal to the cross-sectional area of a plasma column). A_p is the total effective area of probe (including the ion sheath effects), $\bar{v}_0$ and $\bar{v}_+$ correspond to the average thermal velocity of neutral cesium and cesium ions, respectively, L represents the distance between two plasma emitters, and K_n is defined as the probability for ions impinging on the high work-function surface of one of the plasma emitters to be neutralized; it has been evaluated to be less than 0.03 in our experiments.

It should be noted that the above expression has been derived under the assumption that there is uniform plasma density between emitters. In fact, when recombination losses become important as compared with diffusion losses (for $n_+>10^{13}$ ions/cm^3), the plasma density at the midplane between the emitters will tend to be lower than that close to them. Because the plasma density is measured at this midplane, the recombination coefficient evaluated from such measurements will tend to be somewhat larger than the true value. Preliminary measurements of the axial plasma-density distribution indicate that the assumption of uniform plasma density is acceptable and leads to corrections less than 20% for plasma densities up to 2×10^{13} ions/cm^3. Further measurements of axial density distribution up to about 10^{14} ions/cm^3, however, are still required and are being prepared to show the importance of the correction to be applied to Eq. (1) in determining the value of α from the experimental data of Fig. 2. Equation (1) is used here to obtain a first indication for the approximate values of α and their variation with plasma density. At this time it seems to lead to an acceptable first-order approximation for the value of α. These values obtained from the data of Fig. 2(a) through Eq. (1) are shown in Fig. 2(b) as functions of the ion density n_+. Figure 2(b) shows that the recombination coefficient increases monotonically as a function of plasma density. This indicates that either collisional-radiative recombination (electron-electron-ion collision)[5-12] or dissociative recombination[13] is dominant over pure radiative recombination (electron-ion collision). Indeed, the latter is calculated to be about 10^{-12} cm^3/sec over the range of densities shown in Fig. 2(b) and therefore contributes only in a minor way to the total rate of recombination for plasma densities above 10^{12} ions/cm^3 at the electron temperatures considered here (of the order of 2000°C).

We now compare the experimental value of α shown in Fig. 2(b) with the values predicted by the theory of recombination by the collisional-radiative process. This process involves the three-body collisions of the electron-electron-ion type suggested independently by D'Angelo,[5] Bates and Kingston,[6] and McWhirter.[7] More refined and extensive quantitative results of this process have been calculated by Hinnov and Hirschberg,[8] by Bates, Kingston, and McWhirter,[9] and by Byron, Stabler, and Bortz.[10] The results of the latter two papers are almost identical, whereas Hinnov and Hirschberg predicted substantially smaller values of α at low densities for corresponding values of the plasma temperature. It should be mentioned that these theoretical values are computed for H^+ ions. However, extensions of similar calculations to alkali ion plasmas have been made by Bates, Kingston, and McWhirter[9]; their results suggest that the rate of collisional-radiative recombination is not very sensitive to the species of singly charged ions. More recently, Byron, Bortz, and Russell[11] and Dugan[12] have calculated the recombination rates for K^+ and Cs^+ plasmas, respectively. They also found that the values of recombination rates for K^+ and Cs^+ plasmas are on the same order of magnitude as those for the H^+ plasmas.

To determine the physical significance on experimental results, these values are compared [see Fig. 2(b)] with the theoretical values predicted by Hinnov and Hirschberg and by Bates, Kingston, and McWhirter for electron temperatures of 2000°K and 2500°K. (The prevailing electron temperature in our measurements was in this range.) Although our experimental points show some scattering, the density dependence as well as the absolute values of the measured recombination coefficient are quantitatively in agreement with the theory of the collisional-radiative process (more favorably with Hinnov and Hirschberg's results). These results substantiate the theoretical predictions[9,11,12] that the rate of collisional-radiative recombination associated with Cs^+ plasma (an alkali-ion plasma) does not differ much from that of H^+ plasmas. They also indicate that dissociative recombination is of no importance in the range of ion and neutral densities corresponding to our experiments. (This latter conclusion will be further verified by comparing measurements with different degrees of ionization at the same density.)

Subject to further verification (that the correction required in our measurements at plasma densities greater than 2×10^{13} ions/cm^3 is not of major consequence, and that dissociative recombination is of no importance), the values of α obtained in our experiments are in agreement with the values predicted by the theory of collisional-radiative recombination. These measurements then lead to the conclusion that recombination by electron-electron-ion collisions predominates within the investigated ranges of plasma densities (10^{12} to 10^{14} cm^{-3}) and degrees of ionization (7% to 90%).

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NEW AUTOIONIZING ATOMIC ENERGY LEVELS IN He, Ne, AND Ar†

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The NBS 180-MeV electron synchrotron has been used as a continuum light source for absorption spectroscopy in the 180-470Å region. Two-electron transitions to states which autoionize have been observed in He; transitions to autoionizing states have also been observed in Ne and Ar.

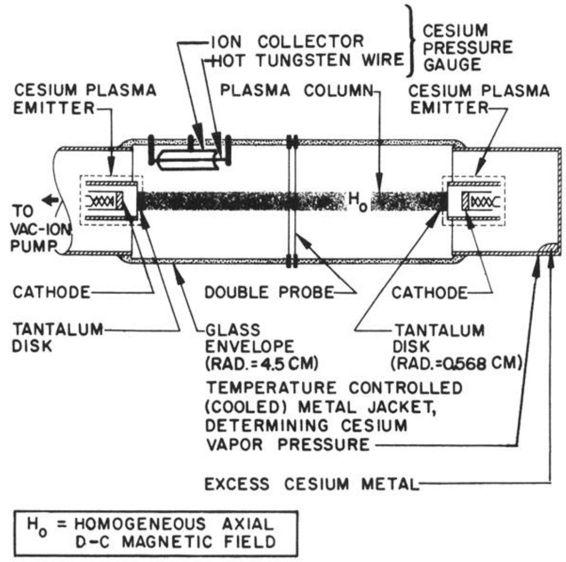
Autoionization may occur when neutral atoms are excited to energy levels which lie above their first ionization limit. The strength of the interaction of these states with the continuum states determines the lifetime of the discrete levels. This interaction results in an interference phenomenon which imparts an unusual shape to the absorption in the region of the discrete level. Transitions to or from autoionizing levels have not previously been observed spectroscopically in He or Ne. Beutler[1] observed autoionized lines in absorption by Ar between the ${}^2P_{3/2}{}^\circ$ and ${}^2P_{1/2}{}^\circ$ edges of the first ionization limit near 780 Å. There has been no previous spectroscopic observation of such levels in Ar at higher frequencies. Autoionization has been observed in absorption for several elements, for example, in alkaline earth metals by Garton and Codling[2] and by Ditchburn and Hudson.[3] Previous workers[4-9] studying the photoionization cross section in He, Ne, and Ar in this wavelength region have been able only to suspect the presence of autoionized levels since emission line sources were used.

Autoionization in He is of special interest, since a transition to discrete levels lying above the first ionization limit requires that both electrons be excited. Evidence for the existence of such states was obtained by Whiddington and Priestley[10] and more recently by Silverman and Lassettre[11] from discrete electron-energy losses in the forward scattering of electrons by He. Using different approximations, the autoionization probability of these levels has been calculated by Wu[12] and by Bransden and Dalgarno[13] with more than an order of magnitude disagreement. A general theory of the shape of autoionized absorption lines has been formulated by Fano,[14] who has shown that the data of Silverman and Lassettre are compatible with the theory.

The light source used in the present experiments was the 180-MeV electron synchrotron at the National Bureau of Standards. The light radiated by the radially accelerated electrons is confined to a narrow cone in the forward direction of the electrons and is continuous in wavelength. The usable intensity extends from the infrared down to below 100 Å. Detailed characteristics of the radiation from this synchrotron will be described in another paper.

A 3-meter grazing incidence vacuum spectrograph, designed to withstand the vibrations attendant to the synchrotron application, was constructed. This instrument is being used with a grating having 15 000 lines per inch to produce a dispersion of 1 Å per mm in the 200Å region at a resolution width somewhat less than 0.1 Å. An aluminum foil filter is used with the instrument to keep light of wavelength less than 170 Å and greater than 800 Å from entering the spectrograph. Commercial tank gas samples are admitted to the spectrograph at pressures ranging up to several hundred microns of Hg. This gas can leak from the spectrograph through the entrance slit, requiring a differential pumping system to maintain the synchrotron doughnut pressure below 5×10^{-6} mm Hg. The gas-sample pressure is measured with a McLeod gauge.

The absorption spectrum obtained for helium



NOTES:

1. ATMOSPHERE: Cs VAPOR, AT PRESSURE BETWEEN 1 AND 10^{-6} MM Hg APPROX.
2. VAC-ION PUMP USED ONLY FOR PROCESSING; IT IS TURNED OFF DURING EXPERIMENT.
3. LENGTH OF THE PLASMA COLUMN IS 10 CM.

FIG. 1. Cesium plasma tube for measurement of recombination coefficient.