result is about 0.08 cm/sec, whereas the experimental value is about 1 cm/sec.[4] The calculated v_c is too low, presumably due to oversimplifications inherent in the model. A better calculation, taking account of the increased compressibility of the real liquid, is very likely to give the right order of magnitude. The model does, however, give the correct dependence on the radius of the tube and shows the qualitative difference between the classical and quantum treatments. The quantum vortex ring still offers a possible explanation of the existence and size of the observed critical velocity in liquid helium II.

I should like to thank Professor P. C. Martin, both for the suggestion that led to this calculation, and for a critical reading of the manuscript.

*National Science Foundation Predoctoral Fellow.

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GIANT MAGNETOSTRICTION IN DYSPROSIUM AND HOLMIUM SINGLE CRYSTALS*

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Large magnetostrictive effects have been observed in dysprosium and holmium single crystals. Maximum linear strains of $(\Delta l/l) = 7500 \times 10^{-6}$ for dysprosium at 20°K and $(\Delta l/l) = 3500 \times 10^{-6}$ for holmium at 45°K are reported here.

The single crystals were grown by Nigh using a method which he has described.[1] Two specimen disks of dysprosium and two of holmium were used; all were about 9 mm in diameter by 1 mm thick. The planes of the disks were perpendicular to the b_1 and c axes[2] for dysprosium, and to the a_1 and c axes for holmium. Standard strain gauge methods[3] were used. A maximum magnetic field of 26 kOe was applied parallel to the plane of the disk, and it was possible to rotate the field direction relative to the specimen through 2π radians in this plane.

The observed strains (relative to the demagnetized state) in the principal directions in dysprosium are shown in Fig. 1 as a function of applied magnetic field along the (easy) a_1 axis. These curves represent composite results of measurements on the two disks. The observed a_1 axis strain, $(\Delta l/l)_a$, shown as a function of the direction of the applied field in Fig. 2, was measured relative to the state with the field parallel to the a_1 axis. The b_1 axis strain, $(\Delta l/l)_b$, observed during this rotation

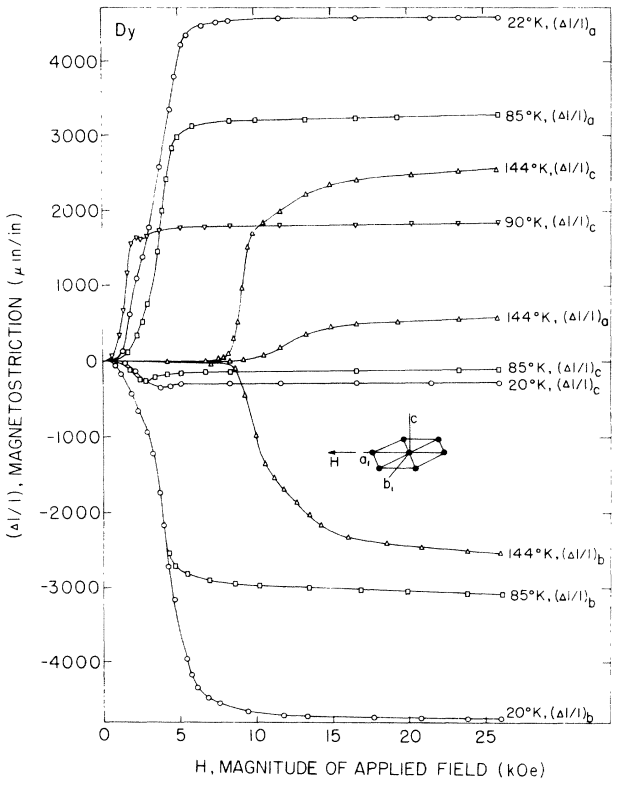

FIG. 1. Magnetostriction along a_1, b_1, and c axes in dysprosium as a function of field along the a_1 axis.

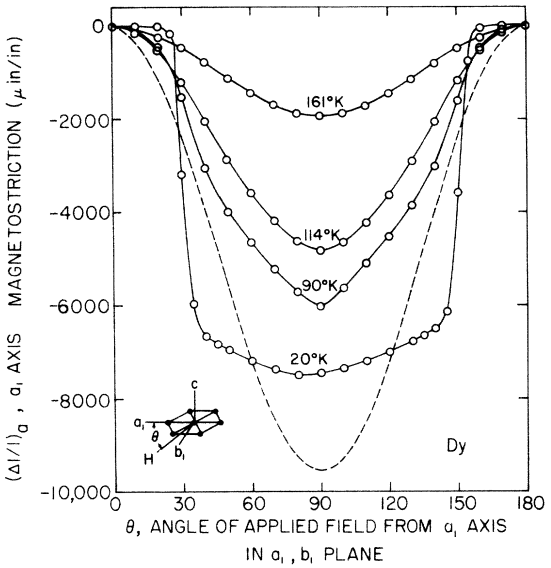

FIG. 2. Magnetostriction along the a_1 axis in dysprosium as a function of applied field direction in the basal plane. The field was 26 kOe.

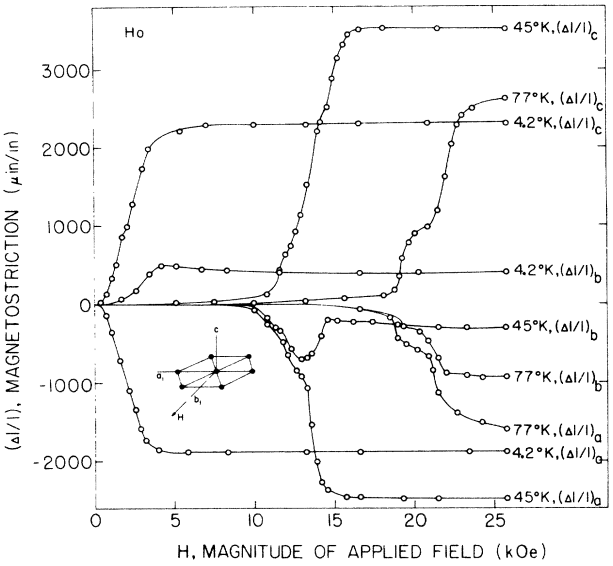

FIG. 3. Magnetostriction along a_1, b_1, and c axes in holmium as a function of field along the b_1 axis.

was opposite in sign and slightly larger in magnitude than the values shown for $(\Delta l/l)_a$. These measurements have been made at various temperatures in the helical antiferromagnetic region (87 to 179°K) and the ferromagnetic region (below 87°K).[4]

It is not possible to attach precise significance to the magnitude of the maximum strain in the ferromagnetic region; this is due to randomness of the demagnetized domain configuration and the resulting uncertainty in the initial strain state. In the antiferromagnetic state, domain effects appear to be of less importance, and this uncertainty was largely removed by annealing between runs at a temperature above the ferroparamagnetic transition. The difficulties in the initial strain state were not present in the field rotation measurements, since all strains were measured relative to the unique saturated state with field along the easy direction.

Similar measurements have been made on holmium, which has a region of helical antiferromagnetism (19 to 133°K) and a region of conical ferromagnetism (below 19°K).[5] For the strain versus applied field curves (Fig. 3), the field was applied parallel to the (easy) b_1 axis. In the case of the field rotation measurements (Fig. 4), the strain was measured relative to the condition with the field along the b_1 axis. During this rotation $(\Delta l/l)_b$ was again opposite in sign and almost equal in magnitude to $(\Delta l/l)_a$.

The dependence of strain on magnetic field for both dysprosium and holmium is strikingly similar in shape to that of the reported magnetization data.[6] The sudden onset of the change in strain which appears at the critical field coincides with the breakdown of the spiral spin

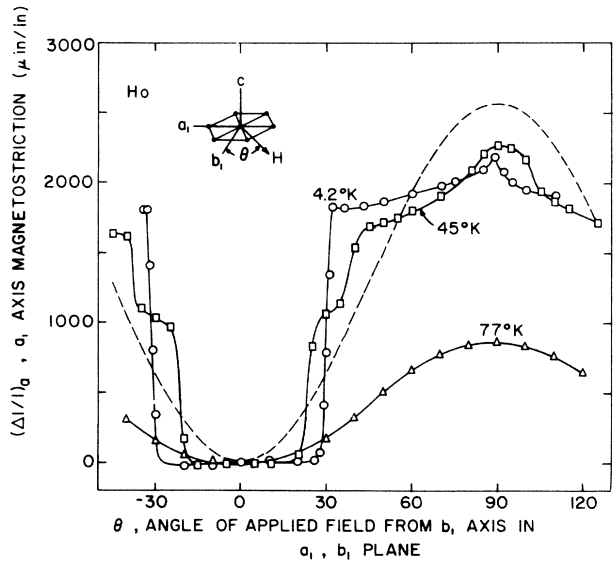

FIG. 4. Magnetostriction along the a_1 axis in holmium as a function of applied field direction in the basal plane. The field was 26 kOe.

structure and the ordering of the spins parallel to the magnetic field. This transformation involves a relative rotation of adjacent layers of spins. The accompanying change in interlayer exchange interaction energy is manifested in the observed change in the magnetoelastic energy.

The dependence of $(\Delta l/l)_a$ on the direction of applied field departs rather markedly from the pure $\sin^2\theta$ dependence in dysprosium below 70°K and in holmium below 45°K (Figs. 2 and 4). The shape of these curves may be explained by assuming that the sixfold anisotropy becomes sufficiently large below these temperatures so that the magnetization vector remains essentially parallel to the easy direction until the applied field makes an angle of almost 30° to this direction. The magnetization then abruptly collapses along this direction and jumps 60° to the next easy direction.

It will be noted that this process does not occur exactly at $\theta = 30°$; it begins for slightly less than 30°, and is completed for θ slightly greater than 30°. This is because the actual internal field is the vector sum of the applied field and the demagnetizing field. A first approximation to the true dependence of the strain on the magnetization direction is given by the dashed curves of $\sin^2\theta$ (Figs. 2 and 4). These have been fitted to coincide at $\theta = 60°$ with the observed a_1 axis strain data for dysprosium at 20°K, and for holmium at 4.2°K. When fitted to the b_1 axis strain data for dysprosium at 20°K, this approximation gave a maximum strain of 10 000 $\times 10^{-6}$.

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[2]Both dysprosium and holmium are hcp. A coordinate system is chosen such that the z axis is parallel to the hexagonal c axis, the x axis parallel to the crystallographic a_1 axis, and the y axis (b_1 axis) perpendicular to the x and z axes.

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THERMAL SCREENING EFFECTS IN A FERROMAGNET

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In considering the theory of spin waves,[1] we have examined elementary excitations of the Hamiltonian

$$H = -\sum J(R_{ij})\vec{S}_i \cdot \vec{S}_j \qquad (1)$$

and lowest-order interactions among these excitations, in cases when $J(R_{ij})$ is a very long-ranged, oscillatory interaction.[2] As numerical computation is required, we are preparing to program the IBM 7094 for this project. In the course of this preparation, we have, however, obtained a result which is independent of the details of the calculation. This result appears to have several interesting consequences, and we have therefore made it the subject of the present communication.

What we have found, to put it simply, is that there exists a sort of "Debye length," $R_0(T)$ (to be defined below), which separates the interactions into a "near zone" and a "far zone," and that although the thermal effects on the interactions in the near zone must be found numerically, the only thermal effect in the far zone is a uniform screening. That is, the interactions $J(R_{ij})$ for values of $R_{ij} \gg R_0(T)$ appear to be multiplied by a universal factor

$$M(T)/M(0), \qquad (2)$$

where $M(T)$ is the magnetization at temperature T, and $M(0)$ its value at absolute zero. We have also been able to extend the Dyson[3] form of the power series expansion for $M(T)$ to the actual case of nonnearest-neighbor interactions, as well as the absence of a term in T^3 and the presence