pions and assuming for $K_{e4}$ decay a reasonable $\nu$ spectrum.[2] Thus at present we estimate the ratio $K_{e4}/\tau$ to be $\sim 10^{-3}$ from this experiment alone.

This result agrees with the emulsion result[1] and several theoretical estimates.[2,3]

The invariant mass of the two pions is 298 MeV, 303 MeV, and 332 MeV. It would be very interesting to know the distribution of this quantity.[3] However, it is difficult experimentally because the detection efficiency for decays at rest depends upon the neutrino energy which depends strongly upon the relative energy between the pions. This decay mode is compatible with the $\Delta Q = \Delta S$ rule. In the same scan we have not found any clear example of the decay $K^+ \to \pi^+ + \pi^+ + e^- + \bar{\nu}$ which violates the $\Delta Q = \Delta S$ rule.[4]

To establish the latter mode we must identify unambiguously all three tracks, while for the present example the $\pi^-$ identity alone removes the chief background, $\tau'$ decay with Dalitz pair. This factor reduces our efficiency somewhat for the negative electron mode.

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[1]E. L. Koller, S. Taylor, T. Huetter, and P. Stamer, Phys. Rev. Letters 9, 328 (1962).

[2]K. Chadan and S. Oneda, Phys. Rev. Letters 3, 292 (1959).

[3]V. S. Mathur, Nuovo Cimento 14, 1322 (1959); G. Ciocchetti, Nuovo Cimento 25, 385 (1962).

[4]R. E. Behrends and A. Sirlin, Phys. Rev. Letters 8, 221 (1962).

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# ELECTRON-PROTON ELASTIC SCATTERING AT 1 AND 4 BeV†

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The elastic electron-proton scattering cross section has been measured at forward electron angles to detect possible deviations from the Rosenbluth formula. Data have been obtained at 4-BeV incident electron energy with $q^2 = 6$, 10, 14, and 18 $F^{-2}$, and at 1 BeV with $q^2 = 6$, 10, and 14 $F^{-2}$. The results are consistent with a photon Regge trajectory of zero slope. A core term is indicated in the values of the proton's charge form factor, $G_{ep}$, extracted from the 4-BeV data.

These measurements were undertaken using the apparatus shown in Fig. 1. The radio-frequency power was turned off and the internal beam of the Cambridge electron accelerator was allowed to spiral in to strike a $CH_2$ target of 0.009 radiation length thickness. Hydrogen counts were then obtained by a carbon subtraction procedure. The bremsstrahlung beam (typically $1.5 \times 10^{10}$ equivalent quanta per second) was monitored by a thin, helium-filled ion chamber. To obtain absolute cross sections, the ion chamber was calibrated at reduced beam against a quantameter similar to that described by Wilson.[1] The recoil protons were momentum analyzed by a quadrupole magnet spectrometer 12 in. in diameter, employing scintillation counters $C_1$ and $C_2$ as detectors. The large $dE/dX$ counter, $C_3$, was used to separate minimum ionizing particles from protons. The integral spectrum obtained with the 5% momentum bite used was 50% wider than the tails of the elastic peak in the worst case.

The raw hydrogen data were treated as outlined in Table I. The cross sections so obtained are listed in Table II. Only relative uncertainties are indicated. The estimated ±7% scale factor uncertainty is common to all measurements.

The data have been analyzed for possible Regge behavior. Following Blankenbecler, Cook, and Goldberger,[2] a factor $Z^a$, with $a = 2[\alpha(q^2) - 1]$, was assumed to multiply the Rosenbluth cross section. To test for a value of $\alpha(q^2)$ different

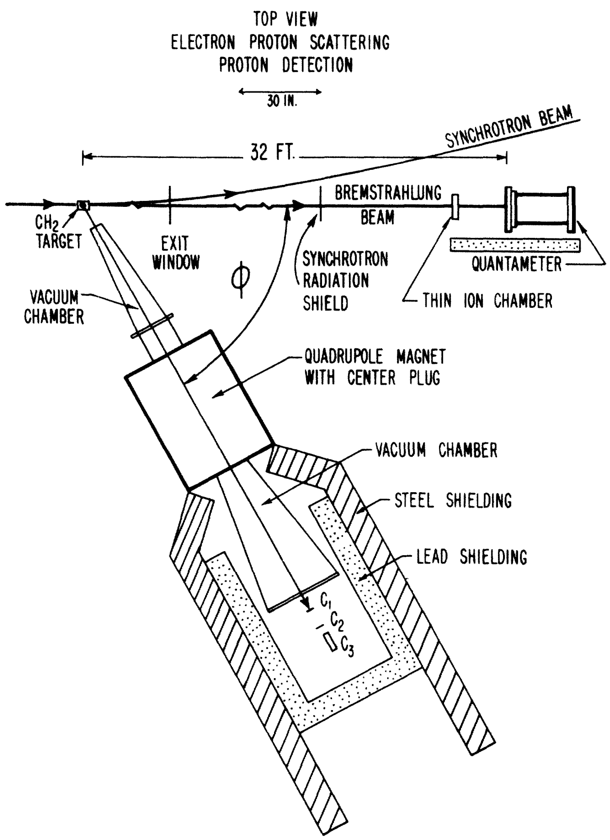


FIG. 1. Plan view of proton detection apparatus.

from unity, the ratio ($R$) of the experimental cross section to the nominal Rosenbluth cross section was computed. This ratio was then compared at one ($R_{1\text{ BeV}}$) and four BeV ($R_{4\text{ BeV}}$) at the same momentum transfer, and a slope, $\alpha'(q^2)$, was extracted:

$$R_{4\text{ BeV}}/R_{1\text{ BeV}}=(Z_4/Z_1)^a\simeq 1+2\alpha'(q^2)\ln(Z_4/Z_1). \quad (1)$$

The slopes so obtained are listed in Table II. A Regge dependence is predicted to decrease the cross section below the Rosenbluth extrapolation, and $\alpha'(q^2)$ should be positive. The observed values of $\alpha'(q^2)$ are, however, consistent with zero or even negative slopes.

Many systematic uncertainties which can influence $\alpha'(q^2)$ cancel when the ratio of experimental cross sections is taken. In the present data, for example, the recoil proton's momentum depends only on $q_1^2$, and the nuclear absorption correction does not influence the ratio at constant $q^2$. A Regge effect can be induced by the logarithmic dependence on incident energy in the radiative corrections.[3] However, the difference in the correction is typically only 3% between one and four BeV. Uncertainties in the relative contributions of $G_{ep}$ and $G_{mp}$ can also affect the ratio. Previous measured values for these functions were used,[4] and a 20% uncertainty in their relative contribution is assumed in the error analysis.

With due consideration to these possibilities the apparent slope of the photon's Regge trajectory is less than 4% of the strong-interaction slope ($1/M_n^2$) at the 90% confidence level.[5]

The validity of the Rosenbluth formula can also

Table I. Corrections and uncertainties.

| | $q^2=10$ | | Fractional uncertainty | |
|---|---|---|---|---|
| | 4 BeV | 1 BeV | Relative | Absolute |
| Nuclear absorption | 1.241 | 1.241 | 0.01 | 0.025 |
| Radiative correction[a] | 1.045 | 1.075 | 0.012 | 0.01 |
| Real bremsstrahlung in target | 1.010 | 1.011 | 0.002 | ... |
| Monitoring | 1.000 | 1.000 | 0.008 | 0.05 |
| Slit scattering | 0.989 | 0.978 | 0.006 | 0.01 |
| Inelastic protons | 0.996 | 1.00 | 0.003 | 0.005 |
| Electronic dead time | 1.015 | 1.010 | 0.002 | ... |
| Solid angle | 1.000 | 1.000 | 0.015 | 0.03 |
| Bremsstrahlung loss at exit aperture | 0.996 | 0.966 | 0.005 | ... |
| Machine energy | 1.000 | 1.000 | 0.002 | 0.02 |
| $CH_2$ radiation length | 1.000 | 1.000 | ... | 0.015 |
| Statistics | (4 %) | (8.4 %) | | |
| | 1.308 ±5.0 % | 1.284 ±8.9 % | 0.028 | 0.069 |

[a]See reference 4.

Table II. Cross sections and form factors.

| $E_e$ (BeV) | $q^2$ ($F^{-2}$) | $(d\sigma/d\Omega)_{\text{electron}}$ ($\times 10^{31}$ cm$^2$) | $\alpha'(q^2)$ (in $M_n^{-2}$) | $G_{ep}$ derived from 4-BeV data | $G_{mp}$ assumed |
|---|---|---|---|---|---|
| 1 | 6.00 ± 0.06 | 3.88 ± 0.22 | -0.09 ± 0.12 | 0.526 ± 0.021 | 1.59 |
| 4 | 6.00 ± 0.04 | 92.1 ± 4.5 | | | |
| 1 | 10.0 ± 0.1 | 0.786 ± 0.07 | -0.02 ± 0.10 | 0.423 ± 0.022 | 1.17 |
| 4 | 10.0 ± 0.08 | 22.14 ± 1.4 | | | |
| 1 | 14.0 ± 0.15 | 0.171 ± 0.022 | -0.19 ± 0.10 | 0.365 ± 0.027 | 0.89 |
| 4 | 14.0 ± 0.10 | 7.90 ± 0.59 | | | |
| 4 | 18.0 ± 0.15 | 3.24 ± 0.28 | | 0.31 ± 0.026 | 0.70 |

be checked by plotting $(d\sigma/d\Omega)d\sigma_{\text{Mott}}^{-1}[1+\tau] \times\tan^2\frac{1}{2}\theta_e$ against $\cot^2\frac{1}{2}\theta_e$. The present results are compared with the Stanford data,[6] as analyzed by Hand, Miller, and Wilson[4] and plotted in Fig. 2. The straight line has been fitted by the least-squares method to the Stanford data. In spite of absolute normalization uncertainties, the results are in impressive agreement at $q^2=6$ (not shown) and 10 $F^{-2}$. The agreement at $q^2=14$ $F^{-2}$ is less good. However, uncertainties in both the Stanford and Harvard data make it difficult to draw a conclusion. The point for which $q^2=18$ $F^{-2}$ also indicated an enhancement of the forward electron angle cross section. However, there is no corresponding low-energy point for comparison within this experiment. If these indications are taken seriously, the apparent deviation is in the direction predicted by Gourdin and Martin[7] for an interference term of negative sign.

On the other hand, assuming the validity of the Rosenbluth formula, computed numerical values for $G_{ep}$ are listed in Table II. To perform this analysis, values for $G_{mp}$ were taken from the two-resonance model fit to the previous data by Hand, Miller, and Wilson.[4] Since the present information is less sensitive than previous data to $G_{mp}$, and since $G_{mp}$ is the most precisely known of the nucleon form factors, this procedure is considered to be the most reliable means of extracting $G_{ep}$ from this information. It does assign all discrepancies to $G_{ep}$ alone.

As indicated in Fig. 3 these values are consistent with the previous Stanford results[6] and the more recent Cornell data.[8] The increased precision is primarily due to the enhanced sensitivity of $G_{ep}$ to the forward electron scattering angles investigated in this experiment. A core term as large as 0.3 $G_{ep}(0)$ is compatible with the present data. Further, a relatively rapid change in slope is required of $G_{ep}$ around $q^2=6$ $F^{-2}$.

The data have been fitted to a pion resonance model of the form

$$G_{ep}(q^2)=1-\sum_i\left(a_i-\frac{a_i}{1-q^2/q_{ri}^2}\right) \qquad (2)$$

similar to that proposed by Bergia et al.[9] With a single pion resonance (at $m_r$), the two parameters can be adjusted to fit the precise low momentum-transfer data ($q^2$ less than 2 $F^{-2}$). They may also reproduce either the shape of the $q^2=5$ to 10 $F^{-2}$ region or the higher momentum-transfer data from Cornell. But the single resonance model cannot reproduce both regions simultaneously. For example, a so-called best fit yields $a_1=1$ (no core at all) and $m_r=570$ MeV. This fit is 3 standard deviations low at $q^2=30$ $F^{-2}$ and 4 high at $q^2=6$ $F^{-2}$.[10]

A more acceptable representation of the data was obtained using a double resonance model and the following parameters[11]:

$$m_{r1}=760\text{ MeV},\quad a_1=2.1,$$

$$m_{r2}=1500\text{ MeV},\quad a_2=-2.0.$$

We do not understand the precise significance of these observations. Information on the neutron's charge form factors would facilitate separation of the scalar and vector components of the nucleon charge form factor. This might clarify the situation. On the other hand, the poor fits of the single resonance model may be

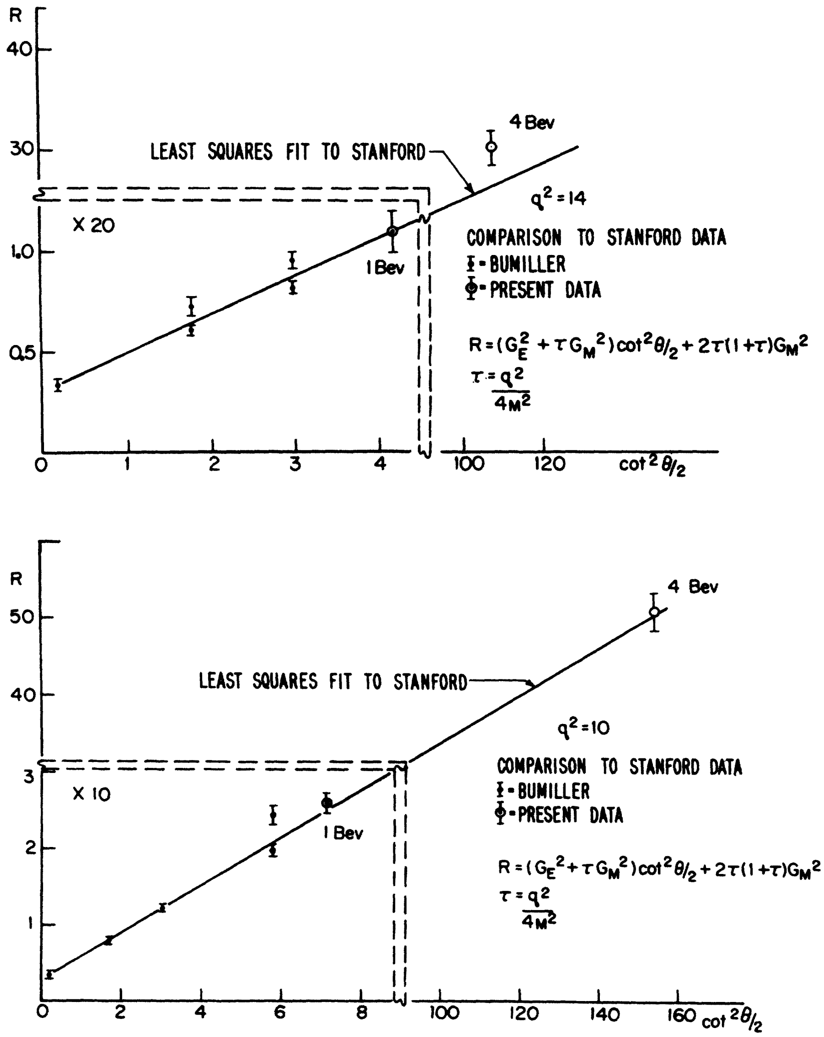


FIG. 2. Experimental values of $(d\sigma/d\Omega)\sigma_{NS}^{-1}\tan^2\frac{1}{2}\theta[1+\tau]$ plotted against $\cot^2\frac{1}{2}\theta$. Straight line is a least-squares fit to the Stanford data.

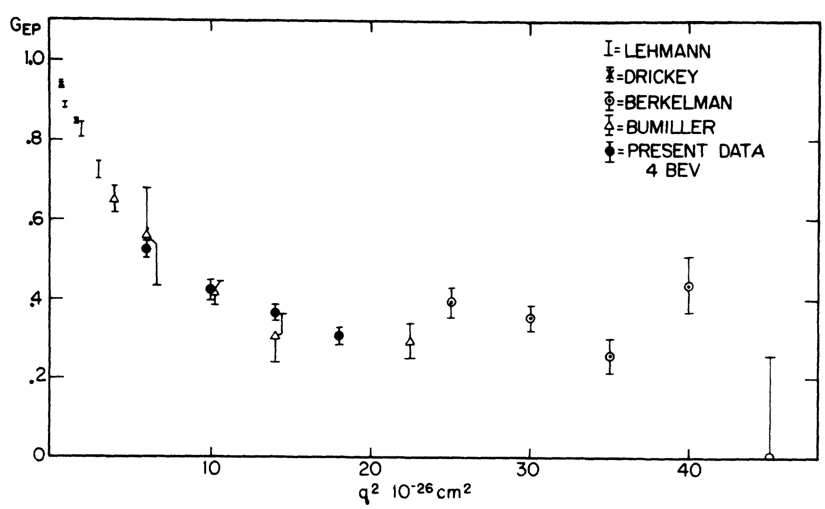


FIG. 3. Charge form factor of the proton. Scale uncertainties are not shown. These were ±4 % in the present data.

attributed to more fundamental difficulties with this type of analysis. Additional information on the form factors of both the proton and the neutron at higher momentum transfers will be available in the near future.

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[1]R. R. Wilson, Nucl. Instr. 1, 101 (1957).

[2]R. Blankenbecler, L. F. Cook, and M. L. Goldberger, Phys. Rev. Letters 8, 463 (1962). Here $Z$ = cosine of the center-of-mass scattering angle in the crossed channel.

[3]Radiative corrections were obtained from N. Abensouer and D. R. Yennie (to be published).

[4]L. N. Hand, D. Miller, and Richard Wilson (to be published).

[5]M. Lévy has demonstrated that a summation of perturbation-theory Feynman graphs does not lead to Regge behavior for elastic scattering [Phys. Rev. 130, 791 (1963)]. A Regge-pole experiment has also been performed at 1.3 BeV at Cornell. No effect has been seen. G. R. Rouse, K. Berkelman, and M. Feldman, Bull. Am. Phys. Soc. 118, 361 (1963).

[6]F. Bumiller, M. Croissiaux, E. Dally, and R. Hofstadter, Phys. Rev. 124, 1623 (1961).

[7]M. Gourdin and C. P. Martin (to be published).

[8]K. Berkelman, M. Feldman, M. Littauer, G. Rouse, and R. R. Wilson (to be published).

[9]S. Bergia, A. Stanghellini, S. Fubini, and C. Villi, Phys. Rev. Letters 6, 367 (1961).

[10]Cornell has obtained fits to their data which are consistent with no core at all in $G_{mp}$ and only 0.08 in $G_{ep}$.

[11]Our sensitivity to $m_{r2}$ is not large. An acceptable fit is obtained with the value of this parameter lying between 1000 and 2000 MeV.